

Compact Thermal Model for Transient Temperature Fields in Electronic Systems

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ABSTRACT

A thermal model is presented, which describes the evolution of the temperature distribution in electronic systems. Introducing a set of deliberately chosen effective time constants, the time dependence is given by convolution integrals with the dissipated power, which are independent of position. Multiplying with a low order matrix which depends on position but not on time, the temperature field is obtained. The matrix constitutes the model and is fitted with a linear and fast algorithm to measurement or simulation. The number of space positions for which the matrix is defined may be reduced arbitrarily to locations of interest to obtain a compact model. As a consequence of the separation of the variables of position and time, a very fast and accurate calculation of the temperature evolution in MCM (multi-chip-modules) is achieved which is beyond the possibilities of FEM-analysis in the case of power pulses over long time intervals. The thermal interaction of different chips in a MCM is investigated.

Keywords: Thermal modeling, compact thermal model, multi-chip-module, thermal interaction.

1 INTRODUCTION

Progress has been made in the past in the development of compact thermal models of different electronic packages with multiple thermal ports (thermal contact areas). Those thermal models are called compact because the number of model parameters is very much lower than the number of parameters describing a "detailed model" as, for instance, a finite element model (fig.1). If the model parameters stay constant for all boundary conditions at the thermal contact areas, a boundary condition independent model is realised. Then several models may be assembled to construct a thermal model system. The model reduction is possible, because usually only the hot spot or "junction" temperatures in the device are of interest in addition to the heat flows at thermal contact areas to the environment. Models for the steady state case [1, 2, 3, 4] only use a few parameters, depending on the degree of accuracy. Models for the time dependent case [5, 6] can be described by thermal impedances (power-step response functions in time) and need much more parameters.

The purpose of this paper is to present a compact thermal model, which allows to calculate the time development of a whole temperature *field* in a set-up as in fig.1, with low com-

putational effort, however the model will no longer be boundary condition independent. So systems with thermal contacts fixed to constant ambient temperature are considered. In power electronic systems the dissipated power in the semiconductor devices perform complicated power pulses over long time intervals, so that computation times with Finite Element methods [7] or other numerical methods [8, 9] become to excessive.

2 THEORY

The starting point used here is an expansion of the temperature field T in a series of eigenfunctions $\psi_j(x)$ of the heat conduction equation [9]

$$\rho(\bar{x}) c(\bar{x}) \frac{\partial T(\bar{x}, t)}{\partial t} = H(\bar{x}, t) + \nabla_{\bar{x}} \cdot (\lambda(\bar{x}) \cdot \nabla_{\bar{x}} T(\bar{x}, t))$$

Many thousand eigenfunctions are needed for an accurate representation of $T(x, t) = \sum_j c_j(t) \psi_j(x)$ with corresponding numerical computation times. The exact expression for T with the help of $h_j(t) = \int_V \psi_j(x) H(x, t) d^3x$ is:

$$T(\bar{x}, t) = \sum_{j=1}^{\infty} \psi_j(\bar{x}) c_j(0) e^{-t/\tau_j} + \int_0^t \sum_{j=1}^{\infty} h_j(\tau) \psi_j(\bar{x}) e^{-(t-\tau)/\tau_j} d\tau$$

in the system volume V . (1)

The time constants τ_j are the inverse eigenvalues of the problem [9, 10]. When the heat generation density H includes several independent heat sources (chips in a MCM) with dissipating power $p_l(t)$ $l = 1, \dots, L$, H is of the form $H(x, t) = \sum_l p_l(t) \cdot H_l(x)$ with $H_l(x)$ describing the *distribution* of the heat generation in chip l , $\int_V H_l(x) d^3x = 1$. Due to (1) the temperature field in case of $p_l(t) = \Theta(t)$ and $p_k(t) = 0$ for all $k \neq l$ for homogeneous starting temperature $T(x, 0) = 0$, i.e. $c_j(0) = 0$ and with $r_{jl}(x) = \tau_j \psi_j(x) \int_V \psi_j(x') H_l(x') d^3x'$ is:

