Improving Trajectory Piecewise-Linear Approach to Nonlinear Model Order Reduction for Micromachined Devices Using an Aggregated Projection Basis

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ABSTRACT

In this paper we present an improved algorithm for nonlinear model order reduction (MOR) based on the trajectory piecewise-linear method proposed in [6] which, as shown before, provides an effective and efficient strategy for automatically generating low-cost macromodels of highly nonlinear dynamical systems. The proposed extension consists of applying a more sophisticated projection basis, which merges multiple reduced order bases in Krylov subspaces generated at different linearization points. As a result we obtain a 'richer', aggregated reduced basis which, as shown on the example of a micromachined switch, enables us to improve the accuracy and further reduce the order of the macromodels for the considered highly nonlinear device.

Keywords: model order reduction, nonlinear dynamical systems, Krylov methods, piecewise-linear approach

1 INTRODUCTION

Recent progress in integrated circuit fabrication technology has enabled digital system designers to integrate analog circuitry and micromachined devices, but such mixed-technology microsystems are extremely difficult to design because of limited verification and optimization tools available. In particular, there are no generally effective techniques for automatic generation of reduced order system-level models from detailed physical descriptions of micromachined blocks. Most existing research on model order reduction (MOR) has focused on techniques for linear systems, and many of these approaches have not been effective when applied to the kind of nonlinear problems associated with micromachined devices. The existing nonlinear MOR techniques, based on linear or quadratic reduction (cf. [1], [2], [4], [7]), are primarily useful for weakly nonlinear systems. In order to overcome this weak nonlinearity limitation, a trajectory piecewise-linear reduction technique (TPWL) was developed [6]. The initial effectiveness of the TPWL technique suggests that the method has promise, and in this paper we investigate ways to improve TPWL's accuracy and efficiency.

We start in the next section by describing a trajectory piecewise-linear nonlinear model order reduction technique. Section 3 shows computational results for a micromachined switch example, verifying the discussed numerical method, and Section 4 presents our conclusions.

2 TRAJECTORY PIECEWISE-LINEAR MODEL ORDER REDUCTION

In this section we summarize the trajectory piecewise-linear MOR technique, discussed in more detail in [6], and then describe the new approach for generating the basis for the reduced order model. We start with presenting the piecewise-linear representation of nonlinear dynamical systems, followed by describing the method of generating the piecewise-linear model. Finally, we propose a new projection scheme using an aggregated reduced order basis.

2.1 Piecewise-linear representation

The main drawback of Taylor-series based MOR methods for nonlinear systems is that these methods typically expand the nonlinear operator about a single state x_0(cf. [1], [2], [4], [7]) and therefore the generated models are only accurate locally. In [6], it was suggested to use a collection of expansions around states visited by a given training trajectory. To present the approach more concretely, consider a general nonlinear system:

$$\frac{dx}{dt} = f(x) + Bu, \qquad (1)$$

where x is an N-dimensional state vector, u is an M-dimensional input vector (typically $M << N$), and B is an $N \times M$ matrix. A set of s linearizations of f about the states $x_0, \ldots, x_{s-1}$ could be used to generate linear systems of the form:

$$\frac{dx}{dt} = f(x_i) + A_i(x - x_i) + Bu$$

where $A_i = \frac{\partial f(x_i)}{\partial x}$ is the Jacobian of f evaluated at x_i. In order to represent a system that transitions smoothly from one linearization to the next, consider a weighted combination of the above models:

$$\frac{dx}{dt} = \sum_{i=0}^{s-1} \tilde{w}_i(x) f(x_i) + \sum_{i=0}^{s-1} \tilde{w}_i(x) A_i(x - x_i) + Bu \qquad (2)$$

where $\tilde{w}_i(x)$ are weights depending on state x. Assuming a convex combination of models implies that the weights are positive and that $\sum_{i=0}^{s-1} \tilde{w}_i(x) = 1$. A simple weighting scheme is to use $w_i(x) = (\exp(\|x - x_i\|))^{-25} / \sum_{i=0}^{s-1}(\exp(\|x - x_i\|))^{-25}$.

