

# Simulation Based Design of O<sub>2</sub> MEMS Sensor

Z. Andjelic, P. Krippner, A. Vogel  
 ABB Corporate Research Center, Ladenburg, Germany  
 e-mail: [zoran.andjelic@de.abb.com](mailto:zoran.andjelic@de.abb.com)

## ABSTRACT

The paper presents the methodology for the Simulation Based Design (SBD) of MEMS sensors influenced by mechanical and electromagnetic phenomena. The complexity of the problems mostly asks for both the efficient 3D numerical technologies and the sophisticated tools to capture the physical phenomena appearing in the operational conditions. In this paper we present the application of the Multipole Based Integral Technology (MBIT) for the simulation of MEMS-based paramagnetic oxygen sensors. A special procedure for the calculation of the forces caused by the molecular pressure is also presented.

*Keywords:* BEM, Magnetostatic, MBIT, Integral Methods, Sensors

## PROBLEM DESCRIPTION

The traditional design of magneto-mechanical MEMS sensors like, for example, paramagnetic O<sub>2</sub> sensors, assumes the magnetic circuit (excited either by permanent magnets or by current-carrying coils) and movable indicating organ. The working principal of a paramagnetic O<sub>2</sub> sensor is based on the fact that in the presence of the magnetic field, the paramagnetic O<sub>2</sub> molecules tend to concentrate in the area of higher magnetic flux density. An eccentrically positioned glass-made probe immersed in such an environment exhibits then the different spatial molecular pressures. This pressure difference results further in a force, i.e. torque that causes the probe rotation. To keep the probe in the initial position an opposite force is generated by current-

carrying filaments attached to the probe surface. Thus, by measuring the current required to keep the probe in the initial position, one can measure the content of O<sub>2</sub> in the surrounding media.

## MBIT

MBIT [2], [3], [4] is applied to the Boundary Element Method with integral formulations of type

$$\varphi(x) = \sum_e \int_{e(y)} k(x,y) \sigma(y) dy \quad (1)$$

where  $k(x,y)$  for the present type of calculation is of a form  $\frac{\vec{r} \cdot \vec{n}}{r^3}$ . Here  $\vec{r}$  is the distance between collocation point  $x$  and integration point  $y$  and  $\vec{n}$  is a normal vector in the collocation point. MBIT is used to approximate the kernel  $k(x,y)$  for large relative distances  $r$ , so called ‘far field’ approximation.. For the ‘near-field’ regions with small  $r$  the standard BEM is applied. Using series Taylor series expansion or Spherical Harmonics the above kernel can be, in the case of ‘far-field’, replaced by:

$$\varphi(x) = \sum_{\substack{\text{cluster} \\ \text{center } c}} X_\mu(x; c_n) \kappa_{\mu\nu}(c_e, c_n) \bullet \sum_e \int_y Y_\nu(y; c_e) \sigma(y) dy \quad (2)$$

Here  $c_n$  denotes the cluster centers of nodes/vertices and  $c_e$  the cluster centers of elements.

$$\begin{aligned}
k(x, y) &\approx k_{MBIT}(x, y; c_e, c_n) \\
&= \sum_{\mu, \nu} \kappa_{\mu\nu}(c_e, c_n) X_\mu(x; c_n) Y_\nu(y; c_e)
\end{aligned} \quad (3)$$

The terms  $\kappa_{\mu\nu}$ ,  $X_\mu$  and  $Y_\nu$  in Spherical Harmonics have a form:

$$\begin{aligned}
\kappa_{\mu\nu}(c_e, c_n) &:= \kappa_{\mu+\nu}(c_e, c_n) := \\
&\frac{1}{4\pi C_{\mu+1}^{\mu+1} |c_e - c_n|^{\mu+1}} Y_{\mu+1}^{\mu+1} \left( \frac{c_e - c_n}{|c_e - c_n|} \right)
\end{aligned}$$

$$X_\mu(x, c_n) := C_{\mu+1}^{\mu+1} |x - c_n|^{\mu+1} Y_{\mu+1}^{-\mu-1} \left( \frac{x - c_n}{|x - c_n|} \right)$$

$$C_l^p := \frac{1}{\sqrt{(l-p)!(l+p)!}}$$

$$Y_l^p(x) := P_l^{|p|}(\cos\theta) e^{ip\phi}$$

Here  $\kappa(c_n, c_e)$  is a correlation matrix between element and node cluster trees. Finally we can evaluate the complete ‘far field’ integral by multiplying and summing up all pre-calculated values of the correlation function  $\kappa$ , of  $X$  and  $Y$ . We only have to evaluate the  $X(x, c_n) \kappa(c_n, c_e) Y(y, c_e) \sigma(y)$  for every cluster center pair  $(c_n, c_e)$  and charge density  $\sigma(y)$ .

