Numerical and Analytical Studies of AC Electric Field in Dielectrophoretic Electrode Arrays

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ABSTRACT

Manipulation of micro-sized particles and biological cells using dielectrophoresis (DEP) is an emerging technique in MEMS and nano technology[3]. This paper presents an exact solution of dielectrophoretic motion of a polarized particle in the vicinity of interdigitated bar electrodes fabricated on planar insulating surfaces which have been widely employed in conjunction with hydrodynamic forces and gravity to separate particles in field flow fractionation device[5]. We solved the electric field using exact mixed boundary conditions on the electrode plane. DEP forces exerted on a spherical particle are calculated and particle levitation is studied. Besides the levitation force, the particle will also experience a lateral force which causes particles clustering at certain locations above the electrode plane depending on the geometry of electrode array. Comparison is made with previous approximate solutions and numerical results.

Keywords: DEP, analytic solution, numerical method

1 INTRODUCTION

It is observed that a polarized particle will move in response to an external non-uniform electric field. Such a phenomenon is well known as dielectrophoresis (DEP) in contrast to electrophoresis of charged particles in electric field[3]. The mechanism of DEP arises from the polarization of particles and non-uniform forces exerted by external electric field. This technique has been widely employed to manipulate micro and nano-sized latex particles as well as other biological particle such as cells, bacteria and virus.

The electrostatics and electrodynamics of both simple homogeneous particles and composite particles enclosed by layered shell structures have received considerable attention in cellular biology, bioengineering, and food industry. Fundamental research studies have sought to establish the significance of microelectrode properties on the equilibrium position and transient motion of particle mixture under various experimental conditions. Since the particles are small compared with electrode dimensions, the study of electric fields generated by the electrode structure becomes essential and can be used to guide the design of microelectrode geometry with desired functions[1].

One typical application of DEP device widely used in separation technology is Field Flow Fractionation (FFF), in which inter-digitated electrode arrays are fabricated on a planar substrate[5]. A simplified model for a FFF chamber is sketched in figure 1 which illustrates how particles of different properties are levitated to different heights by DEP forces and are separated by applying a horizontal flow. Because

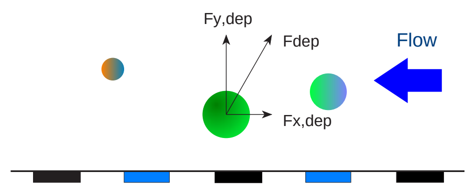

Figure 1: DEP of particles above interdigitated electrodes.

DEP force depends on the arrangement of the electrode and the resulting electric field distribution as well as the dielectric properties of particles and the surrounding medium, this leads to several possible electrode array designs configurations that can be utilzed for particle manipulation and separation. Thus, there are several approaches through which DEP can be employed for the separation of a mixture of particles with different properties.

In this paper, we present an exact solution for DEP forces exerted on spherical polarized particles in the vicinity of interdigitated electrode arrays in order to more accurately characterize the effect of electric field on particle motion. The governing equations are solved analytically using exact mixed boundary conditions on the electrode array. Previous studies ([1], [2], [6]) have assumed a linear distribution of electric potential between adjacent electrodes and thus failed to predict the singular behavior of electric field near the edge of the electrode, which we identified through an asymptotic analysis of electric field near a wedge-shaped electrode. Our solution reveals some new features that have not been predicted in previous studies. The analytic solution also serves as a benchmark to validate numerical DEP models that are applicable to more complicated microelectrode.

2 ELECTRIC FIELD AND DEP FORCE

2.1 Governing equations for DEP

The DEP force can be derived by either effective dipole moment method or Maxwell stress tensor method. In general, the time-averaged DEP force for a spherical particle in

AC electric field is given by

$$\boldsymbol{f} = \frac{1}{2}\Re\left(\boldsymbol{m}\cdot\nabla\right)\boldsymbol{E} \tag{1}$$

where $\boldsymbol{m}$ is the dipole moment. For an isotropic, homogeneous dielectric spherical particle, the DEP force is given by[3]

$$\boldsymbol{f} = 2\pi\epsilon_m a^3\Re(f_{\text{cm}})\nabla E^2 \tag{2}$$

in which f_{cm} is Clausius-Mossotti factor defined by[3]

$$f_{\text{cm}} = \frac{\epsilon_p^* - \epsilon_m^*}{\epsilon_p^* + 2\epsilon_m^*} \tag{3}$$

where the complex permittivity is $\epsilon^* = \epsilon - i\sigma/\omega$ and subscript m and p represent the surrounding media and the particle. The Maxwell equations suitable for DEP study can be written in

$$\boldsymbol{E} = -\nabla\Phi \tag{4}$$

$$\nabla\cdot\sigma\boldsymbol{E} + \frac{\partial\rho}{\partial t} = 0 \tag{5}$$