Given a q-th order ($q << N$) projection basis defined by a $N \times q$ matrix V (cf. Section 2.3), and using the change of variables $x = Vz$, yields a representation of system (2):

$$\begin{cases} \frac{dz}{dt} = (A_r \cdot w(z)^T)z + \gamma \cdot w(z)^T + B_r u \\ y = C_r z \end{cases} \quad (3)$$

where $B_r = V^T B$, $C_r = C^T V$, $A_r = [A_{0r} A_{1r} \dots A_{(s-1)r}]$ and $A_{ir} = V^T A_i V$, $\gamma = [\gamma_0 \dots \gamma_{s-1}] =$

$$[V^T(f(x_0) - A_0 x_0), \dots, V^T(f(x_{s-1}) - A_{s-1}x_{s-1})]$$

and $[z_0, z_1, \dots, z_{s-1}]$ are representations of linearization points $x_0, \dots, x_{s-1}$ in the reduced basis:

$$[z_0, z_1, \dots, z_{s-1}] = [V^T x_0, V^T x_1, \dots, V^T x_{s-1}]$$

2.2 Generation of piecewise-linear models

One of the key issues which need to be addressed in the above piecewise-linear approach is how to generate model (2), or more specifically, how to select linearization points x_i. In the approach proposed in [6] a 'training' trajectory in the state space of the nonlinear system is generated by simulating the response of the nonlinear system to a given 'training' input signal. The linearization points x_i are taken from the training trajectory. In this way the number of linearized models is kept small. The trade-off is that the piecewise-linear model (2) is input-specific, and consequently it approximates well the initial nonlinear system for system trajectories located 'close enough' to the training trajectory. In order to reduce the cost of generating the piecewise-linear model a fast approximate simulation method, employing linearized reduced order models generated 'on the fly', is also proposed (cf. [6] for details).

2.3 Generating the aggregated reduced order projection basis

The last component of the proposed MOR algorithm is to determine the projection matrix V. In [6] the projection matrix was constructed using a Krylov subspace based on a linearization about the initial state x_0. Instead, consider replacing the above approach with the following three-step procedure. First, at each of the linearization points x_i, generate a reduced order basis in a suitable Krylov space, corresponding to a linearized model generated at x_i. Second, form the union of all the bases, and third, reduce the set using the singular value decomposition. The final reduced basis can then be used to generate a collection of reduced order linearized models (cf. (3)), such that each of them will have a transfer function which matches (up to a certain number of moments) the transfer function of the corresponding nonreduced linearized model.

Since we generate a 'richer' basis we expect that it will more accurately approximate the initial state space. One may argue that the above method may generate models of significantly larger order than the simple algorithm. In fact the situation is the opposite. As shown in the next section, the extended algorithm has potential to generate suitable, accurate reduced bases with smaller order than the simple algorithm using a single linearization around the initial state.

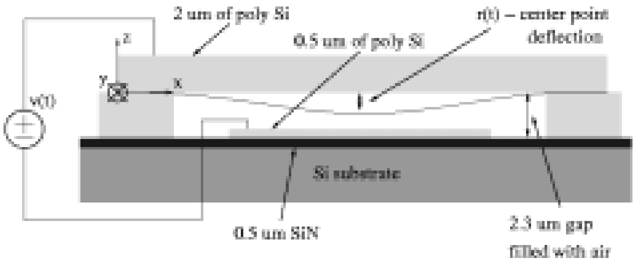

Figure 1: Micromachined switch (following Hung et al. [3]).

3 A MICROMACHINED SWITCH EXAMPLE

In this section we present simulation results using a micromachined switch example shown in Figure 1. The dynamic behavior of the switch can be determined by solving a coupled system of partial differential equations which models the interacting electrostatic, elastostatic and fluidic forces in the switch [3]. Spatial discretization of the coupled system of partial differential equations and a proper choice of state space generate a nonlinear dynamical system in form (1) (cf. [6]), and for the examples considered below we used a spatial discretization that generated an $N = 880$ order system. Note also that the tests were performed using an implementation of the algorithms in Matlab running on a Linux workstation with a Pentium III Xeon processor.

The first group of tests aimed at validating the generated piecewise-linear reduced order algorithms. More specifically, we needed to find out whether a given reduced model correctly approximated the initial nonlinear system for the inputs which were *different* that the 'training' input used to generate that model. The result of one of such tests is shown in Figure 2. In this test the piecewise-linear model of order $q = 26$ was generated for the 9-volt step training input. Then, we tested the model with a cosinusoidal input voltage with 7-volt amplitude. One may note that that the obtained transient matches almost perfectly the reference result. The graph also provides a comparison of the piecewise-linear model with the linear and quadratic reduced order models of the same order and using the same projection basis (cf. [2], [4], [7]). It is clear that the proposed TPWL model provides a significantly better accuracy than the other two models.

