Computer aided modeling of static and dynamic transfer characteristics

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ABSTRACT

When creating mathematical models of microsystem components or microelectronic devices, analytical and experimental modeling concepts can be distinguished. Analytical modeling, often performed on process or device level, can only be applied if the equations and geometries describing the components' behaviour are known. If there is none or insufficient information about the component to be modeled, the transfer characteristics may be determined by using methods of system identification and curve fitting from given input and output characteristics. These methods can be applied to typical domains found in microsystem technology. In this paper a software package is presented which enables the user to generate behavioural models from transfer characteristics given by measurements, simulations or data sheets.

Keywords:
Behavioural modeling, curve fitting, system identification, nonlinear least squares, model synthesis.

INTRODUCTION

The increasing complexity and package density in microsystem technologies and microelectronics requires the increasing usage of modeling and simulation in the fields of design, verification and optimization. Efficient simulation of complex heterogenous systems is coupled with the availability of suitable models for the used components. Modeling microsystem components presumes well-founded knowledge of the object to be described and the simulation tool to be used. The relevant effects being necessary for the specific investigation must be considered, characterized and transformed into a model for the chosen simulator. Usually modeling is performed manually based on heuristic approaches and therefore often it is a time and cost intensive process. In this paper we present an approach for computer aided modeling in addition to the respective software tools. The developed tools offer the recording of system-describing data, approximation and identification methods to determine transfer characteristics and the generation of models for different simulation tools.

MODELING CONCEPT

Due to the complexity of microsystem applications global modeling approaches are not suitable in general. Therefore a complex system to be modeled usually is separated into subsystems. Clues for decomposing a complex system are often given by structural informations but further aspects must be regarded as well [1]. The access to input and output signals of each subsystem must be ensured for the purpose of verification. This is in contrast to the desire to choose small subsystems that are easy to describe. Due to minimized geometric dimensions parasitic effects and cross sensitivities often must be considered. In case of a block oriented modeling approach this results in further input and output quantities to describe these effects. After a model structure that corresponds to these aspects has been found, the subsystems can be parametrized.

If the equations describing the components' behaviour and physical parameters (e.g. geometries) are known analytical models can be created. A correlation between system and model parameters is given within analytical models (white box models). Experimental modeling methods are applied if there is none or insufficient information available regarding the component to be modeled, if the underlying equations are too complex or if no correspondence to the design parameters is needed. Such approaches that are known as black or grey box modeling methods, are based on the determination of transfer characteristics from given input and output data.

The models for each subsystem are put together to the complete system model. Due to different modeling approaches the subsystems often are described on different hierarchical levels while performing a global system simulation.

In contrast to analytical modeling experimental methods offer a wide potential for being implemented into software tools. Especially if the given subsystems can be described as nonlinear static or linear dynamic systems, powerful algorithms are available to determine the transfer characteristics from given input and output data. The integration of these algorithms into easy-to-handle software tools - combined with an interface to simulators - shall enable the user to reduce modeling efforts.

MODELING TOOLS

The usage of the developed modeling tools is described and illustrated with typical application examples within the following chapters.

Obtaining data

As mentioned above the modeling concept and tools are based on the approximation of data given as characteristic curves that may be obtained from measurements, simulations or data sheets.

For this reason a file format as an interface to the following introduced tools was developed [2]. It enables the flexible description of characteristic curves or families of characteristics. Thus, present data for example can either be given as real or complex data.

When modeling components in electronics, it is often referred to data sheets published by the manufacturers. These data sheets also contain information which can only be stated graphically, as they are based on measured values and sometimes cannot be reproduced as simple formulas. If these characteristics are to be integrated into models the problem occurs that the information of the data sheets is not machine-readable. If characteristics are to be modelled with the help of balancing curves, hitherto single measure value pairs have to be extracted manually from the graph of the data sheet. This step can be considerably simplified using *MageSheetscan*. The graph of the data sheet must be read in by a scanner and can be displayed on the monitor with *MageSheetscan* (see figure 1). Then data samples can be marked on the curves by the mouse and stored as value pairs. The values determined in this way can be written in a file in order to be used by approximation tools and thus can be integrated into models more easily.

