RF Measurement Techniques for Micro-cantilever Characterization and Application

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ABSTRACT

The reflection coefficient (S_{11}) of a one-port device is most sensitive to its input impedance if a matched condition is obtained. This implies two methods to measure the deflection-induced impedance change for a 1-port device such as micro-cantilever in our case: 1) selecting an operating frequency at which the input impedance is closest to the characteristic impedance of the measurement system; 2) inserting a matching network to obtain a matched condition at a preferred operating frequency.

Using the first method, the capacitance change can be measured at various DC bias voltages. Using the second method, the input impedance of a DUT is matched to 50 *ohms* at a desired frequency. Because of this matched condition, the reflection coefficient is very sensitive to DC bias on a DUT. A 20 *dB* change of $|S_{11}|$ in 2 *Volts* range without amplification was obtained. Both methods are quite simple and can be very useful for characterizing the capacitance change for a capacitive sensor and/or sensor application.

Keywords: MEMS, pull-in voltage, microstrip line, MUMPs

1 INTRODUCTION

On-chip circuitry is usually desired for a microelectromechanical capacitive sensor to achieve better performance and better economic value. However, for some certain micromachining processes (e.g. LIGA), the integration of electronic circuit and mechanical parts on a single chip is difficult or sometimes impossible. Also, for some special applications, it is sometimes necessary to optimize both the mechanical properties of a sensor and the electronic circuitry. Without on-chip circuitry, micromechanical sensors under development are mostly characterized by optical methods such as laser vibrometer, interferometer, stroboscope, etc. For practical use, the electronic readout is a more favored method; an off-chip circuit is then a valid solution to all the above cases.

However, one of the disadvantages of an off-chip circuit is the relatively large parasitics associated with the system. The desired output signal from a mechanical sensor is usually severely distorted and hard to be discriminate. To overcome this difficulty, an electromechanical amplitude modulation (EAM) technique has been reported [1-2]. Almost all the applications of this technique measure the current through the capacitive sensor and then convert it into voltage by a transimpedance amplifier. In this paper, will first analyze the sensitivity to the input impedance of a 1-port device and hence derive two simple measurement methods for characterizing the deflection-induced input impedance change. Because of a microstrip-line-like structure of a micro-cantilever, a microstrip line model will be used to calculate the capacitance changes at different dc biases, which are then compared with results from measurement.

2 ANALYSIS

The structure of a cantilever above a ground plane is very similar to an open-ended micro-strip transmission line. The dielectric material is now air or vacuum in a vacuum chamber. The equivalent circuit of a cantilever can be modeled as an open-ended micro-strip transmission line as shown in Figure 1. Each differential section contains a pair of shunt conductance *G* and capacitance *C* and a pair of series resistance *R* and inductance *L*. The distributed series resistance *R* per meter is _/*A*, where _ is the resistivity of the conductor and *A* is the cross section area. The distributed capacitance *C* per meter is _$_0$*w/d* plus a small capacitance due to fringing field if *w* >> d, where _$_0$ is the permittivity of air, *w* the width of the differential section, and *d* the gap between conductor and ground plane. The inductance *L* per meter is $1/(c^2 C)$, where *c* is the speed of light in air, and conductance *G* of air is zero. According to MUMPs [3] no. 41 Run Data, the resistivity of the Poly1 structure layer is $2.1\times10^{-3}\ ohm-cm$, which makes the series resistance *R* much larger than the inductance *L*. The transmission line model therefore degenerates to an *RC* circuit. As a cantilever is deflected, the gap is a function of position along the length and so is the capacitance C_i of each differential section in Figure 1. The series resistance R_i of each differential section is primarily the same as the cantilever deflects. Since a set of contact pads is necessary for on wafer measurements, the total input impedance is that looking into the input port of the equivalent circuit in shunt with an equivalent circuit of the contact pads, which is usually approximated by a large capacitor. This model can be used to calculate the deflection-induced input impedance change of a micro-cantilever.

Next we will study how to detect the reflection coefficient change due to the static deflection of a micro cantilever under electrostatic force induced by a dc bias voltage.

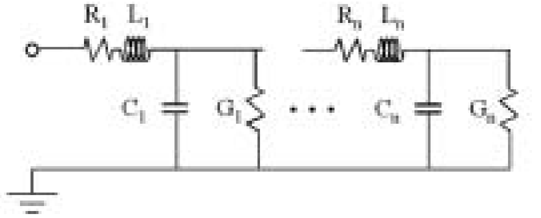

Figure 1: The transmission line model of a micro cantilever.