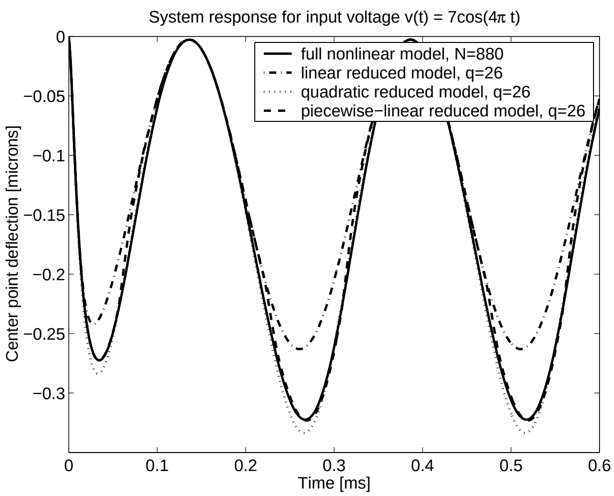

Figure 2: Comparison of system response computed using the linear, quadratic and piecewise-linear reduced order models. Input signal $u(t) = (7\cos(4\pi t))^2$.

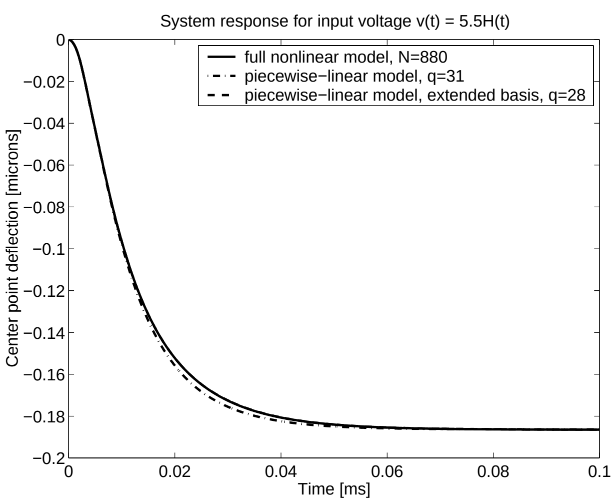

Figure 3: Comparison of system response computed using the piecewise-linear models with a simple and extended algorithm of generating the reduced order basis.

Next, we compared the two algorithms for generating the projection basis – the simple one (used in [6]) and the extended one, proposed in this paper. Figure 3 shows the deflection of the center of the micromachined fixed-fixed beam computed using the two considered methods. In both MOR methods the 5.5-volt step input voltage

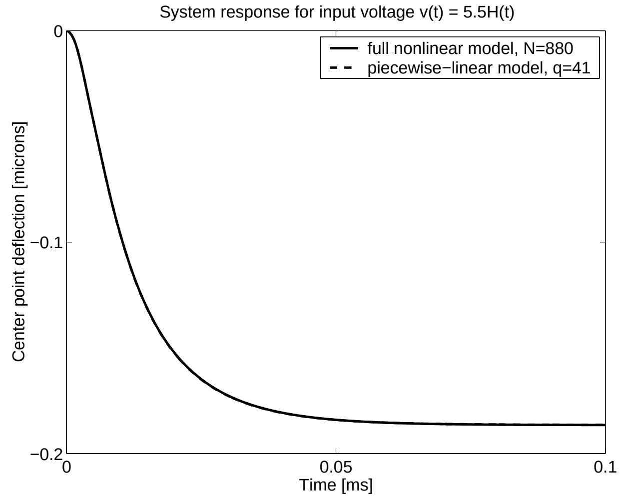

Figure 4: Comparison of system response computed using the full nonlinear simulator and the piecewise-linear reduced order model with a simple algorithm of generating the reduced order basis. The model of order $q = 41$ was generated for the 5.5-volt step input voltage.

was used as a 'training' input and the number of linearization points equaled 6. For the simple algorithm the order of the reduced model equaled $q = 31$. In the extended algorithm, a basis of order 7 was generated at each of the linearization points. Then the size of the aggregate basis V_{agg} was reduced from 42= $6 \cdot 7$ to 28 using the SVD algorithm. One may note that the piecewise-linear model of order $q = 28$, generated with the aggregated reduced order basis gives significantly more accurate results than the piecewise-linear model generated with a simple basis. (On the graph, the dashed line overlaps perfectly with the solid line.) In order to obtain the desired accuracy with the MOR method using the simple basis generation algorithm, the order of the basis has to be increased to $q = 41$ in the considered case (cf. Figure 4).