$$Zth_l(x, t) = \sum_{j=1}^{\infty} r_{jl}(x) (1 - e^{-t/\tau_j})$$

$Zth_l(x, t)$ denotes the unit step response of the system for heating only chip l with unit strength of power and is the thermal impedance for fixed position x . It can serve to calculate $T(x, t)$ for arbitrary $p_l(t)$ by convolution of $p_l(t)$ with the time derivative $\dot{Z}th_l(x, t)$ [10, 11]. The τ_j have been calculated [9] for a multilayer structure corresponding to fig.1. The spectrum (r_j , τ_j) for a fixed hot spot location x is shown in fig.2. Only 3 to 4 largest time constants can be identified clearly from the plot, the other disappearing in a quasi-con-

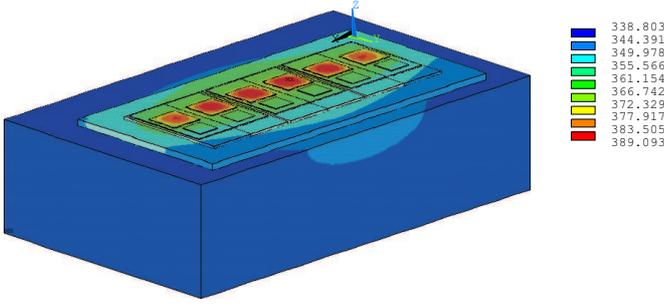


Figure1: FEM-simulation (99,532 grid points) of steady state temperature for IGBT and diode module with radiator. IGBT chips are heated with uniform power, diodes are not heated.

tinuous distribution. Similar results for the (r_j, τ_j) spectrum were obtained in [12], where deconvolution methods are applied to measured $Zth(t)$ curves.

The exponentials $\exp(-t/\tau_j)$ can be approximated by an effective set of $M = 8$ to 20 constants τ_{eff_i} according to

$$e^{-t/\tau_j} = \sum_{i=1}^M a_{ij} e^{-t/\tau_{eff_i}} \quad \text{with} \quad \sum_{i=1}^M a_{ij} = 1 \quad (2)$$

For logarithmically distributed τ_{eff_i} with intervals growing by a factor of ≈ 2 , any function $(1 - \exp(-t/\tau_j))$ with τ_j between $\text{Min}(\tau_{eff_i})$ and $\text{Max}(\tau_{eff_i})$ can be approximated very accurately (to less than 0.3%) by $1 - \sum_{i=1}^M a_{ij} e^{-t/\tau_{eff_i}}$. $Zth(x, t)$ for any single heat source can be written with

$$r_{eff_i}(x) = \sum_{j=1}^{\infty} a_{ij} r_j(x) \quad \text{as:}$$

$$Zth(x, t) = \sum_{i=1}^M r_{eff_i}(x) (1 - e^{-t/\tau_{eff_i}}).$$

The a_{ij} may be determined by a linear least square fit. For regular spaced time points this leads to large errors of the fit function between the sampling points for small times. Therefore an integral norm is used for the distance between $\exp(-t/\tau_j)$ and the fit function which is minimised with respect to this norm. The result is a linear equation system for a_{ij} , $i = 1, \dots, M$, where the matrix elements of the equ. system can be calculated analytically. The distance $\|f - g\|^2 = (f - g, f - g)$ is defined by means of the inner product

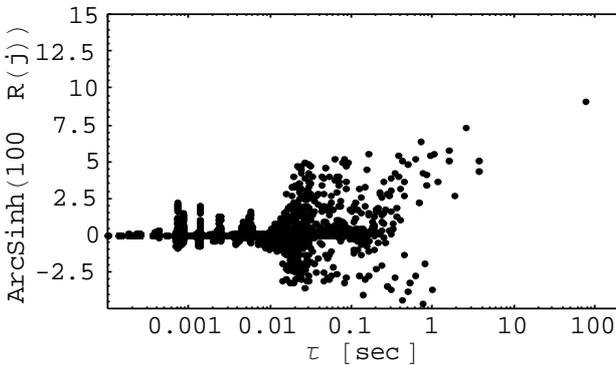


Figure2: (r_j, τ_j) spectrum for multilayer structure similar to fig.1, calculated with the method of [9].

