# Coupled-Energy-Domain Macromodeling of MEMS Devices

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## ABSTRACT

This paper describes a systematic approach to the development of multi-energy-domain macromodels of MEMS devices. This approach is based on a particular form of the coupled algebraic-differential equations describing the dynamics of the device. All the model variables retain their physical meaning, which makes it easier to relate the model to the physics of the device it represents. The models can be easily built into a simulator using their associated stamps, thus improving their computational efficiency, compared to models written in hardware description languages. As a concrete example, the model of an electrostatic wobble motor is described, and numerical results obtained from its simulation are presented.

**Keywords**: microsystems, modeling, simulation.

## 1 INTRODUCTION

Lumped-constant models (or *macromodels*) of MEMS devices, although less accurate than distributed-constant models, offer the advantage of greatly reducing the computational effort required for MEMS simulation. Published results indicate that the accuracy of lumped-constant models is sufficient for many applications [1]. For these reasons, the development of such models has been a focus of research in the area of MEMS simulation.

The representation of MEMS devices' behavior by means of equivalent electrical networks is still the most common macromodeling approach [2]. A likely reason for the popularity of this technique is that it makes it possible to simulate MEMS using ordinary circuit simulators, such as SPICE. There are definite drawbacks, however, to using models of this type. The most serious is that an equivalent electrical network representing the behavior of a MEMS device does not always exist; even when it does, its construction may not be obvious or trivial. Moreover, representing non-electrical variables as voltages or currents in a circuit obscures their physical meaning. As a consequence, it may not be easy to see how well the model captures the physics of the device it represents.

Other models, developed using more recent techniques, rely directly on the algebraic-differential equations that describe the device's behavior. The equations are described using suitable languages such as MAST [1], MATLAB [3], or hardware description languages (HDLs) capable of representing the dynamics of analog continuous-time systems, such as VHDL-AMS. In principle, any lumped-constant model can be described in this way, thus overcoming the most serious limitation of the equivalent-circuit modeling technique mentioned earlier. On the downside, the simulation of HDL models – or models written in other high-level languages – is usually considerably slower than the simulation of equivalent models built into the simulator.

This paper describes a systematic approach to the development of multi-energy-domain macromodels of MEMS devices, based on a particular formulation of the set of coupled algebraic-differential equations that describe the dynamics of the device. The model variables (e.g. position, velocity, charge) represent the state of the device in the energy domains (e.g. mechanical, electrical) that are spanned by the model. There is no need to represent the model equations by means of an equivalent electrical circuit network [4] or to divide the variables into separate groups to be handled differently by the simulator [3]. Instead, all model variables are treated equally from a mathematical standpoint, while at the same time retaining their physical meaning.

The models can be easily incorporated into a simulator by deriving the corresponding element stamp [5]. Representing device models by means of stamps has several advantages. On the one hand, it allows the seamless integration of such models in a simulator capable of handling indifferently mechanical, electrical or electromechanical devices. On the other hand, the use of element stamps greatly reduces the computational effort required for simulation, compared to simulators that rely on HDL models [1], [3].

The theoretical framework on which the development of such models is based is presented in the next section. In Section 3 the technique is illustrated on a specific example, the derivation of the model of an electrostatic motor. Results of simulations of this model are presented in Section 4.

## 2 COUPLED-ENERGY-DOMAIN MODELS

It has been noted that the majority of complex MEMS structures are built from a common set of basic (or atomic) elements [6], [7]. Therefore models of composite MEMS devices can be built form the models of its constitutive elements. The dynamical behavior of some of these elements (for example, beams, anchors and plates) can be fully described in a single energy domain (e.g. mechanical, as in the examples

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just listed). Other devices, such as electrostatic actuators, are best characterized by models that span two or more energy domains. It will be shown that a particular mathematical formulation of the equations describing the device's behavior generates models that can be easily built into a simulator using so-called element stamps. Since the most common MEMS device models involve the electrical and mechanical domains, the proposed technique will be exemplified on this type of models. As will become clear from the discussion, however, this approach can be extended to the development of models involving other energy domains.

