# **Complete Transient Simulations of Electrostatic Actuators**

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#### ABSTRACT

A code for the dynamical simulation of electrostatic actuated micromechanical systems with complex 3D geometry was created. It allows exact electrical force computation at each time step during the dynamic analyses. A finite element method is used to extract the mode shape matrix and the resonance frequency. A boundary element method is used to compute the electrostatic force distribution. The linear normal mode summation method is used for dynamic characterization of the microdevices. In order to test the present code, it is applied to the simulation of an optical microshutter.

**Keywords:** dynamic simulation; microshutter; modeling; electromechanical coupling.

### INTRODUCTION

The fact that a high number of microsystems are designed to work in a dynamic regime created a need for transient simulation. The modeling and simulation of their behavior can help the designer to vary different geometrical parameters and material properties in order to achieve optimized response of their devices.

When a conductive mechanical structure is placed in an electric field, the resulting electrostatic force will deform the movable parts. If the electrostatic field is constant, or its time variation slow enough, after a certain time, the system will reach an equilibrium position, so it can be considered as working in quasistatic conditions. On the other hand, when the electric field has a periodic time variation with a frequency close to the mechanical resonance, the quasistatic approach cannot be applied for the dynamic description.

In the characterization of electrostatic actuated microsystems it is particularly important to study with attention the electromechanical coupling, which determines a nonlinear dynamic behavior. Two approaches were made in order to solve the electromechanical coupling: the quasistatic and the dynamic approach.

Up to now, in the field of electrostatic actuated microdevices, studies were made for systems that reach an equilibrium position when the electrostatic forces

equilibrate the mechanical ones [1]. Zipping actuators, in which an electrode touches another, were also analyzed using MEMCAD system [2]. Electromechanical hystereses in devices that exhibit contact between components were also investigated. A tool, CoSolve-EM, was used in order to solve quasistatic 3D-contact electro-mechanics for a clamped-clamped beam, calculating full displacement, capacitance and contact force versus voltage [3]. In this kind of approach the inertial effects are neglected so that dynamic prediction cannot be made.

The dynamic approach was implemented by some authors. The method of linear mode summation was used to build reduced order macromodels to perform the nonlinear analysis of MEMS [4]. The validity of the approach was illustrated with the example of doubly clamped beam under electrostatic actuation. Using the reduced order macromodels, it was possible to observe nonlinear effects such as the frequency shift due to bias voltage and the amplitude-dependence of resonance frequency [5]. The model used analytical form for electrostatic force calculation, which is not suitable for system with complex 3D geometry.

The code that is presented in this paper will make possible the computation of the electrostatic force distribution for each time step and each actuator position.

### METHODOLOGY

In order to study the dynamic behavior of the electrostatic actuated microsystem, with complex 3D geometry, the normal mode summation method (N.M.S.M.) was chosen. The transient case has recently been addressed using the normal mode summation method. In the frame of this formalism, the movement of the device is projected onto its normal modes. In the normal mode summation method, the deformed shape of the structure is expressed as a linear summation of the linear normal mode shapes, as shown in equation (1):

$$\{y\}_{Nx1} = [S]_{NxN} \{g\}_{Nx1}$$
 (1)

where N is the number of degree of freedom for the finite element model,  $\{y\}_{NxN}$  is the vector of displacement,  $[S]_{NxN}$  is the mode shape matrix that have as columns the mode

shape coordinates for each node (time independent) and  $\{q\}_{NzI}$  is the vector of modal coefficients (time dependent).

The movement equation has as variables the nodal coordinates:

$$[M]_{NxN} \{\ddot{y}\}_{Nx1} + [K]_{NxN} \{y\}_{Nx1} = \{f_{c}(y,t)\}_{Nx1}$$
 (2)

where  $[M]_{N \in \mathbb{N}}$  is the mass matrix,  $\{\bar{y}\}_{N \in \mathbb{N}}$  is the acceleration vector,  $[K]_{N \in \mathbb{N}}$  is the stiffness matrix and  $\{f_e(y,t)\}_{N \in \mathbb{N}}$  is the applied force vector.