$$\nabla\cdot\epsilon\boldsymbol{E} = \rho \tag{6}$$

The complex potential of AC electric field of interest can be expressed as

$$\Phi = \Re(\phi e^{i\omega t}) \tag{7}$$

If we consider problems without phase variation, the modula of the complex potential ϕ satisfies Laplace equation

$$\nabla^2\phi = 0 \tag{8}$$

2.2 Singular behavior of electric potential

We first investigate the behavior of the electric field near a singular point/line. The region of interest consists of piecewise smooth surfaces (2D) or curves (3D). The boundary condition will either be potential, flux or mixed type. There are two possible types of singularities: geometric singularity arising from discontinuity of normal vector of the surface (curve) and flux singularity arising from change in the nature of the homogeneous operator specifying the boundary condition. In this paper, we focus our attention on two dimensional problem. Consider electric potential distribution in a wedge region $0 \le \theta < \theta_0$, $r > 0$ subject to the following boundary condition:

$$\phi(r, \theta = 0) = 0 \tag{9}$$

$$\frac{1}{r}\frac{\partial\phi(r,\theta_0)}{\partial\theta} = f(r) \tag{10}$$

in which K is a constant. The asymptotic solution of the potential close to $r = 0$ can be obtained from Mellin transform

$$\begin{aligned}
\phi &= Kr^{\pi/2\theta_0}\sin\frac{\pi\theta}{2\theta_0} + O(r^{3\pi/2\theta_0}) \\
\frac{1}{r}\frac{\partial\phi}{\partial\theta} &= K\frac{\pi}{2\theta_0}r^{(\pi-2\theta_0)/2\theta_0}\cos\frac{\pi\theta}{2\theta_0} + O(r^{3\pi-\theta_0)/2\theta_0}) \\
\frac{\partial\phi}{\partial r} &= K\frac{\pi}{2\theta_0}r^{(\pi-2\theta_0)/2\theta_0}\sin\frac{\pi\theta}{2\theta_0} + O(r^{3\pi-\theta_0)/2\theta_0})
\end{aligned}$$

In particular, when electrodes are fabricated on a planar surface and $\theta_0 = \pi$, we find that the flux singular behavior near the edge of the electrode has a power $-1/2$. This corresponds to velocity singularity at the tip of a planar barrier in an invicid flow.

2.3 Exact solution of electric field and DEP

We now consider DEP levitation of particles in the vicinity of a parallel electrode array. The width of electrode stripe is d_1 and the spacing between adjacent electrodes is d_2. The configuration exhibits periodicity with period $2d$ defined by $d = (d_1 + d_2)/2$. We apply voltages V_0 and $-V_0$ to adjacent electrode pairs. For mathematical simplicity, let's first non-dimensionalize the Laplace equation by introducing the characteristic potential and length as

$$\phi_c = V_0, \qquad L = \frac{d}{\pi} \tag{11}$$

Due to geometric symmetry, only the region $0 < x < \pi$ needs to be solved. If we choose the origin at the center of electrode, the corresponding boundary conditions are

$$\phi = 1, \qquad y = 0, \quad 0 < x < c \tag{12}$$

$$\frac{\partial\phi}{\partial y} = 0, \qquad y = 0, \quad c < x < \pi \tag{13}$$

$$\frac{\partial\phi}{\partial x} = 0, \qquad x = 0 \tag{14}$$

$$\phi = 0, \qquad x = \pi \tag{15}$$

in which $c = \pi d_1/2d$. The general solution to Laplace equation satisfying boundary conditions at $x = 0$ and $x = \pi$ is given by

$$\phi(x,y) = \sum_{n=1}^{\infty}\frac{A_n}{\lambda_n}\cos\lambda_n x e^{-\lambda_n y} \tag{16}$$

where

$$\lambda_n = n - \frac{1}{2} \tag{17}$$

The boundary condition at $y = 0$ leads to dual series equations

$$\sum_{n=1}^{\infty}\lambda_n^{-1}A_n\cos\lambda_n x = 1, \qquad 0 < x < c \tag{18}$$

$$\sum_{n=1}^{\infty}A_n\cos\lambda_n x = 0, \qquad c < x < \pi \tag{19}$$

These equations can be solved by a standard method[4] for which A_n are given by

$$A_n = \frac{P_{n-1}(\cos c)}{K(\cos(\frac{c}{2})} \tag{20}$$

where P_n is Legendre polynomial of order n and K is complete elliptic function of the second kind. Potential distribu-

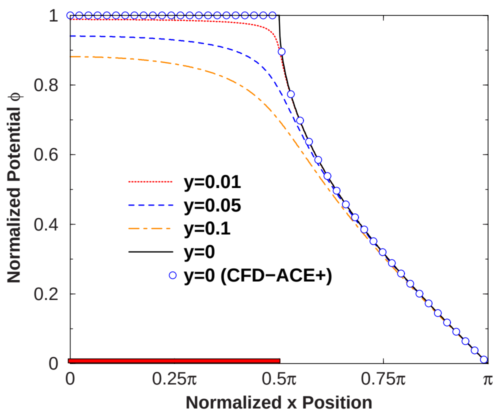

Figure 2: Potential distribution at various heights.