Simulation of the electrical behavior of a system is based on Kirchoff's laws. In particular, Kirchhoff's current law (KCL) states that the sum of all the outgoing currents at any node must be equal to zero:

$$\sum_k i_k = 0. \tag{1}$$

In this equation, each $i_k$ represents the branch current through an element connected to the node.

Similarly, simulation of a system's mechanical behavior is based on the equations of classical mechanics, which in the case of a rigid body can be written as follows:

$$\begin{aligned} \dot{\mathbf{p}} &= \sum_k \mathbf{F}_k \\ \dot{\mathbf{m}} &= \sum_k \mathbf{T}_k, \end{aligned}$$

where $\mathbf{p}$ and $\mathbf{m}$ are the body's linear and angular momenta, respectively, $\sum \mathbf{F}_k$ is the sum of the forces and $\sum_k \mathbf{T}_k$ is the sum of the torques acting on the body. By introducing an inertial force: $\mathbf{F}_p = -\dot{\mathbf{p}}$ and an inertial torque: $\mathbf{T}_m = -\dot{\mathbf{m}}$, the equations above can be rewritten as:

$$\begin{aligned} \sum_k \mathbf{F}_k &= 0 \\ \sum_k \mathbf{T}_k &= 0. \end{aligned} \tag{2}$$

Each term in (1) or (2) represents a quantity associated with one of the elements that make up the system. Therefore it seems natural to cast device models in a form that matches the particular structure of those equations, that is, models expressing currents, forces or torques generated by the device in terms of variables describing the state of the system. In the case of an electromechanical system, the state variables would normally consist of a set of voltages, a set of positional coordinates, and the time derivatives of those coordinates.

The first step in the construction of a model for a particular device is to identify all the quantities associated with the device that contribute to (1) or (2), either directly or through their first-order time derivatives: currents or charges in (1), forces, torques or momenta in (2). Those quantities must then be expressed as functions of the state variables, and these functions must be *algebraic*, i.e. they may not contain differential expressions. Thus this set of expressions will consist of one or more equations of the following type:

$$\begin{array}{ll} i = f_i(\mathbf{x}), & q = f_q(\mathbf{x}), \\ F = f_F(\mathbf{x}), & T = f_T(\mathbf{x}), \\ p = f_p(\mathbf{x}), & m = f_m(\mathbf{x}), \end{array} \tag{3}$$

where $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ is the vector of the system state variables. The simulation of the complete system can then be carried out by substituting these expressions for the terms in (1) and (2), and solving the resulting system of algebraic-differential equations using numerical integration. This substitution need not be done explicitly, but can be carried out indirectly, as explained next.

Denote the variables on the left-hand side of (3) with the generic letter $y$. By construction, any one of these variables appears in (1) or (2) either directly or as first-order time derivative. Therefore substitution of these variables into (1) and (2) generates a set of differential equations of the following form:

$$\sum_k \dot{y}_k = -\sum_j y_j. \tag{4}$$

Numerical integration of this set of equations can be performed using a suitable algorithm, such as the trapezoidal method:

$$y(t_n) = y(t_{n-1}) + \frac{h}{2}\left[\dot{y}(t_n) + \dot{y}(t_{n-1})\right],$$

where $h = t_n - t_{n-1}$. Thus (4) becomes:

$$\sum_k \left[y_k(t_n) - y_k(t_{n-1})\right] + \\ + \frac{h}{2}\sum_j \left[y_j(t_n) + y_j(t_{n-1})\right] = 0,$$

or, equivalently:

$$\sum_k \left(f_k[\mathbf{x}(t_n)] - y_k(t_{n-1})\right) + \\ + \frac{h}{2}\sum_j \left(f_j[\mathbf{x}(t_n)] + y_j(t_{n-1})\right) = 0, \tag{5}$$

because $y_k = f_k(\mathbf{x})$, where $f_k$ is one of the model equations in (3).