By introducing (1) in equation (2), the new variables become the modal coordinates (time-dependent coefficients of the mode shapes):

$$\begin{split} & [M_G]_{NxN} \{\ddot{q}\}_{Nx1} + [K_G]_{NxN} \{q\}_{Nx1} = \\ & = [S]_{NxN} \{f_{\epsilon}(q,t)\}_{Nx1} \end{split} \tag{3}$$

where  $[M_G]_{NxN}$  and  $[K_G]_{NxN}$  are diagonal matrices. Considering that only the first P modes are important for the dynamic analysis of the system and the others are negligible, equation (4) is obtained.

$$\begin{split} & \left[ M_G \right]_{PxP} \left\{ \ddot{q} \right\}_{Px1} + \left[ K_G \right]_{PxP} \left\{ q \right\}_{Px1} = \\ & = \left[ S \right]^T PxN \left\{ f_e \left( q, t \right) \right\}_{Nx1} \end{split} \tag{4}$$

If the modes shapes are normalized to the mass matrix:

$$[S] N_{xN} [M]_{N_{xN}} [S]_{N_{xN}} = [I]_{N_{xN}}$$

$$[S] N_{xN} [K]_{N_{xN}} [S]_{N_{xN}} = [\Omega^{2}]_{N_{xN}}$$
(5)

 $[I]_{NxN}$  is the identity matrix and  $[\Omega]^2]_{NxN}$  is the square resonance frequencies matrix. So equation (4) becomes:

$$\begin{split} & [I]_{PxP} \{\ddot{q}\}_{Px1} + [\Omega^2]_{PxP} \{q\}_{Px1} = \\ & = [S]^T P_{xN} \{f_e(q,t)\}_{Nx1} \end{split} \tag{6}$$

In that way, the problem is reduced to a system of P uncoupled equations. The flow information diagram for the developed code is presented in Figure 1. The simulation cycle is started with the modal analysis for the mechanical characterization of the microsystem using the finite element method. The finite element model is built in order to extract the mode shapes and the resonance frequencies for the mechanical structure. displacements for each mode are stored in ASCII files to be used in further computations. These data together with the boundary conditions (applied voltage) are used to built the entry files for a boundary element method in order to compute the electrostatic forces acting on the device at

each time step and for each position. To integrate the motion in time a fourth order Runge-Kutta algorithm is used. At each time step the solution is obtained by averaging about four anterior points.

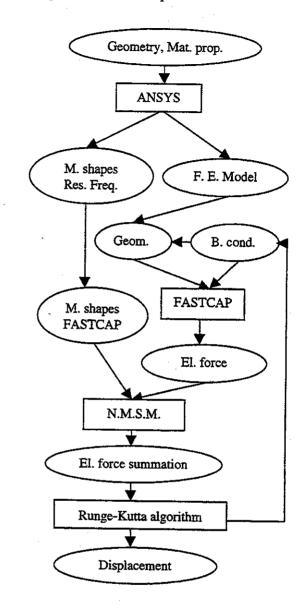


Figure 1: Information flow in the code starting from the geometry and material properties and ending with the time dependent deformation

#### **TESTING**

An optical microshutter manufactured at CSEM, Neuchatel, is used for testing the approach [6]. It is an electrostatic actuated device and it is part of a high-speed light modulation system that uses dense arrays of matrix addressable microshutters. A double polysilicon surface micromachining process is used to build the shutter.

## Microshutter Working Principle

The microshutter consists in a one-end fixed beam, ended with a planar plate. These components can oscillate between two electrodes and two stoppers that avoid the direct contact, Figure 2.

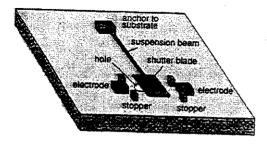


Figure 2: Optical microshutter

When the plate is attracted by one of the electrodes, it covers a hole that let the light pass through it and when the other one attracts it, it lets the hole uncovered. The electrostatic force is used to keep the shutter in one of the two positions described and the elastic energy stored in the beam is used to switch the shutter from one state to the other. In order to be attracted to the electrodes the shutter is taken into resonance by applying a periodic signal to one of them and a constant voltage to the other. The possibility to excite more vibration modes appears due to the levitation phenomena caused by the presence of the ground plate. Once the shutter is initialized, taking the electrode to which it is attracted to a small voltage and the other to a higher voltage performs the switching. Table 1. show the design feature for the microshutter.

Beam width	3 μm		
Beam height	2 μm		
Beam length	160 μm		
Shutter width	35 μm		
Shutter plate length	166.5 μm		
Displacement	30 µm		

Table 1. Main geometric parameters of the shutter

This kind of operating principle and the phenomenon that can appear in the device working cycle makes it appropriate to test the developed code.

# Results from Finite Element Analysis

The finite element model of the shutter is obtained with ANSYS software tool. Modal analysis is performed for the specified microshutter. The first four vibration

modes are computed, and their resonance frequencies are shown in Table 2.