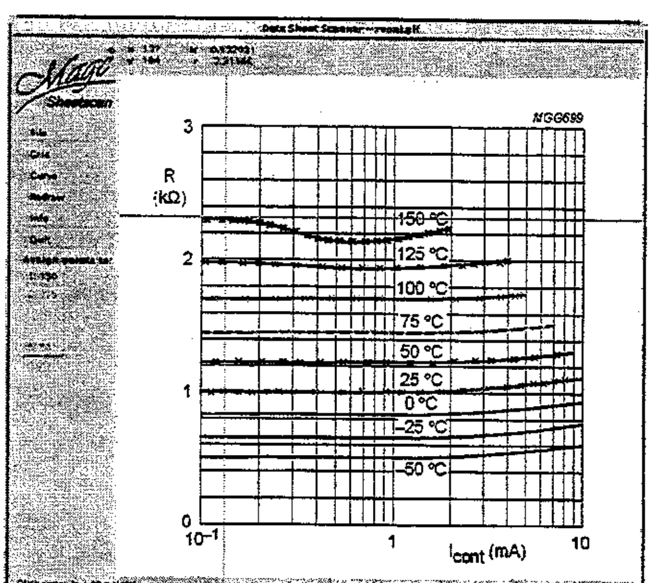

Figure 1. Graphical user interface of *MageSheetscan*

Nonlinear static transfer characteristics

To determine nonlinear static transfer characteristics a tool called *MageStatic* was developed. It enables the user to perform approximations of characteristic curves with up to two input quantities and to create models within a graphical user interface.

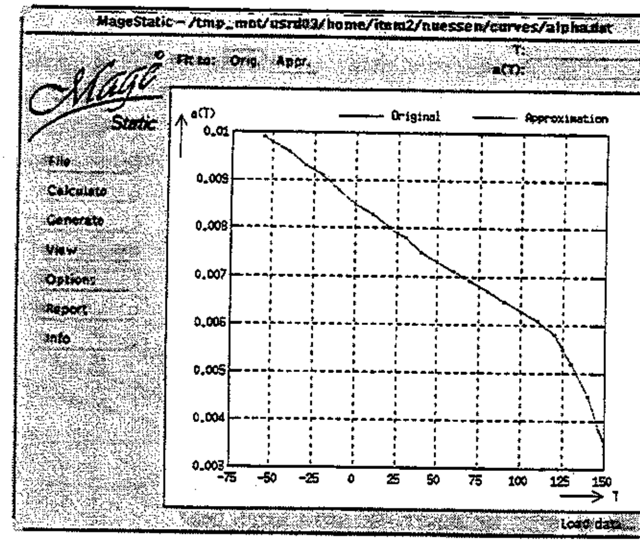

Figure 2. Graphical user interface of *MageStatic*

Algorithms

A given nonparametric model $y = f(\underline{x})$ with the ordinate values $y_i = [y_1 \ldots y_m]^T$ and the independent variables $\underline{x}_i = [x_1 \ldots x_l]^T$ is to be approximated with an analytical function $Y = f(\underline{x}, \underline{p})$ containing the vector of independent parameters $\underline{p} = [p_1 \ldots p_n]^T$. This function is called target function. To determine a parameter vector that fits the given data the quadratic error function

$$S(\underline{p}) = \sum_{i=1}^{m} \left[Y_i(\underline{p}, \underline{x}) - y_i \right]^2 = \left| \underline{Y}(\underline{p}) - \underline{y} \right|^2 \qquad (1)$$

is minimized. This is achived by setting the partial derivatives of the error function to zero. The resulting system of equations depends on the given target function. If this function is linear in the parameters a linear system of equations can be solved in one step, otherwise iterative nonlinear least squares algorithms must be applied. Using *MageStatic* the user can choose between a linear least square algorithm and the nonlinear algorithms of Gauss-Newton, Levenberg-Marquardt, Hartley and Meyer-Roth [3]. Furthermore the target function and starting values must be specified by the user.

Model synthesis

The approximation results can be used to create model descriptions for simulation purpose. *MageStatic* offers an extensive library of optional model descriptions to which the calculated transfer function can be assigned. Both active and passive electrical components, like controlled sources and resistors or capacitors, are available. Nonlinear controlled capacitors may be used in case of capacitive detection methods found for example in pressure and acceleration sensors. Nonlinear controlled resistors may be chosen when piezoresistive components must be described.