$(f, g) = \int_0^{\infty} f(t) g(t) e^{-bt} dt$. The weighting factor $\exp(-bt)$

for $b > 0$ gives more weight to small time values in the fitting process. A value of $b \approx 1/\tau_j$ is appropriate, but also for $b = 0$ much better results are obtained than by the least square fit method for the same size of the equation system. The ordinary least square fit method works well for time sampling points distributed densely for small time values and having increasing distance for large times. The inclusion of the subsidiary condition (2) for the a_{ij} in both cases leads to a reduced linear system of degree $M-1$.

Now (1) can be presented by replacing the $\exp[(\tau-t)/\tau_j]$ by the approximations with τ_{eff_i} . In case of multiple heat sources $H(x, t) = \sum_l p_l(t) \cdot H_l(x)$ with again homogeneous starting temperature $T(x, 0) = 0$, the temperature field is:

$$T(x, t) = \sum_{i=1}^M \sum_{l=1}^L \left(\sum_{j=1}^{\infty} a_{ij} \psi_j(x) \int_V H_l(x') \psi_j(x') d^3x' \right) \int_0^t p_l(\tau) e^{(\tau-t)/\tau_{eff_i}} d\tau$$

$$\text{or} \quad T(x, t) = \sum_{i=1}^M \sum_{l=1}^L M_{il}(x) \cdot P_{li}(t) \quad (3)$$

$$\text{with} \quad M_{il}(x) = \sum_{j=1}^{\infty} a_{ij} \psi_j(x) \int_V H_l(x') \psi_j(x') d^3x'$$

$$\text{and} \quad P_{li}(t) = \int_0^t p_l(\tau) e^{(\tau-t)/\tau_{eff_i}} d\tau$$

For $p_l(t) = \Theta(t)$, we have $P_{li}(t) = \tau_{eff_i} \cdot (1 - \exp(-t/\tau_{eff_i}))$ and $T(x, t) = \sum_{i=1}^M r_{eff_i}(x) \cdot (1 - \exp(-t/\tau_{eff_i})) = Zth_l(x, t)$ with $r_{eff_i}(x) = M_{il}(x) \tau_{eff_i}$.

3 CALCULATION METHODS

$M_{il}(x)$ may be computed directly by evaluation of the eigenfunctions and time constants according to [9]. However, this method is restricted to multilayer structures where all layers have equal lateral extension. For general geometries $M_{il}(x)$ is calculated by adjusting $r_{eff_i}(x)$ linearly to $Zth_l(x, t)$ for the interesting positions x with a given set of τ_{eff_i} . The algorithm used is the same as described in section 2 for the determination of the a_{ij} . The matrix and right hand side of the equation system is calculated analytically for linearly interpolated $Zth_l(x, t)$ curves. For moderate spatial resolution this can be done fast, but also high resolution is still tractable. In defining $M_{il}(x)$ the density of locations x can be chosen at will and does not influence the accuracy of the calculated time evolution. The $Zth_l(x, t)$ are obtained by FEM simulations which are in part verified by measurements. The $M_{il}(x)$ obtained in this way are stored and constitute the compact model. The $P_{li}(t)$ are evaluated by methods for convolution integrals [10, 11] and are also stored. $T(x, t)$ at any time can then be obtained rapidly by (3). Since the variables for position and time are separated, the convolution integrals $P_{li}(t)$ have only to be calculated once and not for every position x .

