Capillary interaction between a small thin solid plate and a liquid

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ABSTRACT

Micromechanical systems are increasingly being used in microfluidic and biological applications where solid-liquid interaction phenomena become important. Here, we present an analytical/computational study on the interaction force between a thin solid plate interfacing with a liquid, and forming a meniscus under uniform gravitational field. The height of the plate from the liquid surface is prescribed. We assume that the contact angle is fixed, and must be satisfied by all possible menisci. The interaction force and the profile of the meniscus are determined using Young-Laplace equation represented in an Eulerian coordinate. For a long narrow thin plate, the results are obtained in closed form. For a circular plate, the solution is obtained numerically, where we find that the stability of the meniscus is strongly dependent on the radius of the plate. The findings of this study have relevance to the quantification of force between an AFM tip and the thin layer of moisture on the surface that it scans, as well as in the design of MEMS actuators that manipulate objects in liquids without inundating themselves.

Keywords: MEMS, meniscus, capillarity, surface-tension

1 INTRODUCTION

Interaction between a liquid and a solid forming a meniscus with the liquid has intrigued scientists for centuries. Laplace [1] first formalized the relation between the curvature at any point of the meniscus and the pressure differential on either side of the meniscus surface. During the last few decades, capillary interaction has drawn substantial attention. Several studies have focused on the shape and stability of the menisci formed between a thick solid and a liquid or between two thick solids [2]. The corresponding force on the solids is also determined. A variety of micro mechanical systems interface with liquids, such as during fabrication when sacrificial layers are removed by wet etching [3], [4], during self assembly by capillary forces [5], while probing biological species using micro actuators [6], to name just a few. In micro mechanical applications, the interfacing solids are typically thin plates with thickness on the order of a micro meter.

Here, the interaction force between a thin plate and a liquid is studied analytically. First, a mechanism is proposed to describe the formation of the meniscus at the edge of the plate as it is moved in or out of the liquid. Then the interaction force is determined for a long plate with finite width (e.g., a beam) and for a circular plate. First, we review the essential basics.

2 SURFACE ENERGY

Surface energy is the reversible work associated with creating new surfaces in a medium in vacuum. If W_{11} is the work done in creating 2 unit surfaces in the medium by its separation, then the surface energy or surface tension, $\gamma_{11} = W_{11}/2$. Thus γ_{11} is also the energy needed to increase a given surface of the medium in vacuum by unity. γ has the dimension of $energy/area$ or $force/length$, i.e., if the surface of the medium is viewed as a membrane, then surface tension is the force per unit length of a cut in the membrane. The force is tangential to the membrane and normal to the line of cut.

The interface energy γ_{12} per unit area of an interface formed by media 1 and 2 is defined similarly, i.e., the energy required to increase the interface area by unity.

Let $\mathcal{L}$ represent a line where the three media, solid-liquid-vapor, meet (Figure 1). There is a core region around $\mathcal{L}$ of radius less than $100\mathring{A}$ [7] beyond which the surface energies are fixed at a static thermodynamic equilibrium condition. Also, the contact angle ϕ is a constant and is the joint property of the three media. It is related to the surface energies by

$$\gamma_{sl} - \gamma_s - \gamma \cos\phi = 0 \qquad (1)$$

where γ_s, γ_{sl} and γ are the interface energies between the solid and the vapor, the solid and the liquid, and the liquid and the vapor respectively. Within the core region, the surface energies and angle may vary as shown in Figure 1. If Δp is the pressure differential on both sides of the liquid-vapor interface, and R_1 and R_2 are its principal radii of curvature, then

$$\gamma(\frac{1}{R_1} + \frac{1}{R_2}) = \Delta p \qquad (2)$$

which is known as the Young-Laplace equation [1], [8].