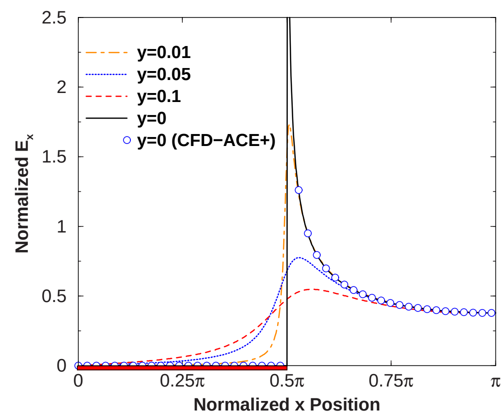

Figure 3: Lateral electric field at various heights.

tion at various heights above the electrode plane for $c = 0.5\pi$ is shown in figure 2. The components of electric field can be calculated by differentiating the series term by term as

$$E_x = -\frac{\partial \phi}{\partial x} = \sum_{n=1}^{\infty} \frac{P_{n-1}(\cos c)}{K(cos\frac{c}{2})} \sin \lambda_n x e^{-\lambda_n y} \tag{21}$$

$$E_y = -\frac{\partial \phi}{\partial x} = \sum_{n=1}^{\infty} \frac{P_{n-1}(\cos c)}{K(cos\frac{c}{2})} \cos \lambda_n x e^{-\lambda_n y} \tag{22}$$

with A_n given by (20). From asymptotic behavior of the Legendre polynomial for large value of order

$$P_n(x) \sim \sqrt{\frac{1}{n}} \cos x + O(n^{-3/2}) \tag{23}$$

we see that (21) and (22) converge for any x and y. In particular, the electric field at the electrode plane is given by

$$E_x = \begin{cases} 0 & 0 < x < c \\ \sum_{n=1}^{\infty} \frac{P_{n-1}(\cos c)}{K(cos\frac{c}{2})} \sin \lambda_n x & c < x < \pi \end{cases} \tag{24}$$

$$E_y = \begin{cases} \sum_{n=1}^{\infty} \frac{P_{n-1}(\cos c)}{K(cos\frac{c}{2})} \cos \lambda_n x & 0 < x < c \\ 0 & c < x < \pi \end{cases} \tag{25}$$

Figure 3 shows the x-component of electric field E_x normalized by $V_0\pi/d$ for four values of height. Note E_x as well as E_y (not shown) diverges at the edge of the electrode.

Once the components of the field are obtained, the square of electric field strength normalized by $V_0^2\pi^2/d^2$ is calculated as

$$E^2 = \frac{1}{K^2\left(cos\frac{c}{2}\right)} \left\{ \left[\sum_{n=1}^{\infty} P_{n-1}(\cos c) \sin \lambda_n x e^{-\lambda y} \right]^2 + \left[\sum_{n=1}^{\infty} P_{n-1}(\cos c) \cos \lambda_n x e^{-\lambda_n y} \right]^2 \right\} \tag{26}$$

We have thus obtained an analytic solution for the electric field based on exact boundary condition along the electrode plane. The associated DEP force on a spherical particle can now be calculated and applied to computer particle motion.

3 RESULTS AND DISCUSSION

Previous studies have assumed that the potential varies linearly between adjacent electrodes and the resulting lateral electric field E_x is a step function between adjacent electrodes. Figure 2 indicates non-linearity behavior of potential near the electrode edge. As a results, the local field exhibits square-root-type singularity which has not been predicted previously. In particular, when $d_1 = d_2$, i.e., $c = \pi/2$, from property of Lengdre function $P_n(0) = 0$ for odd n, we find that

$$E_x(x,\, y) = E_y(\pi - x,\, y). \tag{27}$$

The DEP force exerted on a polarized spherical particle of radius a with electric conductivity σ_p and permittivity ϵ_p suspended in another dielectric media $(\sigma_m,\, \epsilon_m)$ is calculated from (1) by substituting (21) and (22) and perform differentiation term by term. The resulting series expressions ∇E^2 converges for any x provided $y > 0$ and decays exponentially as the particle is far removed from the electrode plane. If we normalize the DEP by choosing characteristic force

$$f_0 = 2\pi^4 \epsilon_0 \epsilon_m a^3 \Re \left[\frac{\epsilon_p^* - \epsilon_m^*}{\epsilon_p^* + 2\epsilon_m^*} \right] \frac{V_0^2}{d^3} \tag{28}$$

then we have

$$\boldsymbol{f} = f_0 \nabla E^2 \tag{29}$$

We plotted dimensionless levitation force f_y and lateral drift force f_x in figure 4 for $c = \pi/2$. Equation (27) demonstrates that the E^2 and f_y are symmetric with respect to both $x = 0$ and $x = d_1/2$, while f_x is anti-symmetric.