The FE-model of fig.1 includes 99,532 grid points. For every grid point x the heating curves $Zth_l(x, t)$ are created and

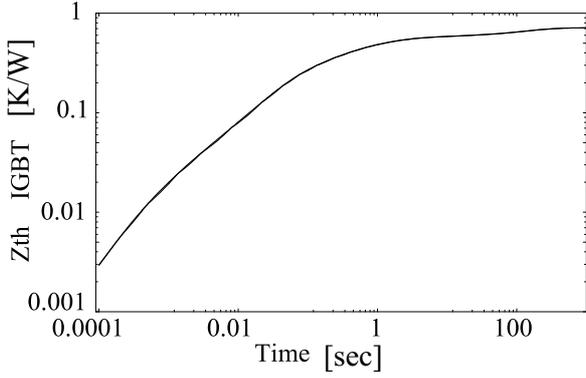


Figure 3: Thermal impedance (heating curve) for the centre of the 3rd IGBT with same IGBT as single heat source. Congruent lines of FE-result and model fit according to (3).

stored by 12 FE-computations for the different chip heating cases. This takes appreciable computation time of many hours. From the $Zth_i(x, t)$ the $M_i(x)$ matrix is deduced by linear fits for every grid point x with i running up to $M = 20$. This can be done in half an hour on a medium fast PC/workstation. Because the $M_i(x)$ tensor is very large, it is not appropriate to speak of a “compact model”, however, the set of interesting x may be reduced at will.

There is a characteristic difference between the $Zth(x, t)$ curves depending on the position x . If x lies within an active heating region, the initial slope $\dot{Zth}(x, t=0)$ is always > 0 . If x is located outside the heating region, the initial slope is 0 and the temperature at x will rise with a delay according to the heat spreading effects in the set-up. This can be inferred from (1): $\partial T(x, t=0)/\partial t = \sum_{j=1}^{\infty} h_j(0)\psi_j(x)$. Expressing $h_j(t)$ by an inner product $(f, g) = \int_V \rho c(x) f(x) g(x) d^3x$ with weighting factor $\rho(x) c(x)$, for which the complete set of eigenfunctions $\psi_j(x)$ is orthonormal [9], the sum can be written as:

$$\sum_{j=1}^{\infty} h_j(0)\psi_j(x) = \sum_{j=1}^{\infty} \psi_j(x) \left(\psi_j, \frac{H(t=0)}{\rho c} \right) = \frac{H(x,0)}{\rho c(x)}$$

From this the proposition follows immediately. As example the Zth -curve of the 3rd IGBT centre of the module in fig.1 with the IGBT itself as heat source is shown in fig.3. The picture contains both the original FEM result and the fit ac-

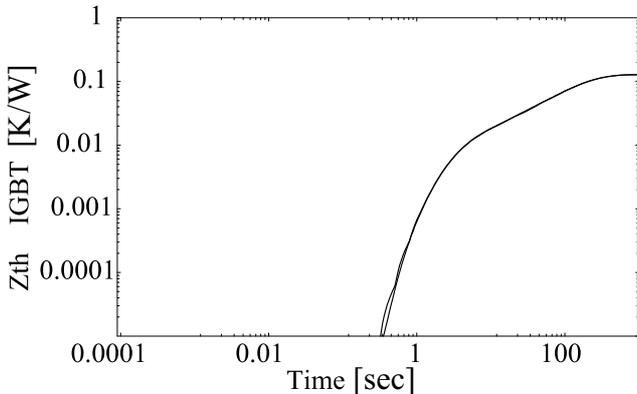


Figure 4: Same as in fig.3, but 5th diode as single heat source.