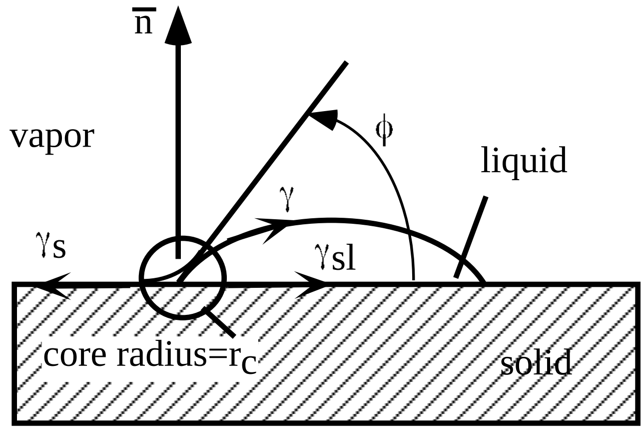

Figure 1: A liquid forms a droplet on a hydrophilic solid. Hence the contact angle $\phi < \pi/2$. The radius of the core region $r_c \sim 10nm <<$ all relevant radii of the problem. The surface energies and the contact angle are constants. Within the core region, the angle of contact may vary.

In the following we assume that all relevant radii of curvature, such as that of the solid surface, is larger than r_c, and the thickness of solid plates is small. A consequence of the first assumption is that the contact angle is fixed and so are the surface energies.

3 MENISCUS AT THE EDGE OF A THIN PLATE

Since the contact angle is fixed, all possible menisci that can be formed for various heights of the thin plate must meet the solid with the same contact angle, ϕ. Then, the angle, θ_0, of the meniscus at the triple point with the vertical becomes variable, and is determined by the height of the plate. Figure 2(a) shows the edge of a hydrophilic plate forming menisci with a liquid as the plate is moved into or out of the liquid. The plate can be of any shape, and the profile of the menisci will depend on the shape of the plate. But the contact angle, ϕ, is always respected. Here the lowest meniscus corresponds to the highest elevation of the plate from the far field liquid surface. As the plate is lowered, the triple line gradually moves up along the edge of the plate until it reaches the top surface when the plate is below the far field liquid surface. Note that, at any height of the plate, the capillary force is the vertical component of γ (force per unit length of the triple line) acting on the entire length of the triple line. For a hydrophilic plate ($\phi < \pi/2$), as the plate is raised above the far field liquid surface, there exists a meniscus which is aligned with the vertical at the triple point. This meniscus applies the maximum possible capillary force, γ, (downward) per unit length of the triple line. If the plate is a long beam or circular in shape, then the meniscus is same along the entire length of the triple line. If the plate has variable radius of curvature along the edge, the meniscus shape will change along the triple line.

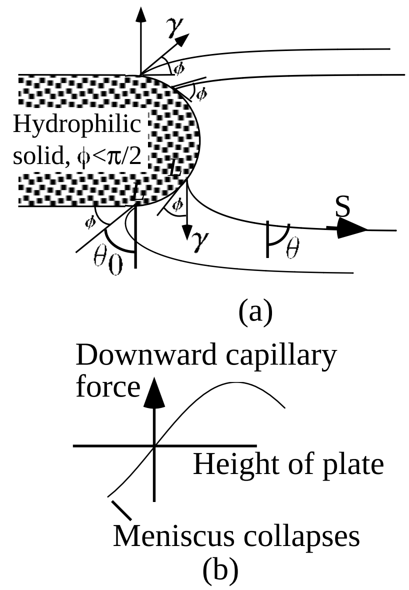

Figure 2: (a) A family of menisci at the edge of a hydrophilic plate as it is moved into or out of the liquid. (b) Capillary force on the thin plate for various heights of the triple point.

When the plate is lowered below the liquid surface, the maximum upward vertical force that the meniscus can apply is $\gamma \sin\phi$, a fraction of γ. Thus for a hydrophilic plate, it is expected that the force to break the meniscus while the plate is removed from the liquid is larger that the force required to break the meniscus prior to submersion as shown in Figure 2(b).

4 PROFILE OF A MENISCUS

The profile is governed by Eq. 2. Let H_0 denote the height of the triple point above the far field liquid surface and Y denote the vertical distance (downward) from the triple point. Then at any point on the meniscus, $\Delta p = \rho g(H_0 - Y)$. $H_0 > 0$ when the triple point is above the far field liquid surface, $H_0 < 0$ when it is below. Let l_0 denote the capillary length defined as $l_0^2 = \frac{\gamma}{\rho g}$. Then Eq. 2 can be non-dimensionalized as

$$\frac{1}{r_1} + \frac{1}{r_2} = h_0 - y \qquad (3)$$

where $r_i = R_i/l_0$, $i = 1, 2$, $h_0 = H_0/l_0$, $y = Y/l_0$. Let S denote the length along the meniscus with $S = 0$ at the triple point, and θ (counterclockwise positive) denote the angle of the meniscus with the vertical with $\theta(S = 0) = \theta_0$. Then $d\theta/ds = 1/r_1$, where $s = S/l_0$.