Consider a micro-cantilever as a voltage-controlled capacitor in parallel with contact pads (modeled as a capacitor). At a static state, the equivalent series capacitance of a micro-cantilever is C_s and the equivalent series resistance is R_s. At a certain bias voltage, an electrostatic force deflects the cantilever such that C_s changes with the bias voltage. The capacitance of contact pads is represented by C_p as shown in the schematic in Figure 2 and the equivalent circuit in Figure 3. Assume that R_s is frequency independent, and C_s of the micro cantilever and the parasitic capacitance are also assumed frequency independent. We can derive the equivalent input admittance Y_{in} ($1/Z_{in}$) and calculate the reflection coefficients in a frequency range with characterization impedance Z_0 (50 *ohms*) as follows

Input admittance $Y_{in} = j\omega C_0 + (R_s + 1/j\omega C_s)^{-1}$, (1)

Reflection coefficient $\Gamma = \dfrac{y_0 - Y_{in}}{y_0 + Y_{in}}$, (2)

where $y_0 = 1/Z_0$. Typical normalized absolute changes in magnitude and phase of reflection coefficient have their maxima at certain frequency since the input impedance at this frequency is closest to the characteristic impedance, 50 *ohms*, which makes the reflection coefficient a minimum and most sensitive to the change of the input impedance of the device under test. This example implies that if the input impedance can be matched to 50 *ohms* at a selected frequency, the reflection coefficient will be most sensitive to the change of the input impedance at that selected frequency.

The sensitivity of the reflection coefficient to the input admittance is defined as

$$S(\omega) = \left.\frac{\partial\Gamma/\partial Y_{in}}{\Gamma/Y_{in}}\right|_{\omega} . \qquad (3)$$

If we assume $\omega R_s C_s << 1$, which is usually the case, the input admittance can be approximated as

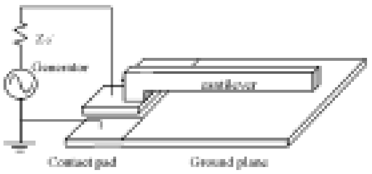

Figure 2: The setup for the measurement of reflection coefficient.

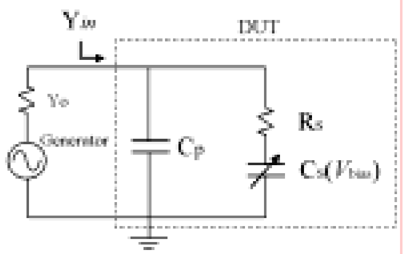

Figure 3: The equivalent circuit of the setup in Figure 2.

$$Y_{in} = j\omega C + G , \qquad (4)$$

where $C = C_0 + C_s$ and $G = (\omega C_s)^2 R_s$. Since Y_{in} is a function of ω, the sensitivity can be rewritten as

$$S(\omega) = \left.\frac{(\partial\Gamma/\partial\omega)(\partial\omega/\partial Y_{in})}{\Gamma/Y_{in}}\right|_{\omega} . \qquad (5)$$

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Note that $dG/d\omega = 2(\omega\, C_s R_s)^2/(\omega\, R_s) \to 0$, which means that G is a slow-varying function of ω, and we can just assume that G is a constant. Plugging equations (2) and (4) into equation (5), we have

$$S(\omega) = \left.\frac{y_0 G(1 + j\omega C/G)}{(G^2 - {y_0}^2)[1 - \dfrac{\omega^2}{{\omega_S}^2} + j\,\dfrac{\omega}{Q_S\omega_S}]}\right|_{\omega} , \qquad (6)$$

where $\omega_S = \sqrt{G^2 - {y_0}^2}/C$ and $Q_s = \sqrt{G^2 - {y_0}^2}/2G$. It is similar to the frequency response of a second order system with an additional zero in the numerator. As ω approaches ω_S, $|S|$ approaches its maximum value of

$\sqrt{[(1+(\omega_s C/G)^2]/(G^2-y_0{}^2)}$, and the value of G is closest to y_0.

To summarize from the discussions above, the RF measurement should be able to provide alternative ways to detect the deflection-induced input admittance change without on-chip circuitry by:

1st method: selecting an operating frequency at which the sensitivity is the maximum.
2nd method: matching the input impedance of device under test to the characteristic impedance Z_0 as closely as possible at any desired operating frequency.