At sufficient height above the electrode plane, we retain only the first term and obtain the approximate expression for E^2 as

$$E^2 \sim \frac{e^{-y}}{K^2\left(cos\frac{c}{2}\right)} \tag{30}$$

These results provide the fundamental governing equations for flow field fractionation devices which are widely used to separate or manipulate a variety of biological particles. The first order DEP force is independent of the lateral position

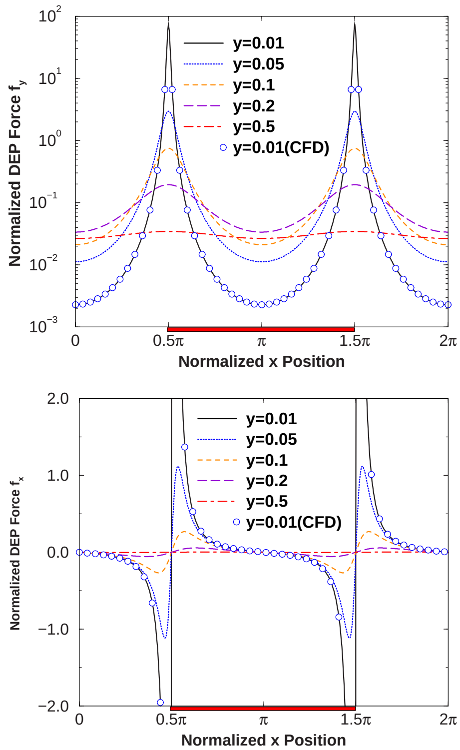

Figure 4: Normalized levitation and the lateral forces.

and directs particles away from the electrode plane for negative DEP. However, detailed numerical simulation indicates that particles will accumulate laterally when they are levitated and the cluster of particles accumulates at center of the electrodes or their gaps. This can be demonstrated by examining higher order terms for DEP force. By expanding equation (26), we obtain

$$E^2 = \frac{1}{\left[K\left(cos\frac{c}{2}\right)\right]^2}\left[e^{-y} + 2\cos c\cos x e^{-2y}\right. + \left.(\cos^2 c + 2\cos 2x P_2(\cos c))e^{-3y} + O(e^{-4y})\right]$$

This equation shows that there is a lateral force which is much smaller than the vertical force. It drives the particles toward the region right above the center of the electrode gaps when $c < \pi/2$ or toward the electrode themselves when $c > \pi/2$.

If the interactions between particles are neglected, we can simulate the evolution of particle ensemble by tracing particle trajectory individually. Particles that experience different DEP force will be levitated to different heights where gravity is balance by the DEP force. Particle collection is achieved by applying a lateral flow or travelling wave DEP force. In order to solve the dynamic equation for particle motion in a coupled manner with the fluid flow equations, we have developed a numerical model available in CFD-ACE+, CFDRC's commercial multi-physics simulation package. Simulation of particle evolutions for a mixture which contains three types of particle of different conductivities are shown in figure 5.

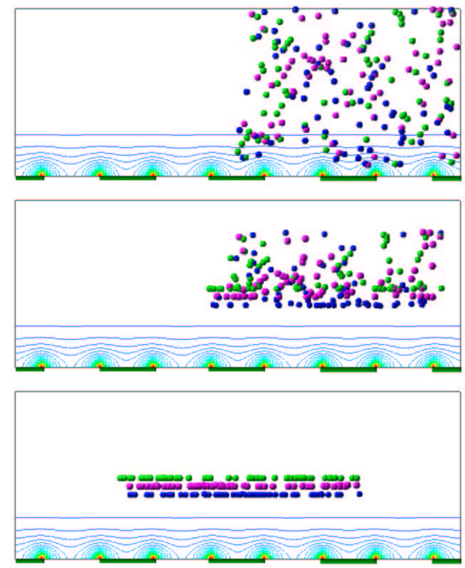

Figure 5: Particle evolution for a mixture containing three types of species at t=0, 1s, 5s. The contours show E^2. Different type of particles are shown in different color.

We have also compared the present analytic results for electric field with numerical solutions based on CFD-ACE+ and have found excellent agreement. Detailed computational approach and its application will be published elsewhere.

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