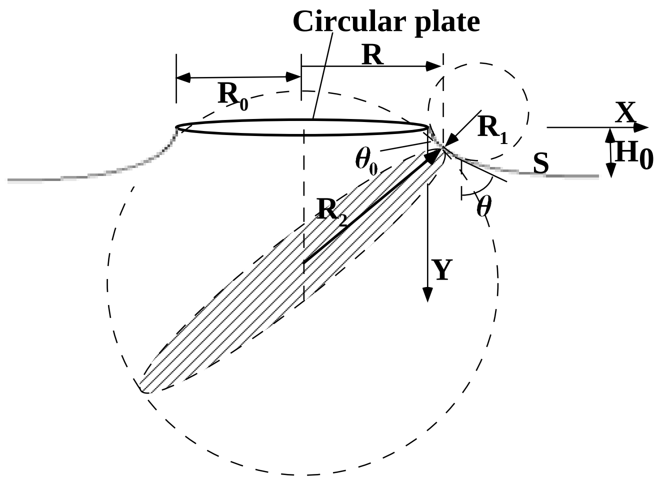

Figure 3: A circular plate forming a meniscus with a liquid. The two principal curvatures are shown.

The second curvature $1/r_2$ in Eq. 3, which depends on the shape of the plate, needs to be determined.

4.1 A long plate with finite width

Here $1/r_2 = 0$, and Eq. 3 simplifies to [9]

$$\frac{d\theta}{ds} = h - y \tag{4}$$

with the boundary condition that as $s \rightarrow \infty$, the meniscus becomes horizontal, i.e., $\theta(s \rightarrow \infty) = \pi/2$ and $\theta' = d\theta/ds = 0$. Thus, for a given h_0, the initial condition $\theta = \theta_0$ must be chosen such that the far field condition is satisfied. The relation between h_0 and θ_0 is given by [9]

$$h_0 = \pm\sqrt{2}\sqrt{1 - \sin\theta_0} \tag{5}$$

Thus, for a prescribed height of a long plate, θ_0 is known, and the profile of the meniscus can be determined analytically. The result can be found in various text books, such as in [10], where the meniscus profile is given for a *vertical* plate partially submerged in a liquid, and the meniscus meets the plate with the contact angle ϕ. Thus, in order to get the profile for a long plate with meniscus height h_0, one needs to simply replace ϕ by θ_0 in the expressions provided in the text books. The capillary force per unit length of the triple line is given by $\gamma\cos\theta_0$.

4.2 Circular plate

Let R_0 be the radius of the circular plate, and R is the radius of the meniscus at any point S. Then, $1/r_2 = r/\cos\theta$, where $r = R/l_0$ (Figure 3). Note that R_1 is the radius of the circle on the plane of the paper. R_2 is the radius of the shaded circle (normal to the plane of the paper) that disects a sphere of radius R_2 which meets the meniscus at R tangentially. Due to symmetry, the center of the sphere lies on the center line of the circular plate. Let the triple point on the right edge of the circular plate be the origin of the coordinate system with $S = 0$, $\theta = \theta_0$. Then, Eq. 3 takes the form

$$\frac{d\theta}{ds} - \frac{\cos\theta}{r_0 + x} = h - y \tag{6}$$

where $x = X/l_0$ and $y = Y/l_0$ are the non-dimensional distance along the horizontal and vertical directions respectively from the origin. For a given initial condition $\theta(s=0) = \theta_0$, $\theta(s)$ can be solved numerically from Eq. 6. But θ_0 is yet unknown, and should be chosen such that as $s \rightarrow \infty$, $\theta = \pi/2$, $\theta' = 0$, $x \rightarrow \infty$, $y = h_0$.

Here we solve for θ_0 by shooting method. For a guessed value of θ_0, we solve for θ and θ' from Eq. 6 by numerical integration at a large value of s (~ 6). The procedure is repeated for various values of θ_0 until a value is reached for which both θ and θ' vanish at large s. Once the required θ_0 is obtained, the profile of the meniscus is evaluated by numerical integration from Eq. 6. Capillary force on the plate is given by $\gamma\cos\theta_0$ per unit length of triple line.