Application example

The methods described above are illustrated with an application example. The family of characteristics depicted in figure 3 represents the equivalent capacitance of a capacitive pressure sensor as a function of pressure and the membrane thickness obtained by FEM simulations.

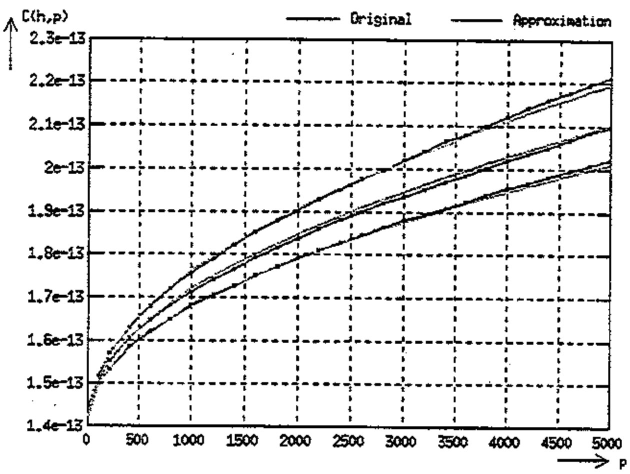

Figure 3. Equivalent capacitance characteristics

```
# Signal controlled capacitor
template sccap p1 p2 p = h
 input nu p
 electrical p1, p2
 number h
{
  number x0 = 1.4105776618254859e-13
  number x1 = 1.8876812765603871e-15
  number x2 = -2.2191812491300969e-09
  number x3 = 1.1273342279323440e-03
  branch _v_in = v(p1, p2)
  branch _i_out = i(p1->p2)
  val c Cap
  Cap = x0+(x1+x2*h+x3*h**2)*sqrt(p)
  _i_out = d_by_dt(Cap*_v_in)
}
```

Listing 1. SABER-MAST model

Based on this characteristic curve a behavioural model for the simulator SABER was to be created. In the first step the target function

$$F(h,p) = x_0 + (x_1 + x_2 \cdot h + x_3 \cdot h^2) \cdot \sqrt{p} \tag{2}$$

was chosen to fit the given data with a sufficient accuracy. Approximation results are also shown in figure 3. In the second step the SABER-MAST model of a nonlinear controlled capacitor was created (see listing 1). For this step the user has to choose the target device capacitor and the input quantities. They can be realized as voltages, currents, non-kirchhoff-signals or parameters given to the template.

Linear dynamic transfer characteristics

The investigation of the dynamic behaviour of linear systems can be performed in the frequency domain. Respective approaches are known from control or filter theory. In microelectronics frequency domain methods for example are applied to characterize the dynamic behaviour of components like operational amplifiers. Within microsystem applications the dynamic behaviour of microstructures like membranes or beams is often characterized in the frequency domain. Based on this *MageDynamic* (see figure 4) was developed. This tool enables the user to perform parameter estimation methods for linear dynamic systems in the frequency domain.

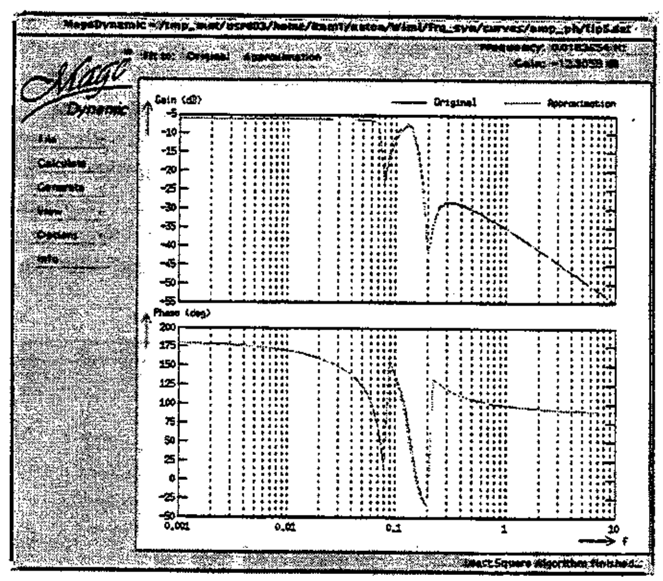

Figure 4. Graphical user interface of *MageDynamic*

Algorithms

If the laplace transform is applied to the linear differential equation

$$\begin{aligned} a_n y^{(n)}(t) + a_{n-1} y^{(n-1)}(t) + \ldots + a_1 y'(t) + a_0 y(t) \\ = b_m u^{(m)}(t) + b_{m-1} u^{(m-1)}(t) + \ldots + b_1 u'(t) + b_0 u(t) \end{aligned} \tag{3}$$