### FORCES ON A GLASS-MADE PROBE

Above described technique is applied to find a magnetic field distribution in the sensor structures given in Fig. 1. The force acting on the glass-made probe is caused by the inhomogeneity of the molecular pressure acting on the probe surfaces. The spatial energy density for such problem can be calculated as:

$$W_m = \int_0^B H dB \quad (3)$$

The energy in the small volume  $dV$  is than

$$dW_m = dV \int_0^B H dB \quad (4)$$

The energy change due to movement for small increment  $dr$  is than

$$\frac{dW_m}{dr} = 0.5\mu_0 \Delta\chi dV \frac{dH^2}{dr} \quad (5)$$

Here  $\mu_0$  is the magnetic permeability of the air and  $\Delta\chi$  is a difference in the magnetic susceptibility. This results further in a force acting on the probe surface S:

$$\vec{F} = -0.5 \int \mu_0 \Delta\chi H^2 d\vec{S} \quad (6)$$

Here  $H$  is a module of a magnetic field  $\vec{H}$  calculated as:

$$\begin{aligned}
\vec{H}(x) &= \frac{1}{4\pi} \int_S \sigma(x) \frac{\vec{r}_{x\zeta}}{r_{x\zeta}^3} dS_x + \\
&\frac{1}{4\pi} \int_V \rho(N) \frac{\vec{r}_{xN}}{r_{xN}^3} dV_N + \vec{H}^0(x)
\end{aligned} \quad (7)$$

In Eq. (7)  $\sigma(i)$  is the magnetic surface charge density,  $\rho(x)$  is the magnetic volume charge density,  $\vec{n}$  is the normal vector in the calculation node  $x$ ,  $r = |x - \zeta|$  and  $\vec{H}_n^0$  is the normal component of the known excitation field caused by the permanent magnets. More about the procedure for the calculation of static magnetic field in the non-linear materials using BEM can be found in [1].

### SOME RESULTS

The calculation is performed on the simplified model of an O<sub>2</sub> sensor. It consists of four permanent magnets serially oriented to produce maximal flux density in the gaps between the steel poles, Fig. 1. **Fig. 2** shows the calculated flux density distribution acting on the O<sub>2</sub> molecules. The force on a glass-made probe is calculated using Eq. (6), **Fig. 3**.

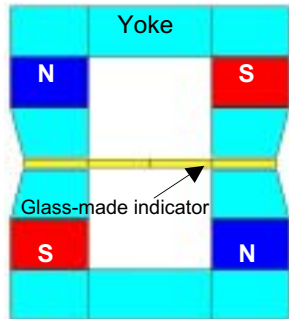


Fig. 1: Model of the MEMS O<sub>2</sub> sensor with four permanent magnets connected in 'series' to produce maximal flux density around the glass-made probe

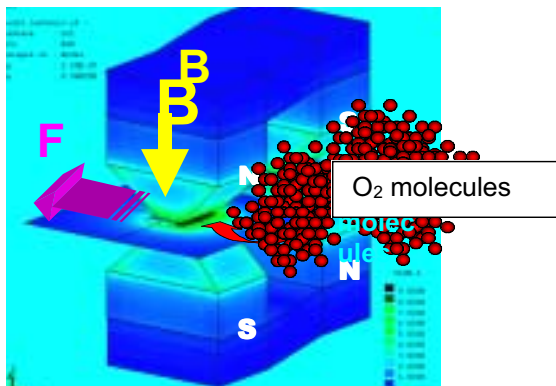


Fig. 2: Calculated flux density distribution on the magnetic structures. The paramagnetic O<sub>2</sub> molecules tend to concentrate in the regions with maximal flux density.