3 MEASUREMENT RESULTS

3.1 DC measurement of a micro cantilever by the 1st method

In this section, we will use the 1st method to measure the DC transfer function of reflection coefficients to bias voltage applied on micro cantilevers fabricated by MUMPs processes [3]. The measurement setup is shown in Figure 4. A HP 8753C Network analyzer with S-parameters test set HP 85047A is capable of generating signals from 300 *kHz* to 6 *GHz* and calculating the reflection coefficient in this frequency range. A microwave coplanar probe mounted on a three-axis stage was used to apply RF signals to contact pads of the microstructures. Before measurement, a standard 1-port calibration procedure (Short-Open-Load) is necessary to move the reference plane to the tips of the probe. The frequency range was selected from 300 *kHz* to 3 *GHz* and the dc bias voltage was applied from a dc power supply to the bias input port on the rear panel of the network analyzer.

The measurement results of reflection coefficient of a sample cantilever are shown in Figure 5 and Figure 6. Both the magnitude and phase exhibit a decreasing trend as bias voltage increases. At the pull-in voltage of a micro-cantilever, both the phase and the magnitude change abruptly to very different values, which provide an alternative way of measuring the pull-in voltage of a micro-cantilever. The changes of the reflection coefficient are due solely to the deflection of the cantilever, which primarily changes the capacitance between itself and the ground plane. The corresponding capacitance changes of two sample cantilevers at various bias voltages are shown in Figure 6. From measurements, the transition of reflection coefficients from a higher limit close to one to a lower limit always occurs in the range from around $100\ MHz$ to $4\ GHz$. A suitable choice of the carrier frequency for the measurement is the midpoint in this range, which is about $1.7\ GHz$. The measurement ability of the Network analyzer gives the equivalent circuit of the input admittance of a cantilever. In order to have a better measurement, a smaller bandwidth for integration and a higher number for averaging of the reading should always be used. The measured capacitance changes at different bias voltages with respect to zero bias for two cantilevers are shown in Figure 6.

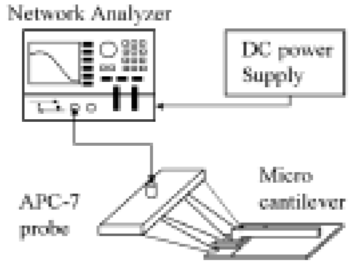


Figure 4: The measurement setup for the reflection coefficient at different DC voltages.

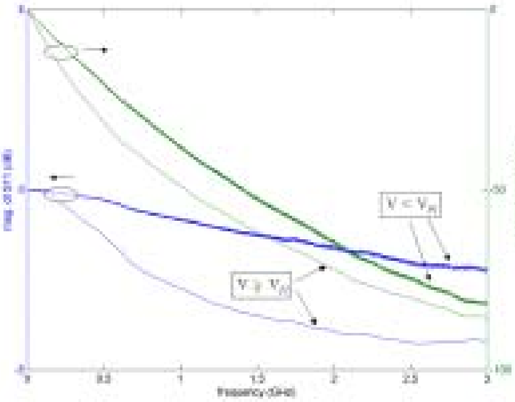


Figure 5: The magnitudes and the phases of measured S_{11} of a sample cantilever at various bias voltages.

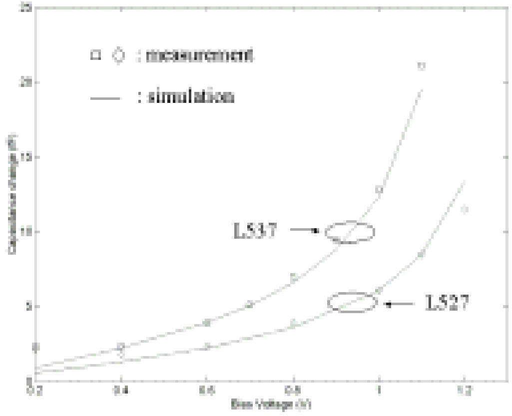


Figure 6: The capacitance change as a function of bias voltage for two different micro-cantilevers. It shows good agreement between measured and calculated data.

