

# A Study of the Behavior of Magnetic Microactuators

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## ABSTRACT

The behavior of an idealized magnetic microactuator is modeled and analyzed. A two parameter mass-spring model is shown to exhibit a bifurcation from one to three steady states as the geometry of the device is altered. In addition we obtain solutions of the differential equation governing the motion of a forced elastic membrane and find similar phenomena. Stability analysis determines that in the case of three steady states that two are stable and one is unstable.

**Keywords:** Magnetic microactuator, modeling, bifurcations.

## 1 INTRODUCTION

Many different driving mechanisms are used for microscale devices, including electromagnetic, electrostatic, chemical, piezoelectric and thermopneumatic actuation [1]. Magnetic actuators have the ability to produce large forces, which allows large deflection. Electromagnetic actuation also has the advantage of contactless movement. In this paper we examine a simple magnetic microactuator, following the treatment of papers [2,3,4] of electrostatic actuation. Significant differences between magnetic actuators and electrostatic mechanisms are that dust particles are attracted to electrostatic devices and they often require large voltages to achieve significant actuation. Low-frequency magnetic fields do not attract dust particles and will pass through a non-magnetic materials. Also they can operate in a conductive fluid, which is a clear advantage in the field of microfluidics. The chief disadvantage is the unfavorable scaling of the force law at smaller dimensions.

The paper is organized as follows. In section 2 we formulate the governing equation for a simple mass-spring model of the magnetic forcing. In section 3 we analyze the model to demonstrate the existence of multiple steady-state solutions, and introduce the full membrane model in section 4. It is shown that there are either one, two or three steady solutions, depending upon the value of the parameters in the model. The stability of solutions is briefly addressed in section 5, and conclusions are discussed in section 6.

## 2 THE MODEL

We consider an idealized device consisting of a circular elastic membrane suspended above a rigid plate. A cylindrical magnet of volume  $V$  and magnetic remanence  $B_r$  is attached on top of the center of the membrane. A coil of wire with  $N$  turns of average radius  $R$  is located beneath the rigid plate at a distance  $l$  below the center of the magnet. The membrane is clamped firmly along its edges. This model is illustrated in figure 1.

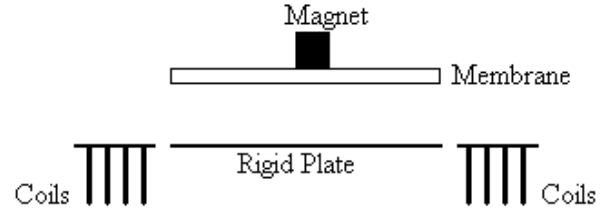


Figure 1: Model of magnetically actuated device.

When a direct current  $I$  flows through the wire the resulting magnetic field causes the plate to deflect by  $u$  from its equilibrium position at  $u=0$ . The magnetic force acting on the membrane is determined from the Biot-Savart law to be

$$F_{mag}(u) = \frac{\gamma(l-u)}{\left(1 + (l-u)^2/R^2\right)^{3/2}}, \quad (1)$$

where  $\gamma = \mu_0 N^2 I^2 R^2 / 4\pi$ . The restoring force of the membrane is assumed to take the standard mass-spring form

$$F_{res}(u) = -ku, \quad (2)$$

where  $k$  is the spring constant for the membrane.

We introduce the dimensionless variables  $w=(l-u)/l$ ,  $\alpha=l/R$  and  $\beta=\gamma/k$ . The parameter  $\alpha$  characterizes the

geometry of the model and  $\beta$  the ratio between magnetic and mechanical forces in the system. Equating the magnetic and restoring forces leads us to the following expression for the steady state deflection as

$$\alpha \dots \dots \dots (3)$$

Physically relevant solutions exist in the region  $0 < w < 1$ .

### 3 ANALYSIS

#### 3.1 Numerical Solutions

We numerically solve equation (3) for  $w$  as a function of  $\alpha$  and  $\beta$ . A bifurcation plot is shown in figure 2. For values of  $\alpha$  less than the critical value of  $\alpha^*$  only one solution exists in  $0 < w < 1$ . Beyond  $\alpha^*$  three solutions exist in  $0 < w < 1$  for a range of  $\beta$ . We have determined the first 8 digits of  $\alpha^*$  to be 1.6237976.