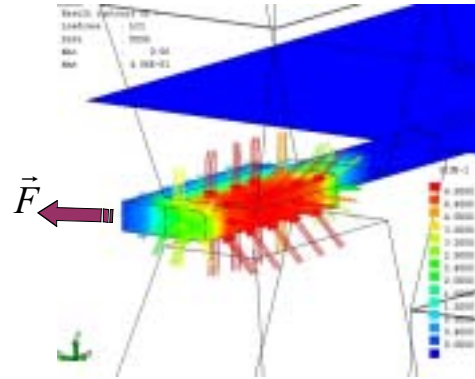


Fig. 3: The force distribution on the glass-made probe is caused by the inhomogeneity of the molecular pressure acting on the probe surfaces

### BEM vs. MBIT: SOME COMPARISONS

The above analysis is done using our 3D-simulation software POLOPT. Both the standard BEM and MBIT are applied for electromagnetic field analysis. The diagrams in Fig. 4 show the difference in the memory requirements between these two. The MBIT calculation is performed for two different values of parameter  $\eta$  (0.5; 0.9) that represents the ration between "near field" and "far field".

Also, the calculations are repeated for different expansions orders  $m=3,4,5$  and 6.

Fig. 5 shows the comparison in CPU between standard BEM and MBIT. Application of MBIT has enabled more than 10 times faster simulation using 10 times less memory simultaneously.

For the present test it can be seen that the increasing of the expansion order  $m$  leads to the increasing of the memory, Fig. 4, but does not lead to the increasing of the CPU, Fig. 5.

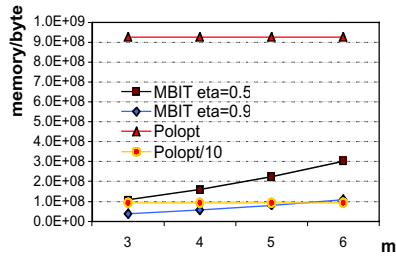


Fig. 4: Memory requirements for standard BEM (POLOPT) and MBIT for different “distance” parameter  $\eta$  and different expansion orders  $m$ .

Finally, we can say that application of MBIT requires more than 10 time less memory in comparison with standard BEM. The reason is that when using MBIT it is necessary to store only “near-field” matrix coefficients. This results in memory reduction from  $O(n^2)$  (standard BEM) to  $O(n \log^4 n)$  for MBIT.

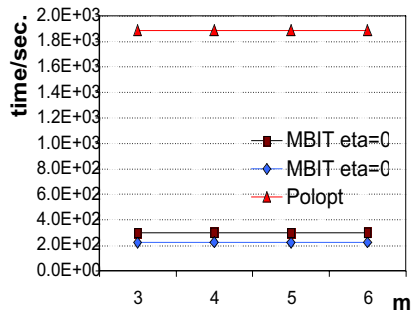


Fig. 5: CPU time measurement: BEM versus MBIT

## CONCLUSION

The paper presents a procedure for a simulation of the paramagnetic  $O_2$  MEMS sensors. Applied MBIT technique enables efficient treatment of fully 3D geometry. Application of MBIT has enabled more than 10 times faster simulation using 10 times less memory simultaneously.

A special procedure for analysis of the forces caused by a molecular pressure on a glass-made probe immersed into magnetic field is presented.

## REFERENCES:

- [1] B. Krstajic, Z. Andjelic, S. Milojkovic, S. Babic, S. Salon: “Nonlinear 3DMagnetostatic Field Calculation by the Integral Equation Method with Surface and Volume Magnetic Charges”, IEEE Transaction on Magnetics, vol.28, NO.2, March 1992
- [2] Greengard, L.:”The Rapid Evaluation of Potential Fields in Particle Methods”,MIT Press, Cambridge Ma., 1988
- [3] Lage, C.:”Softwareentwicklung zur Randelementmethode:Analyse und Entwurf effizienter Techniken”; Dissertation Christian-Albrechts-Universität Kiel, 1995
- [4] Fischer, U.: “MBIT-POLOPT”, ABB internal Report , C.01.T.54, July 2000.