the transfer function in the s-domain

$$G(s)=\frac{b_0+b_1(s)+b_2(s)^2+\ldots+b_m(s)^m}{1+a_1(s)+a_2(s)^2+\ldots+a_n(s)^n} \qquad (4)$$

with $s=\delta+j\omega$ is obtained. Applying the limit value $s\rightarrow j\omega$ results in the complex transfer function

$$G(j\omega)=\frac{b_0+b_1(j\omega)+b_2(j\omega)^2+\ldots+b_m(j\omega)^m}{1+a_1(j\omega)+a_2(j\omega)^2+\ldots+a_n(j\omega)^n}. \qquad (5)$$

The algorithms implemented in *MageDynamic* enable the user to estimate the coefficients a_i and b_i from given data in the frequency domain.
With

$$G(j\omega)=|G(j\omega)|\cdot e^{j\varphi(\omega)}=R(\omega)+jI(\omega) \qquad (6)$$

present approximation data may either be given in gain and phase or real- and imaginary part. Presumed a physical realizable minimum phase (all-pass free) system it is further possible to estimate the parameters from a given gain plot.

Root locus parameter estimation

If the data describing the system is given as complex values (real- and imaginary part or gain and phase) the following algorithm is used for parameter estimation. Like shown in [4] first the error criteria

$$e(j\omega_v)=G(j\omega_v)-\frac{\hat{B}(j\omega_v)}{\hat{A}(j\omega_v)} \qquad (7)$$

with the given data

$$G(j\omega_v)=|G(j\omega_v)|\cdot e^{j\varphi(\omega_v)}=R(\omega_v)+jI(\omega_v) \qquad (8)$$

and the transfer function to be determined

$$\hat{G}(j\omega)=\frac{\hat{B}(j\omega)}{\hat{A}(j\omega)}=\frac{\hat{B}_R(\omega)+j\hat{B}_I(\omega)}{\hat{A}_R(\omega)+j\hat{A}_I(\omega)} \qquad (9)$$

is defined. It describes the deviation from the original and approximated data for a discrete frequency value ω_v. As equation (7) is nonlinear concerning the parameters to be determined, a linear least square algorithm to minimize the deviation cannot be applied. Rearranging the equation (7) results in

$$e(j\omega_v)\cdot\hat{A}(j\omega_v)=\hat{A}(j\omega_v)\cdot G(j\omega_v)-\hat{B}(j\omega_v) = \varepsilon(j\omega_v). \qquad (10)$$

The extended error criteria $\varepsilon(j\omega_v)$ is obtained by multiplying $e(j\omega_v)$ with the denominator polynomial of the desired transfer function. This correlation leads to false estimated parameters that can be corrected by introducing weighting factors. This leads to the weighted least square error function

$$Q=\sum_{v=0}^{N} w_v\cdot|\varepsilon(j\omega_v)|^2=\sum_{v=0}^{N} w_v\cdot\left|\hat{A}(j\omega_v)\cdot e(j\omega_v)\right|^2 \qquad (11)$$

with the weighting factors

$$w_v=\frac{1}{|A(j\omega_v)|^2}. \qquad (12)$$

Solving the system of linear equations by minimizing the partial derivatives of equation (11) gives the first estimation of the parameters. The obtained coefficients are put into the error function and a new minimization is performed. This iterative procedure is repeated until either the quadratic error remains under a given value or a maximum number of iterations is reached.