Equation (5) is a set of algebraic equations in $\mathbf{x}(t_n)$, the values of the system state variables at $t_n$, which can be written compactly as:

$$\mathrm{g}(\mathbf{x}_n) = 0, \tag{6}$$

where $\mathbf{x}_n = \mathbf{x}(t_n)$.

This system of equations can be solved numerically using Newton's method, which generates a sequence of successive approximations $\mathbf{x}_n^l$ to the solution of (6) according to the following recursive relationship:

$$\begin{aligned} \left.\frac{\partial \mathrm{g}}{\partial \mathbf{x}}\right|_{\mathbf{x}_n^l} \Delta\mathbf{x}^l &= -\mathrm{g}(\mathbf{x}_n^l) \\ \mathbf{x}_n^{l+1} &= \mathbf{x}_n^l + \Delta\mathbf{x}^l. \end{aligned} \tag{7}$$

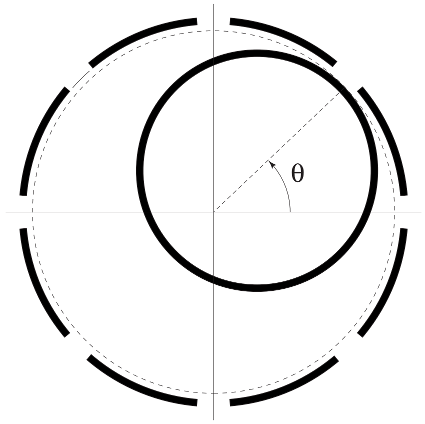


Figure 1. A planetary electrostatic motor

Equation (7) is a set of linear equations. Its right-hand side is $-\mathrm{g}(\mathbf{x}_n^l)$, and its coefficient matrix is $\dfrac{\partial \mathrm{g}}{\partial \mathbf{x}}$, the Jacobian of $\mathrm{g}(\mathbf{x})$, evaluated at $\mathbf{x}_n^l$. As is clear from (5), $\mathrm{g}(\mathbf{x})$ is a linear combination of model equations (3). Therefore the coefficient matrix and right-hand side of (7) can also be obtained from linear combinations of terms derived from the model equations. Specifically, the model equations, evaluated at $\mathbf{x}_n^l$, contribute to the right-hand side of (7), while the partial derivatives of the model equations, also evaluated at $\mathbf{x}_n^l$, contribute to the coefficient matrix. These contributions, collected in a two-dimensional array, are known as *device templates* or *stamps* [8]. Stamps give a complete, compact and easily understandable representation of the contributions of a device model to the system of equations that must be solved to simulate the system. For this reason they are ideally suited for building device models into a simulator.

## 3 ELECTROSTATIC MOTOR MODEL

The modeling technique described in the previous section is now exemplified on the derivation of the model of an electrostatic motor. A *planetary* or *harmonic* electrostatic motor [9] consists of a single cylindrical rotor placed inside a stator that is divided into a number of electrodes, as shown in Fig. 1. The rotor is electrically insulated from the stator by one or more layers of dielectric material. The application of appropriate voltages between the rotor and the stator electrodes creates a torque on the rotor that forces it to roll inside the stator. The rotor and stator voltages, the angle $\theta$ identifying the rotor position inside the stator, and the time derivative of $\theta$ form a complete set of state variables for the motor.

A motor with *N* stator electrodes has $N+1$ electrical terminals: one corresponding to the rotor, and one for each stator electrode. The current through each terminal is the time derivative of the charge on the corresponding electrode. According to the methodology described in the previous section, a model of the motor's electrical behavior consists of a set of $N+1$ functions that specify the relationships between the electrode charges $q_i$ and the motor's state variables:

$$q_i = f_i(v_0, v_1, \ldots, v_N, \theta), \qquad i = 0, 1, \ldots, N. \tag{8}$$