Mode no.	1	2	3	4
Frequency (Hz)	7945.0	11948.0	94353.0	149870.0

Table 2. Resonance frequencies with ANSYS

Figure 3 show the deformed shape as a linear combination of the first two modes. For a qualitative representation of the deformation, the modal coefficients were chosen exaggerated to underline the levitation phenomena. These first tow modes are the first mode out of plane and the first mode in plane.

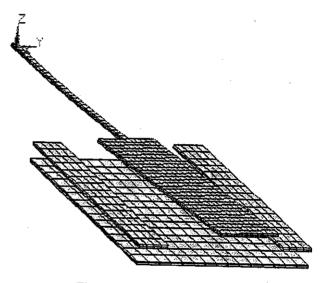


Figure 3: Finite element model

# Results from Boundary Element Analysis

The FASTCAP software tool, which implements a boundary element method, using multipole expansion, is used to compute the charge density on the conductor surfaces and further to calculate the electrostatic force distribution [7-8]. Using the formulas (7):

$$dF_n = \frac{\sigma^2}{2\varepsilon_0} da \tag{7}$$

the electrostatic force  $dF_n$  acting normally on area da with a local charge density  $\sigma$  on the conductor surface can be computed in vacuum, with a  $\varepsilon_0$  electrical permittivity.

The charge distribution for a given applied voltage and a certain position of the components is presented in Figure 4. It can be seen that charges are

accumulated on the ground plate making possible the levitation forces to occur.

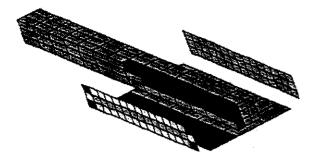


Figure 4: Charge density distribution with FASTCAP

### **Dynamic Analysis Results**

By applying a sinusoidal voltage to one of the electrodes and a constant voltage to the other, the evolution of the system can be determined. Figure 5 presents the time evolution for the second mode coefficient. The second mode is the one that has to be excited for catching the plate to one of the electrodes. For an applied 60V sinusoidal signal and an 80V constant voltage, a time of 1.5ms is needed for catching the plate. The number of cycle during the initialization process can also be determined and it is around 30.

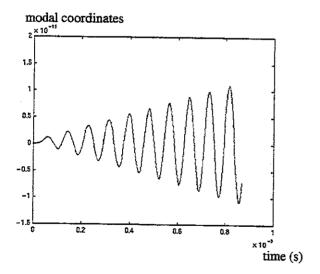


Figure 5: Second mode shape coefficient as a function of time for a sinusoidal applied voltage with a frequency of 11948 Hz

The influence of the other modes that can be excited during the initialization process is studied comparatively to the second mode. It can be seen from Figure 6 that the amplitudes of these modes are negligible in comparison with the second mode amplitude, when the applied voltage has a frequency equal to the second mechanical resonance frequency.

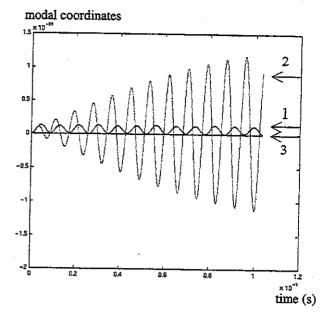


Figure 6: First three modal coordinates as a function of time for a sinusoidal applied voltage with a frequency of 11948 Hz: 1 – modal coordinate for first mode; 2 – modal coordinate for second mode; 3 – modal coordinate for third mode

If a signal with a frequency of 11900 Hz is apply, it can be seen from Figure 7 that the amplitude increases faster

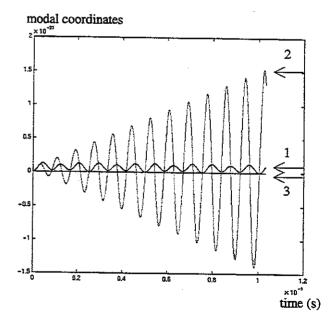


Figure 8: First three modal coordinates as a function of time for a sinusoidal applied voltage with a frequency of 11900 Hz: 1 - first mode; 2 - second mode; 3 - third mode

This can be explained by the fact that the electric field acting between the electrodes modifies the resonance frequency of the shutter. The electric field acts as a restoring spring and so it decreases the mechanical spring constant. That is a consequence of the nonlinear coupling between the mechanical and electrostatic domains.

### CONCLUSIONS

The coupling problem was treated for complex 3D geometrical structures. A code was written to simulate the dynamic behavior of the device for applied potentials varying with time. The mode shapes and the resonance frequencies were computed with ANSYS. This model was transformed in a boundary element model by taking only the surface elements and creating panels. The electrostatic force was computed at each step with FASTCAP. A Runge-Kutta algorithm was used to solve the differential equations. The CPU time is still high (several hours) for systems with complex 3D geometry.

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