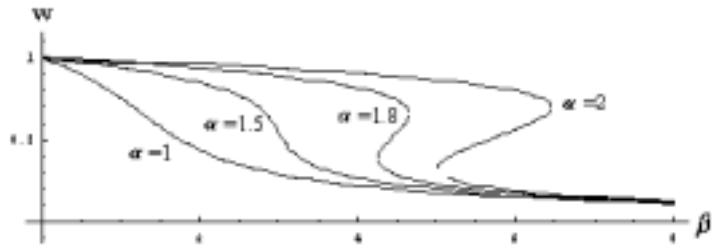


Figure 2: Bifurcation Diagram.

#### 3.2 Functional Analysis

We wish to examine the number of solutions of equation (3) in the region of  $0 < w < 1$ . This is equivalent to finding the roots of the polynomial

$$f(w) = (w-1)^2(1 + \alpha^2 w^2) - \beta^2 w^2. \quad (4)$$

We note that  $f(0)=1$  and that  $f(1)=-\beta^2$ . As  $\beta^2$  is always positive, we realize that there is at least one root within  $0 < w < 1$  (if  $\beta \neq 0$ ).

We define the functions  $f_1(w)=(w-1)^2(1+\alpha^2 w^2)^5$  and  $f_2(w)=\beta^2 w^2$ . The polynomial  $f_1(w)$  is of order twelve, but has only three roots:  $w=1$  has multiplicity two, and the complex conjugate roots  $w=\pm i/\alpha$  have multiplicity five.

The turning points of  $f_1(w)$  are  $w=1, \pm i/\alpha$  and  $(5\alpha \pm \sqrt{25\alpha^2 - 24})/12\alpha$ . We require  $w$  to be real, which means that we have either one or three turning points.

If  $\alpha < 2\sqrt{6}/5 = 0.979796$  we have one turning point at  $w=1$ . If  $\alpha \geq 2\sqrt{6}/5$  we have three turning points at  $w=1, (5\alpha \pm \sqrt{25\alpha^2 - 24})/12\alpha$ , we also find that there is one point of inflection between  $(5\alpha - \sqrt{25\alpha^2 - 24})/12\alpha$  and  $(5\alpha + \sqrt{25\alpha^2 - 24})/12\alpha$  as well as one between  $(5\alpha + \sqrt{25\alpha^2 - 24})/12\alpha$  and 1. Hence, we can determine the shape of the curve of  $f_1(w)$  to take one of the forms shown in figure 3 depending on the value of  $\alpha$ . The roots of  $f(w)$  are the points at which the curve of  $f_1(w)$  intersects with the parabolic curve of  $f_2(w)$ . These two curves can only intersect a maximum of three times, and so there are at most three possible steady solutions for the mass-spring model.

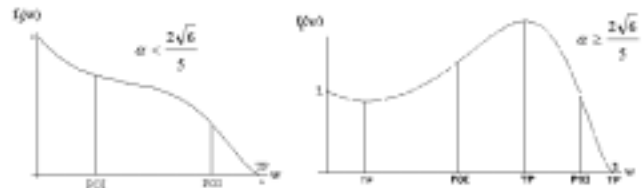


Figure 3: Shape of  $f_1(w)$ . TP is a turning point and POI is a point of inflection.

### 4 FULL MEMBRANE MODEL

The steady state equation for the displacement  $u(r)$  of a circularly symmetric membrane is given by

$$u_{,rr} + \frac{1}{r}u_{,r} = -\frac{f(u(r))}{T}, \quad (5)$$

where  $f(u(r))$  is the external force per unit area, which in this case is magnetic. Assuming that the force exerted by the magnet in equation (1) acts uniformly over the whole area of the membrane, we obtain the following expression for  $f$

$$\dots \dots \dots (6)$$

where  $a$  is the radius of the membrane and  $u_o$  is the displacement of the center of the membrane. We non-

dimensionalize with  $v=(l-u)/R$ ,  $s=r/a$  and  $\lambda=\gamma T\pi$ , where  $\lambda$  is a positive number. We also define  $v_o=(l-u_o)/R$ . Hence equation (5) becomes

$$v_{ss} + \frac{1}{s}v_s = \lambda \frac{v_o}{(1+v_o^2)^{3/2}}. \quad (7)$$

We must also satisfy the boundary conditions  $v_s(0)=0$  (as the membrane is symmetric about  $r=0$ ) and  $v(1)=\alpha$  (since the displacement  $u$  is zero at the clamped edge  $r=a$ ). Noting that equation (6) has the solution

$$v = \frac{\lambda v_o s}{1 + v_o^2 s^2}, \quad (8)$$

which satisfies the condition  $v_s(0)=0$  and has  $v(0)=v_o$ , we impose the condition