Gain plot parameter estimation

The following algorithm must be applied if the present data is given as a gain plot.
Multiplying a transfer function given in equation (5) with its conjugate complex G(-jω) results in

$$S(\omega)=|G(j\omega)|^2=\frac{B_0+B_1\omega^2+\ldots+B_m\omega^{2m}}{A_0+A_1\omega^2+\ldots+A_n\omega^{2n}}=\frac{B(\omega)}{A(\omega)} \qquad (13)$$

which is a real function of ω. $S(\omega_v)$ symbolizes the squared gain value at a given frequency ω_v. The deviation from the given values $S(\omega_v)$ and the transfer function to be determined is defined as the error criteria

$$e(\omega_v)=S(\omega_v)-\frac{B_0+B_1\omega_v^2+\ldots+B_m\omega_v^{2m}}{A_0+A_1\omega_v^2+\ldots+A_n\omega_v^{2n}}. \qquad (14)$$

This criteria again is nonlinear in the parameters and thus is rearranged to

$$e(\omega_v)=\varepsilon(\omega_v)d(\omega_v) \qquad (15)$$

like sugessted in [5]. With

$$\varepsilon(\omega_v)=(A_0+A_1\omega_v^2+\ldots+A_n\omega_v^{2n}) - \frac{1}{S(\omega_v)}(B_0+B_1\omega_v^2+\ldots+B_m\omega_v^{2m}) \qquad (16)$$

and

$$d(\omega_v)=\frac{S(\omega_v)}{A_0+A_1\omega_v^2+\ldots+A_n\omega_v^{2n}} \qquad (17)$$

the least square error function

$$Q=\sum_{\nu=0}^{N}\varepsilon^2(\omega_\nu)\cdot d^2(\omega_\nu). \tag{18}$$

again is minimized with an iterative procedure. Factorizing the obtained rational polynomial

$$S(\omega)=\frac{B_0+B_1\omega^2+\ldots+B_m\omega^{2m}}{A_0+A_1\omega^2+\ldots+A_n\omega^{2n}} \tag{19}$$

yields the desired transfer function

$$G(j\omega)=\frac{b_0+b_1 j\omega+\ldots+b_m j\omega^m}{a_0+a_1 j\omega+\ldots+a_n j\omega^n}. \tag{20}$$

Model synthesis

The task of *MageDynamic* is not only to create mathematical models based on given data, but also to create netlists and behavioural models. For this reason a model description is generated that reproduces the terminal behaviour of the calculated transfer function. With different synthesis methods either an impedance (two-pole network) or a transfer function (two-port network) can be generated from the calculated rational polynomial (see figure 5).

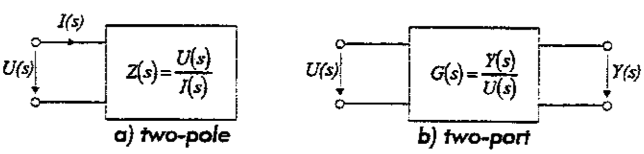

Figure 5. Two-pole and two-port networks

Two-poles

In case of a two-pole network to be realized, the netlists are obtained by developing continued fractions from the given transfer function following the method of CAUER and BRUNE [6]. These networks contain resistors, capacitors, inductors and transformers.

Two-ports

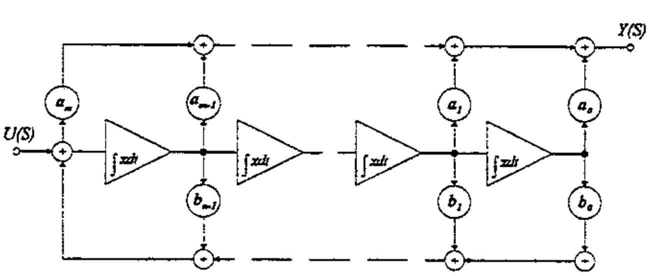

Figure 6. General block diagram for a transfer function

If the given transfer function symbolizes the quotient of an output and an input signal with a common ground node, a network as shown in figure 6 containing integrators, adders and multipliers is used to realize a model description [7]. The order of the transfer function determines the number of integrators needed. The multipliers contain the coefficient values of the numerator and denominator polynomial.

Application examples

Two examples shall be presented in order to show typical applications of the methods described before.

Operational amplifier output impedance

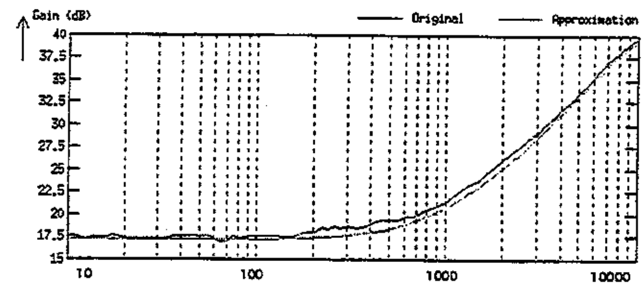

Figure 7. Operational amplifier output impedance

Based on the gain plot shown in figure 7 representing the output impedance of an operational amplifier [8], the transfer function

$$Z(s)=\frac{7.15554+0.00135478\cdot s}{1+1.08812e-05\cdot s+1.5639e-10\cdot s^2} \tag{21}$$

was calculated. The resulting two terminal equivalent circuit and the SPICE netlist stored as a subcircuit are given in figure 8 and listing 2.