Figure 5 shows the simulated pull-in effect for the micromachined beam example. Again in this case the algorithm employing the extended algorithm to generate the reduced basis provides the best accuracy among the considered MOR techniques. The extended algorithm also generates a model with the lowest order. One should note that in the extended algorithm we generated a collection of very low order bases at different linearization points rather than a larger basis at a single linearization point, as in the initial approach. As shown by the presented results this may lead to a model with a smaller order, which is faster to simulate. The trade-off is that the extended basis generation algorithm is computationally more expensive.

We have also used the same reduced order piecewise-linear model of order $q = 26$ to find the sinusoidal steady state of the micromachined switch. To this end we used

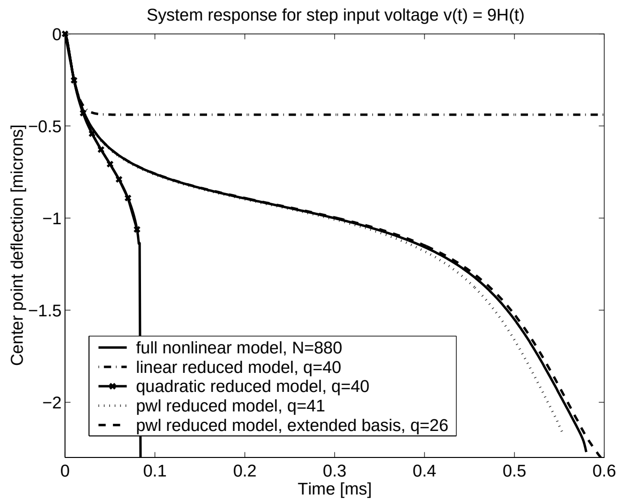

Figure 5: Comparison of system response computed with linear, quadratic and piecewise-linear reduced order models to the step input voltage $u(t) \equiv 9^2$ $(t > 0)$.

Table 1: Comparison of the subsequent harmonics of the sinusoidal steady state, computed using a full order nonlinear model and the reduced order piecewise linear (TPWL) model. The input signal to the system was $u(t) = (9\cos(\pi t))^2$. The piecewise-linear model of order q=26 was generated for the 9-volt step input voltage.

Harmo -nics	Full nonlinear model	Reduced order TPWL model	Error [%]
dc	1.9587	1.9526	0.4
1st	-0.1967+0.0351i	-0.1935+0.0352i	1.6
2nd	-0.0289+0.0283i	-0.0263+0.0263i	8.2
3rd	0.0004+0.0143i	-0.0017+0.0155i	17.6

the shooting method with our reduced model and then computed the subsequent harmonics of the resulting periodic signal. Table 1 compares the first three harmonics computed with the reduced order model and the full nonlinear model. One may note good agreement in the results, which indicates that our technique may be effectively applied e.g. to analyze harmonic distortion of nonlinear systems.

Finally, some performance tests were made for the discussed method. Table 2 shows model generation times as well as simulation times with reduced order models for linear, quadratic and piecewise-linear MOR techniques. One may note that the piecewise-linear reduced order model, although more expensive to generate, provides a comparable performance as the linear reduced order model. It is also apparent from the table that the quadratic MOR is significantly more expensive than the proposed piecewise-linear MOR.

Table 2: Comparison of the times of generation of the reduced model and reduced order simulations for the linear, quadratic and piecewise-linear MOR techniques ($N = 880$, $q = 26$).

MOR method	Model generation time [s]	Simulation time [s]
linear MOR	14.8	1.0
quadratic MOR	3712.3	33.3
TPWL MOR	293.5	8.3

4 CONCLUSIONS

In this paper we have proposed an algorithm for generating the reduced order projection basis, which aggregates a number of bases computed at different states of the system. The results of numerical tests indicate that application of this new scheme allows one to further increase the accuracy and reduce the order of macromodels extracted with the trajectory piecewise-linear MOR algorithm. Furthermore, the extended algorithm does not increase excessively the reduced model generation time, therefore providing a good alternative for the basic method presented in [6].

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