Figure 4 shows the profiles of the menisci for circular plates with varying radii of $r_0 = 5$ to $r_0 = .21$. The height of the triple point is fixed at $h_0 = .5$. For large r_0, the second curvature $1/r_2$ is negligible, and the profile is similar to that formed by a long plate. As r_0 decreases, $1/r_2$ increases near the edge of the plate and the meniscus becomes increasingly steep near the triple point. For $h_0 = .5$, no meniscus exists for $r_0 < .21$. To form a meniscus the plate must be lowered. Thus, the meniscus between a MEMS probe and a liquid is expected to be shallow, but could be steep. Figure 5 shows the force (non-dimensional) on a circular plate as a function of the plate radius for $h_0 = .5$ and $h_0 = .01$. For the former, the meniscus breaks at $r_0 = .21$. It is interesting to note that the force dependence on r_0 is almost linear except very close to r_0 where the meniscus breaks. This implies (and observed in the numerical values of θ_0) that θ_0 remains almost independent of r_0 until r_0 is close to the critical r_{0c} at which the meniscus breaks. Thus for $r_0 >> r_{0c}$ the one dimensional analysis of the meniscus is sufficient.

5 CONCLUSIONS

The mechanism by which a thin plate forms a meniscus on the surface of a liquid is explored. Two geometries of the plate are considered: a long plate with finite width which forms a 1D meniscus (one of the curvatures vanishes), and a circular plate. The profile of the meniscus and the capillary force on the long plate are obtained analytically. For the circular plate, the meniscus profile is determined numerically by shooting method. It is found that the maximum possible meniscus height with the circular plate decreases as the radius of the plate decreases. Thus the meniscus between a MEMS probe and a liquid is shallow, although the slope of the meniscus near the triple point may be steep, i.e., the capillary force may be high.

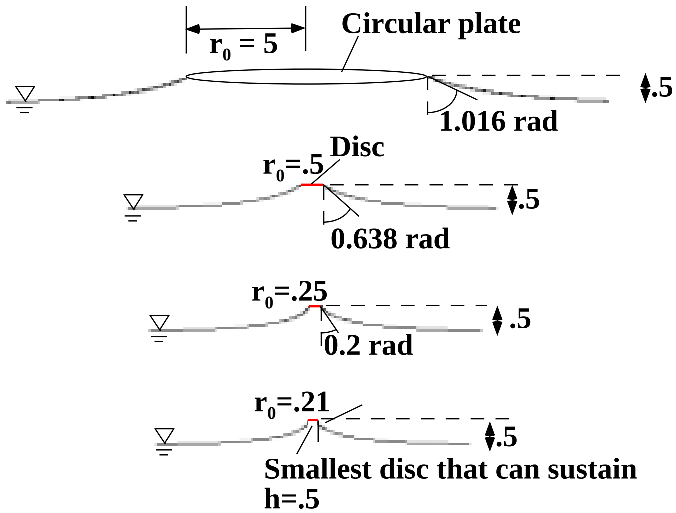

Figure 4: Circular plates with varying radii forming a menisci with a liquid. For the non-dimensional height $h_0 = .5$ of the triple point, the minimum radius that can sustain a meniscus is $r_0 = .21$.

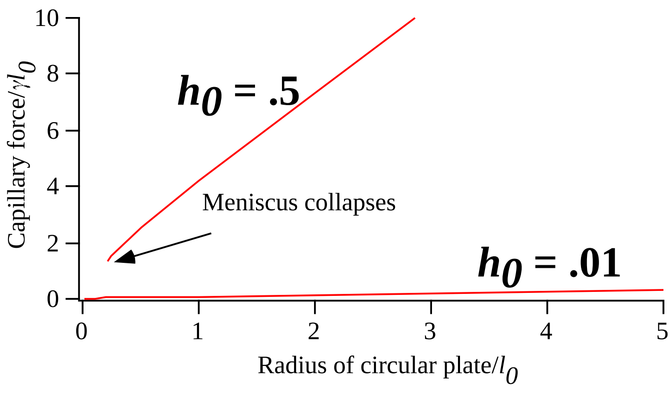

Figure 5: Capillary force on a circular plate with varying radius for two prescribed non-dimensional heights h_0 of the triple point.

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