Implementation of a nonlinear control law for the magnetostrictive beam vibrations by an amplifier of current to current

G. Senouci Bereksi

Center of Advanced European Studies And Research
Friedensplatz 16, Bonn 53111, Germany, bereksi@caesar.de

ABSTRACT

In this work, we start by giving a weak formulation of an amplifier of current to current network model and some estimations for the electrical current using graph theory. These estimations are interpreted in functional analysis to ensure the existence and uniqueness of the solution of the electrical network model. A coupling model including a magnetostrictive beam and an amplifier of current to current is given and a nonlinear control law is used to ensure a stabilization of the electro-magnetostrictive beam vibrations.

Keywords: amplifier of current to current, graph theory, magnetostrictive beam, electromechanical coupling, nonlinear control law, stabilization.

1 INTRODUCTION

This investigation is done in view of applications of the modelling of smart materials systems. The smart materials systems considered here, are mechanical structures including magnetostrictive actuators and sensors and an electronic system including impedancies, current sources, voltage sources and current to current amplifiers.

Let us recall that an implementation of a distributed control law for a piezoelectric shell vibrations by a distributed voltage to voltage amplifiers, has been done by Michel Lenczner and Ghouti Senouci Bereksi [1, 2, 3, and 4]. This investigation seems to be new and very succesful.

As for many applications, the magnitude of the generated strains and forces caused by an application of magnetic fields makes the magnetostrictive transducers advantageous over other smart material transducers such as piezoceramics and electrostrictives, we have tried to extend the investigation already done for piezoelectric transducers to magnetostrictive ones.

The transducer as in [6] reused here, Fig 3, is constituing by a magnetostrictive rod, a wound wire solenoid and a cylindrical permanent magnet. The current to current amplifier is related to the wound wire solenoids.

In the presence of an applied magnetic field provided by the cylindrical permanent magnet, magnetic moments are rotated and affect the solenoid where a current is created. This current is amplified using the electronic device. Acting on the coefficient of amplification we can control the energy of the whole system.

First in this investigation, we establish a weak formulation of electrical network including amplifier of current to current. A theorem of existence and uniqueness of the solution under sufficient conditions based on graph theory is established. The conditions are not included in this paper. For more details see [5]. In second, we set the structural couplling model including magnetostrictive transducer and the electrical network. The model is formulated on energy formulation for magnetic domain and domain wall dynamics. The electronic device is designed in oder to act as a stabilizer of the magnetostrictive transducer vibrations. In third, a control law is formulated using an approximation method inspired from the framework of Ralph Smith [6, 7 and 8]. The method is applied to a cantilever beam clamped in one side and submitted to a uniform periodic mechanical force.

Some numerical simulations have been done and show, in Fig 4. the attenuation of the displacement at the point $x = \frac{4l}{5}$, when the amplifier is used with a coefficient of amplification equals to 4 (we can choose other coefficients) and in Fig 5., the global energy of the whole system with and without amplifier of current to current. Let us recall that no details are given in this paper.

2 ELECTRICAL VARIATIONAL FORMULATION

We state the general variational formulation which is satisfied by the electrical current in the electrical network. The network Fig 1 includes resistors, current sources, voltage sources and current to current amplifiers. Some sufficient conditions, posed on the network for the existence and the uniqueness of the solution, are used. They are based on the conditions stated in [9] and are interpreted in terms of conditions posed on the electrical network. The network denoted by E is divided into five disjointed parts E_0, E_1, E_2, E_3, and E_4. They are occupied respectively by the voltage sources, the current sources, the resistors, the input and output of the amplifiers. Let us note that $E_i^l = \{e_i^l\}$, where

i varies from 0 to 4 and l varies from amplifier to the number of amplifiers. σ_0 is a set of nodes of the electric network related to the ground. β is a circuit which is considered as a path which is a sequence of edges e as shown in Fig 2. for example. Let us recall that this design of circuits is for current only. For voltage, the circuits are defined differently. The two extremities of an edge e are vertices or nodes which are noted s^+ and s^- respectively.