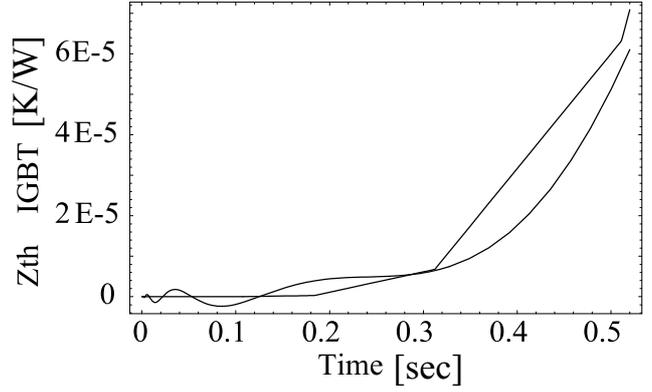


Figure 5: Non-logarithmically detail of fig.4; Zth from FEM linearly interpolated, model fit oscillatory near zero.

ording to the algorithm of sect.2. The lines are indistinguishable in the plot resolution with an error below 1%, but the accuracy is not as high as it was achieved for the monomial in (2). Figure 4 shows again the IGBT heating curve with the 5th diode as heat source. The temperature rise below 1 sec is practically zero. The *relative* error of the fit in this time domain can be large, but is of minor importance. Figure 5 shows this time domain on a non-logarithmic scale. Values of Zth close to 0 are approximated by small amplitude oscillations of the fit function.

4 RESULTS

As an application example the alternating heating of one IGBT and one diode chip in the module of fig.1 is considered. Rectangular power pulses with 10ms period are applied. The temperature evolution is calculated rapidly (some seconds) by (3). Fig.6 shows the temperature evolution of the 3rd IGBT and 5th diode centre, whereas fig.7 shows the temperature contours of the state at 150ms. For comparison the same alternating heating is calculated by the FE-program [7] with the result displayed in fig.8 and 9. A little higher temperature of the FE results is observed. The hot spot temperature of the IGBT at 150ms is higher by 0.339°C than for the model (3) which amounts to 3.1%. This cannot be explained by inaccuracies of the fitting process for $M_i(x)$. The error is attributed to inaccuracies of the FE calculation due to too large time step size in automatic time stepping.

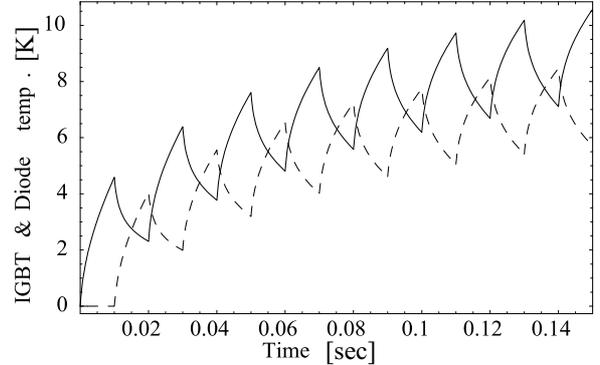


Figure 6: IGBT (full line) and diode (dashed line) hot spot temperature evolution calculated with model (3).

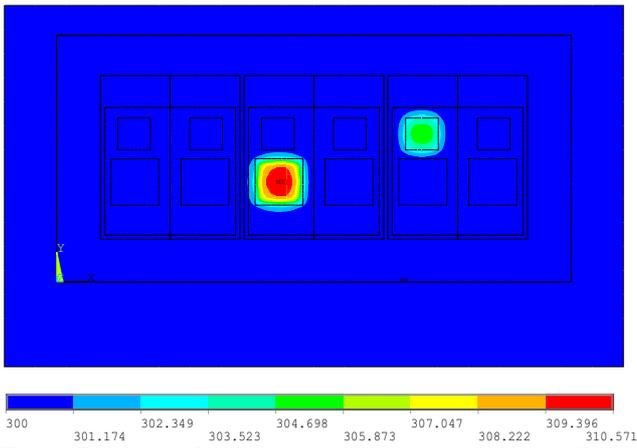


Figure 7: Model (3) temperature contours at 150ms (fig.6).