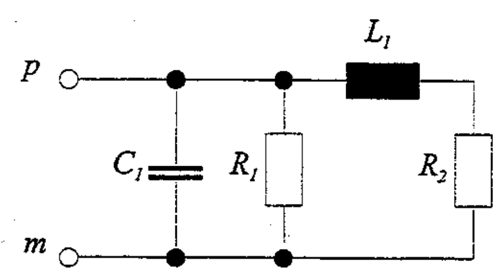

Figure 8. Output impedance equivalent circuit

```
.subckt opamp_output_impedance p m
C1 p m 1.5449e-07
R1 p m 86.0603
L1 p imp_12 0.00129866
R2 imp_12 m 7.8046
.ends opamp_output_impedance
```

Listing 2. SPICE listing of the output impedance equivalent circuit

Acceleration sensor

In figure 9 the sensitivity of a capacitive acceleration sensor [8] as a function of frequency is shown. Due to the applied force feedback linearization a linear modeling approach is valid.

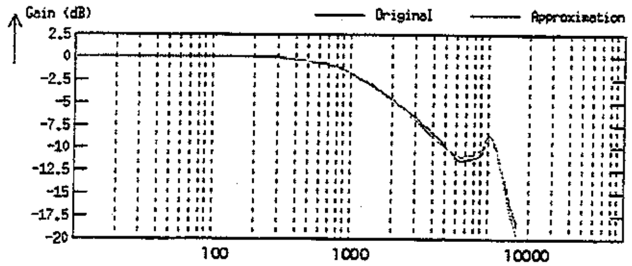

Figure 9. Acceleration sensor sensitivity gain plot

The transfer function

$$G(s)=\frac{1.0077+1.34799e-05\cdot s+1.61867e-10\cdot s^2}{1+0.00012\cdot s+7.726e-10\cdot s^2+2.3167e-14\cdot s^3} \qquad (22)$$

was found suitable to give the best fit to the original data. According to this transfer function the SABER-MAST model shown in listing 3 was created.

```
template adxl_gain i o
  input nu i
  output nu o
{
  val nu o_s
  var nu s0, s1, s2, s3
  values {
    if (dc_domain) {
      o_s = 1.00771 * i
    }
    else {
      o_s = 6986.84*s1 + 5.81845e+8*s2 +
            4.34966e+13*s3
    }
  }
  equations {
    s0: s0 = d_by_dt(s1)
    s1: s1 = d_by_dt(s2)
    s2: s2 = d_by_dt(s3)
    s3: i  = s0+3335.7*s1 + 5.106e+9*s2 +
             4.316e+13*s3
    o: o   = o_s
  }
}
```

Listing 3. SABER-MAST model

In this case the input and output quantities are realized as non-kirchhoff-signals. Models can also be created for the simulators ELDO-HDL/A, HSPICE, PSPICE and SPICE3.

In case of SPICE models the integrators are realized as RC-networks with controlled sources.

SUMMARY

In this paper a software package was introduced that enables computer aided modeling of microsystem components. Characteristic curves that may be taken from measurements, simulation or data sheets serve as input data. The presented tools offer methods of curve fitting and system identification to determine the transfer characteristics from given nonparametric input and output data. Depending on the calculated transfer function, models for different simulators can be created.

Important future tasks to increase the efficiency of the presented tools will be the further development of the implemented approximation algorithms regarding their convergence and accuracy. Besides, methods that support the user finding suitable target functions or estimating the system order must be developed and implemented.

Throughout the support of further simulation tools the performance of the modeling tools shall be increased. Especially, the implementation of synthesis methods to enable the generation of VHDL-AMS models is aspired.

So far the calculated transfer functions can only be assigned to models for electrical components. A further task is the development of synthesis methods concerning models for nonelectrical components.

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