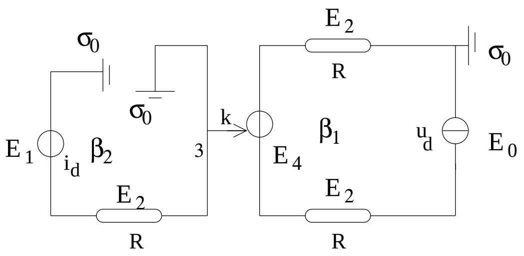

Figure 1: An example of electrical network

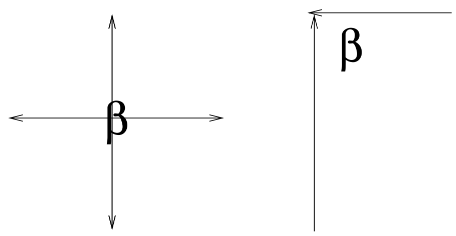

Figure 2: Types of circuits β

In this case of the circuit used here the current is cuted on E_3, so for $v \in \mathbf{P}^0(E_3)$ such that

$$\sum_{e \subset \beta \cap E_{3_s+}} v_{|e} - \sum_{e \subset \beta \cap E_{3_s-}} v_{|e} = 0 \ \forall \ \beta \ \ of \ E_1 \cup E_3 \cup E_4, \tag{1}$$

we need to construct a solution $i \in \mathbf{P}^0(E - E_0)$, relative to v, of the linear system :

$$\begin{gathered} i_{|\Xi} = 0 \\ i_{|e_4^l} - ku_{|e_3^l} = v_{|e_3^l} \ for \ every \ l \\ \sum_{e \subset \beta} i_{|e} - \sum_{e \subset \beta} i_{|e} = 0 \ for \ each \ circuit \ \beta \ \ of \ E - E_0, \end{gathered} \tag{2}$$

where Ξ is a subset of $E - E_0$.
Let us define the sets $\mathbf{P^0}(E)$ or $(\mathbf{P^0}(E_k))_{k=0..4}$ (respectively $\mathbf{P^1}...(E)$) of functions constant on each edge $e \subset E$ or $(e \subset E_k)_{k=0..4}$ (respectively affine on each edge $e \subset E$ and continuous on E). The current i and the voltage u are some distributed fields belonging to $\mathbf{P^0}(E)$. The electrical potential is also a distributed field, it belongs to $\mathbf{P^1}(E)$. The tangential derivative of a function ψ defined on E is denoted by $\nabla_\tau \psi$.

For $i_d \in \mathbf{P}^0(E_1)$, let us define the admissible functions set for the variational problem:

$$\Psi_{ad}(i_d) = \{(\psi, j) \in \mathbf{P}^1(E) \times \mathbf{P}^0(E_4 \cup E_2),$$

$$\psi = 0 \ on \ \sigma_0 \ and \ -|e|\nabla_\tau \psi = i_d \ on \ E_1\},$$

and the following variational formulation.
Consider $(\varphi, i) \in \Psi_{ad}(i_d)$ solution of :

$$\begin{aligned} &\int_{E_2} ijdl(\mathbf{x}) + \int_{E_4} \nabla_\tau \varphi jdl(\mathbf{x}) = -\int_{E_0} u_d jdl(\mathbf{x}) \\ &\int_{E_3} ki\nabla_\tau \psi dl(\mathbf{x}) - \int_{E_4} i\nabla_\tau \psi dl(\mathbf{x}) = 0 \\ &\forall (\psi, j) \in \Psi_{ad}(0). \end{aligned} \tag{3}$$

Let us remark that $j \in \mathbf{P}^0(E_4)$ is used on E_2. We adopt the rule that j takes the same value on e_2^l and on the output e_4^l of an amplifier.
This variational formulation has the form :

$$(\varphi, i) \in \Psi_{ad}(i_d)$$

$$\begin{aligned} a(i,j) + b_1(\varphi, j) &= l(j) \\ b_2(\psi, i) &= 0 \end{aligned}$$

$$\forall (\psi, j) \in \Psi_{ad}(0).$$

Here $b_1(.,.)$ and $b_2(.,.)$ are different.
Theorem *Under sufficient conditions, the variational formulation (3) has an unique solution.*