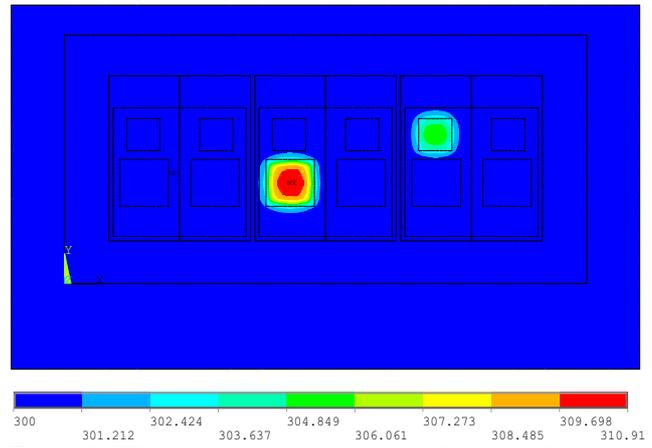


Figure 9: FE temperature contours at 150ms (fig.6).

Figures 7 and 9 show clearly that there is no thermal interaction (mutual influence) of the two chips at 150ms. It takes several seconds before the heat spreading of one chip influences the temperature of the other. This is supported by the impedance in fig.4. Figure 10 shows a result of the model (3) at $t = 50$ seconds or 5000 cycles which is beyond practicability for a FE-calculation. Because of analytical summation of the contributions of the rectangular power pulses in the convolution integrals $P_{l_i}(t)$, the calculation time for model (3) is for $t = 50$ sec (or 5000sec) as short as for $t = 150$ ms.

Usually the dissipated power $p_l(t)$ depends on the chip temperature itself: $p_l(t, T_l(t))$. In this case one node of every chip may be chosen to represent its average temperature $T_l(t)$. The convolution integrals $P_{l_i}(t)$ then are not completely independent of x and larger computation times arise. The $P_{l_i}(t, T_l)$ may be calculated with the method presented in [11]. Investigations on this will be presented elsewhere.

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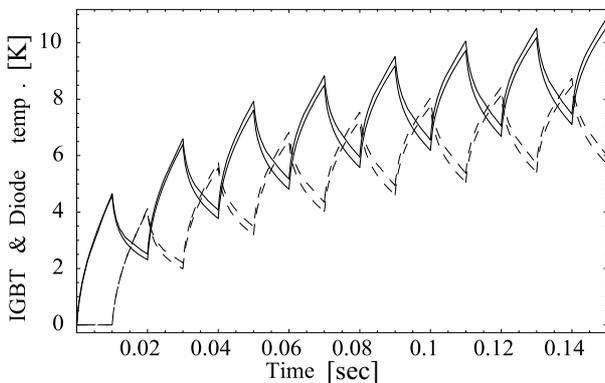


Figure 8: Comparison of FE-result with model-result of fig.6 (lower curves). The error is due to large FE time step sizes.

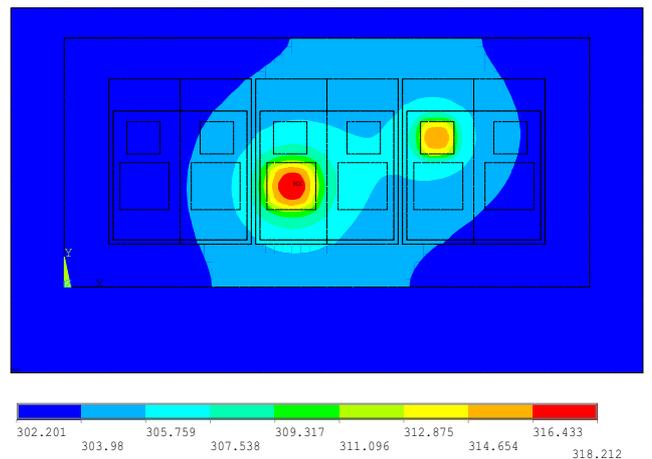


Figure 10: Model (3) result for temperature contours after 50s or 5000 cycles of alternating IGBT and diode heating.