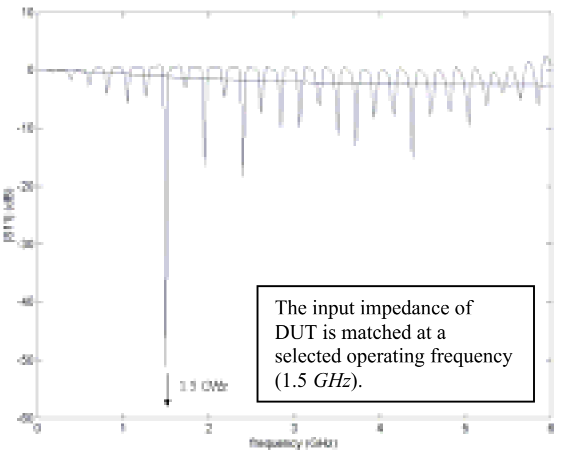
3.2 DC measurement of a micro cantilever by the 2nd method

The concept of the 2nd method is to match the input impedance (or admittance) to the characteristic impedance such that the reflection coefficient is most sensitive to the deflection-induced input impedance change. For a micro cantilever under external force, the gap between ground plane and itself will change along the length and hence the equivalent capacitance looking to the input port is changed. The measurement setup is the same as that in the 1st method with an additional tuner as a matching network to match the input impedance of the DUT to 50 *ohms*. A standard procedure for 1-port calibration was also followed to move the reference plane to the probe tip. In Figure 7, a micro-cantilever is matched at 1.5 *GHz*, at which the magnitude of the reflection coefficient is around –50 *dB*. A more careful adjustment of the tuner can make the matching network work better. However, the reflection coefficient will be smaller and comparable to the noise floor, which is around 55 *dB* as shown in Figure 8. A lower noise floor can be obtained if averaging is applied and a narrow bandwidth is selected at the selected operating frequency. As bias voltage increases, the deflection of the cantilever increases and so does the deviation from the matched condition at a selected operating frequency. Hence the reflection coefficient increases at that operating frequency.

4 CONCLUSIONS AND DISCUSSION

From the derivations of the measurement methods and the measurement results, it is clear that the reflection coefficient is close to 1 for the first method because the input impedance of DUT (associated with parasitics) is large. The reflection coefficient change due to the deflection of the micro-cantilever is small relative to that at zero-bias. Though there exists the most sensitive operating frequency from the sensitivity analysis of the reflection coefficient, it does not give a significantly large signal. This may be because of a wide bandwidth of the sensitivity curve, which makes the sensitivity a slow-varying function of frequency. To simplify our measurement, the capacitance change was measured at $1.7\,GHz$, which is roughly at the center of the transition region of reflection coefficients.

As to the second method, the reflection coefficient can be very small if the DUT is well matched. However, the reflection coefficient change due to the deflection of the micro cantilever is large, though the signal level is very low in a matched condition. This method is best to use on a conjugate-matched 2-port device where the transmitted signal (S_{21}) is a maximum. A weak signal buried in the noise floor is not of interest for a practical application. Fortunately, a DC bias voltage can slightly modify the matched condition and move the signal out of the noise floor as in Figure 8. Besides, a DC bias voltage also moves a micro cantilever to a more sensitive equilibrium position to the external force. As a force sensor, a feedback control circuit is required to keep the micro cantilever at the equilibrium position. The feedback voltage V_{out} is then proportional to the external force. A careful calibration is required to identify the relationship between V_{out} and the external force.



The input impedance of
DUT is matched at a
selected operating frequency
(1.5 *GHz*).

Figure 7: A sample micro-cantilever is matched to the characteristic impedance at 1.5 *GHz*. The smooth line is the magnitude of the reflection coefficient before matching, which does not change very much in the frequency range. All the reflection coefficient measurement can therefore be carry out at this particular frequency.

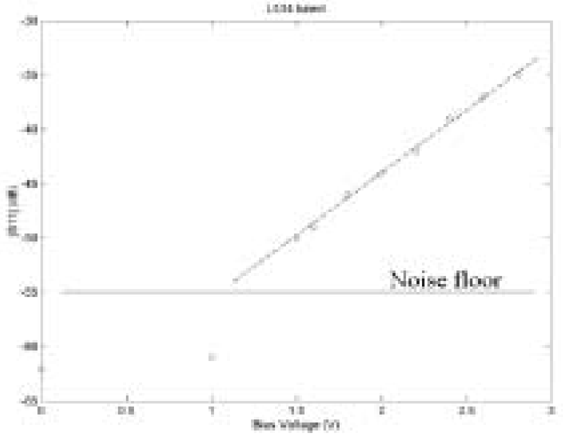


Figure 8: The magnitude of the reflection coefficient of a sample micro-cantilever as a function of the DC bias voltage. As voltage increases the deflection of the cantilever increases the deviation from the matched condition. Hence the reflection increases.

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