3 A STRUCTURAL COUPLING MODEL

We considere a cantilever beam with end-mounted actuators which are related between themselves by an amplifier of current to current as depicted in Fig.3. For modelling purposes, the beam is assumed to have length l, width, thicknes, density, Young's modulus, Kelvin-Voigt damping coefficient and air damping coefficient γ. Finally the transverse beam displacement is given by w while $f(t, x)$ denotes a surface force to the beam.
Moment and force balancing yields the strong form of the Euler-Bernoulli equations

$$\rho(x)\frac{\partial^2 w(t,x)}{\partial t^2} + \gamma\frac{\partial w(t,x)}{\partial t} + \frac{\partial^2 M_{int}(t,x)}{\partial x^2} \tag{4}$$

$$= f(t,x) + \frac{\partial^2 M_{mag}(t,x)}{\partial x^2}$$

$$w(t,0) = \frac{\partial w}{\partial x}(t,0) = 0$$

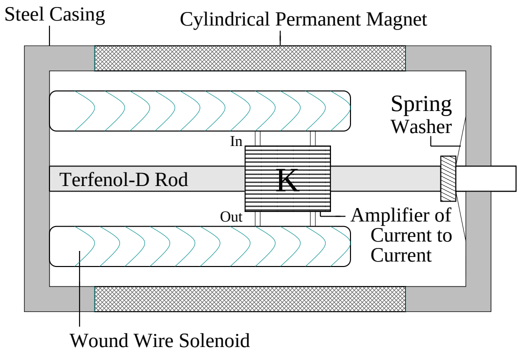

Figure 3: A magnetostrictive apparatus

$$M_{int}(t,l) = \frac{\partial M_{int}}{\partial x}(t,l) = 0,$$

along with appropriate initial conditions, as a model for characterizing the transverse beam dynamics.
$\rho(x)$ and $M_{int}(t,x)$ are the composite density and internal bending moment respectively.
For the case when the Terfenol rods are driven diametrically out-of-phase, the external moment is noted by $M_{mag}(t,x)$.
A weak form of the coupling model is given by :

$$\int_0^l \rho w'' \eta dx + \int_0^l \gamma w' \eta' dx + \int_0^l M_{int} \eta'' dx = \qquad (5)$$

$$\int_0^l M_{mag} \eta'' dx + \int_0^l f \eta dx,$$

$\forall \eta \in \{\mu \in H^2(0,l)/\mu(0) = \mu'(0) = 0\}$.
As we can see, this equation seems to be the same obtained in [7] because of some simplifications we have done, but the control input is different as in [7].

4 Control problem

We consider here the problem of controlling the nonlinear system

$$y'(t) = Ay(t) + [B(u)](t) \text{ with } y(t_0) = y_0, \qquad (6)$$

where

$$y(t) = [w_1(t), ..., w_{m+1}(t), w_1'(t), ..., w_{m+1}'(t)].$$

Note that $u(t) = coef(k)I(t)$ denotes the control input to the system, where k is the coefficient of amplification of the amplifier and coef is a function which depends also of other parameters of the circuit. I(t) is the current input. The system (6) provides the constraints employed in the control problem.

5 Application and numerical simulation

We considere a cantilever beam Fig.3, which is excited by a uniform force

$$f(t,x) = 80 sin(10 \Pi t).$$

The application of the force is stopped after 0.33 seconds.

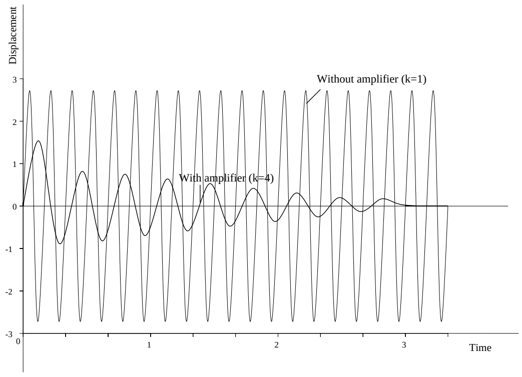

Figure 4: Uncontrolled and Controlled displacement

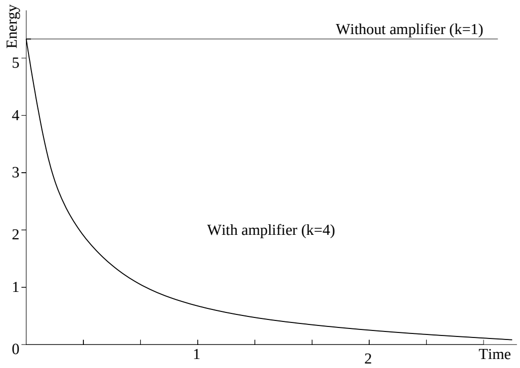

Figure 5: Global energy without and with amplifier

The coupled model was modeled through the modified Euler-Bernoulli model given above and the dynamics were approximated by numerically integrating the system (6).

5.1 References

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