$$v(1) = \alpha, \quad (9)$$

to ensure the boundary condition  $v(1)=\alpha$  is satisfied. Writing  $v_o = \alpha w$  and rearranging equation (9) returns us to equation (3), with  $\beta=\lambda/4$ . The bifurcation analysis of section 3.1 then holds, i.e., for  $\alpha > \alpha^*$  there exist three possible steady states of the membrane for certain  $\lambda$  values. Figure 4 shows an example of the three steady states of the membrane corresponding to  $\lambda=100$  and  $\alpha=3$ . The maximal deflections correspond to  $v_o$  values of approximately 0.12, 0.82 and 2.62.

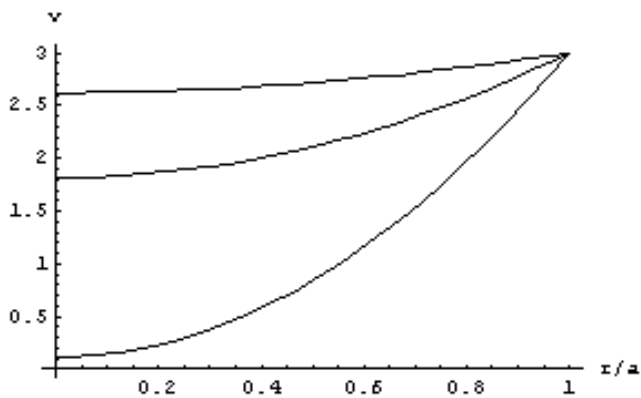


Figure 4: Multiple steady states for a cylindrically symmetric membrane. Here  $\lambda=100$  and  $\alpha=3$ .

## 5 STABILITY ANALYSIS

To examine the stability of the system we apply Newton's second law to the membrane

$$m \ddot{w} = -\frac{\partial V}{\partial w}, \quad (10)$$

where  $m$  is the mass of the membrane. Transforming to the variable  $w=(l-u)/l$  as in section 2 and defining the velocity of the magnet

$$\dot{w} = y, \quad (11)$$

$$\dot{y} = -\frac{\partial V}{\partial w} - cy, \quad (11)$$

where  $c$  is a constant. This equation describes the orbits of the system in phase space. By adding a damping term in equation (10) and numerically integrating we obtain solutions such as those indicated in figure 5. This shows one unstable and two stable equilibrium points.

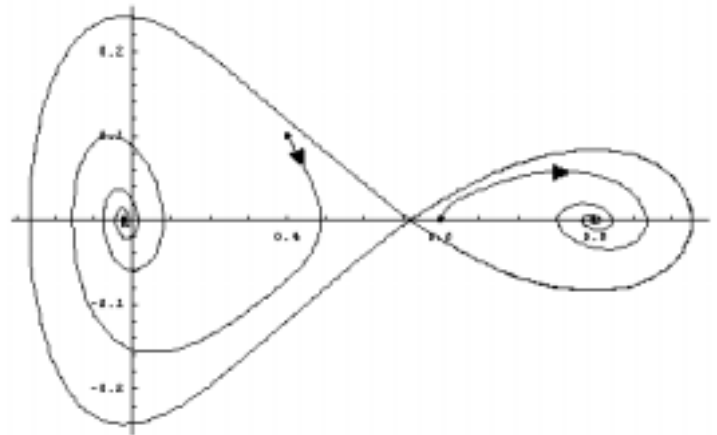


Figure 5: An example of a stability diagram showing two stable and one unstable steady states.

## 6 CONCLUSION

We have presented an analysis of a magnetic microactuator similar to that discussed in [1]. Using a simple mass-spring model we showed that up to three steady states may exist. Similar bifurcation behavior was

shown to exist in a more sophisticated membrane model. The analysis reveals that the geometry of the membrane is highly influential in the bifurcation behavior. The stability of solutions was examined and revealed the presence of either one stable solution or two stable and one unstable solutions. A similar model was developed to describe experimental results in [1]; however in that work a fixed geometry was examined and only one steady state was reported.

The bifurcation analysis for an electrostatic micro-actuator has been addressed in [2,3,4]: it was found that zero to two steady solutions may exist, only one of which is stable. Thus magnetic actuation is shown to have qualitatively different behavior to electrostatic actuation. By constructing a device of a suitable geometry it may be possible to observe hysteresis effects by switching between stable steady states, permitting the development of novel MEMS applications.

## 7 ACKNOWLEDGEMENTS

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