| | $v_0$ | $v_1$ | $\cdots$ | $v_N$ | $\theta$ | $\omega$ |
|---|---|---|---|---|---|---|
| $i_0$ | $C_{0,0}$ | $C_{0,1}$ | $\cdots$ | $C_{0,N}$ | $\frac{\partial q_0}{\partial \theta}$ | |
| $i_1$ | $C_{1,0}$ | $C_{1,1}$ | $\cdots$ | $C_{1,N}$ | $\frac{\partial q_1}{\partial \theta}$ | |
| $\cdots$ | | | $\cdots$ | | | |
| $i_N$ | $C_{N,0}$ | $C_{N,1}$ | $\cdots$ | $C_{N,N}$ | $\frac{\partial q_N}{\partial \theta}$ | |
| $\theta$ | | | $\cdots$ | | $1$ | $-\frac{h}{2}$ |
| $\omega$ | $\frac{h}{2}\frac{\partial T_e}{\partial v_0}$ | $\frac{h}{2}\frac{\partial T_e}{\partial v_1}$ | $\cdots$ | $\frac{h}{2}\frac{\partial T_e}{\partial v_N}$ | $\frac{h}{2}\frac{\partial T_e}{\partial \theta}$ | $I + \frac{h}{2}b$ |

Figure 2. Stamp of electrostatic motor model

The motor's mechanical behavior is described by the following pair of equations:

$$\begin{aligned} m &= I\omega \\ T_r &= -b\omega - T_e(v_0, v_1, \ldots, v_N, \theta), \end{aligned} \tag{9}$$

where $\omega = \dot{\theta}$, *m* is the rotor's angular momentum with respect to the motor's central axis, *I* the rotor's moment of inertia with respect to the same axis, $T_r$ the total torque acting on the rotor, *b* the damping coefficient, and $T_e$ the torque generated on the rotor by the electrostatic field.

The partial derivatives of (8) and (9) with respect to state variables $v_0, \ldots, v_N, \theta, \omega$ contribute to the coefficient matrix of (7), and are collected in the stamp shown in Fig. 2. Specifically, the first $N+1$ rows of the stamp correspond to the KCL equations at the nodes which the motor's electrical terminals are connected to. Interelectrode capacitances $C_{ij}$ are given by:

$$C_{ij} = \frac{\partial q_i}{\partial v_j} = \frac{\partial f_i}{\partial v_j}, \qquad i, j = 0, 1, \ldots, N.$$

The last two rows in the stamp correspond to the mechanical dynamics equations:

$$\begin{aligned} \dot{\theta} - \omega &= 0 \\ T_m + T_r &= 0. \end{aligned}$$

Expressions for $f_0, f_1, \ldots f_N$ and $T_e$ can be obtained from an approximate two-dimensional analysis of the electrostatic field inside the motor [9], which is omitted here for space reasons. The model's contributions to the right-hand side of (7) can be collected in another stamp containing $N+3$ rows and one column, which is also not shown because of space reasons.

## 4 NUMERICAL RESULTS

Numerical results obtained from the simulation of a 12-pole motor are presented. Figure 3 shows the rotor angular position (solid line) and velocity (dashed line) when the stator electrodes are driven by periodic rectangular-wave voltages with duty cycle equal to 1/12 of their period. After an initial start-up transient, the rotor's motion stabilizes and its angular velocity settles at a constant value, as expected. Figure 4 compares the electrostatic torques acting on the rotor under

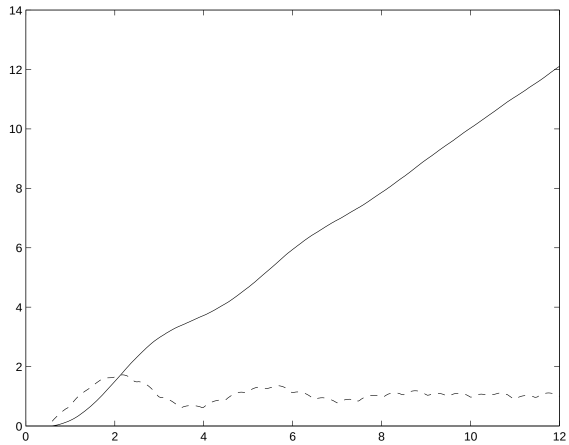


Figure 3. Rotor angle and angular velocity vs. time

two different excitation conditions. The top graph shows the torque when the duty cycle of the rectangular-wave voltages driving the stator electrodes is 1/12 of their period, while in the bottom graph the duty cycle is increased to 1/6 of the period. It can be seen that in the second case the net effect of the stator voltages results in a negative torque during part of the cycle. These results demonstrate the model's effectiveness in simulating the motor's performance under a variety of excitation and load conditions.

## 5 CONCLUSION

The modeling technique presented in this paper lends itself to the systematic development of coupled-energy-domain models for a wide variety of devices. By means of the associated stamps, these models can be easily built into a simulator for increased computational efficiency. This presents a significant advantage in the simulation of mixed-technology systems containing hundreds or even thousands of electronic and MEMS devices.

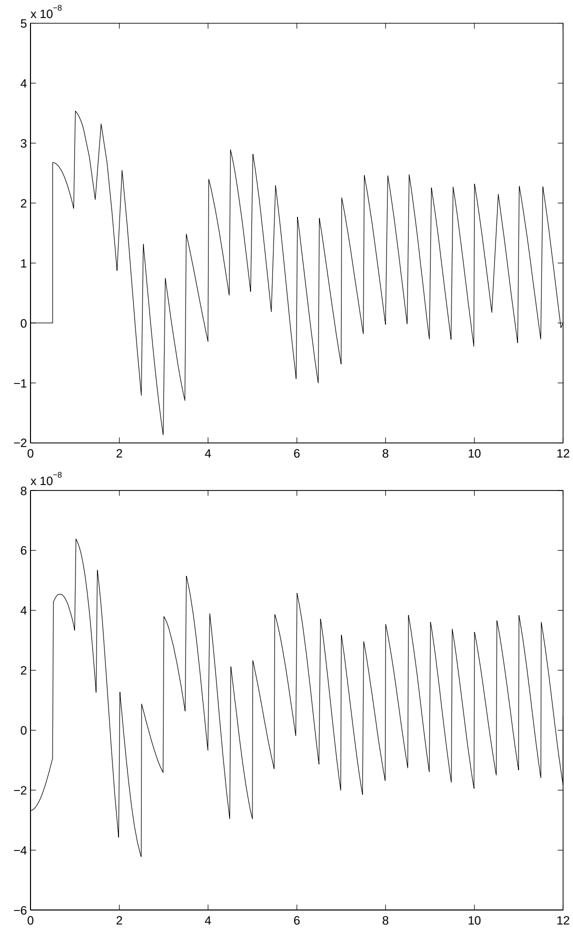


Figure 4. Electrostatic torque on rotor: 1/12 duty-cycle square wave (top) and 1/6 duty-cycle square wave (bottom)

## REFERENCES

[1] J. E. Vandemeer, M. S. Kranz, and G. K. Fedder, "Hierarchical representation and simulation of micromachined inertial sensors," in *Proceedings of the International Conference on Modeling and Simulation of Microsystems*, (Santa Clara, CA), pp. 540–545, Apr. 1998.

[2] S. D. Senturia, "CAD challenges for microsensors, microactuators, and microsystems," *Proceedings of the IEEE*, vol. 86, pp. 1611–1626, Aug. 1998.

[3] J. V. Clark, D. Bindel, N. Zhou, S. Bhave, Z. Bai, J. Demmel, and K. S. J. Pister, "Sugar: Advancements in a 3D multi-domain simulation package for MEMS," in *Proceedings of the SEM Annual Conference and Exposition*, (Portland, OR), June 2001.

[4] A. Dewey, H. Ren, and T. Zhang, "Behavioral modeling of microelectromechanical systems (MEMS) with statistical performance-variability reduction and sensitivity analysis," *IEEE Transactions on Circuits and Systems – II*, vol. 47, pp. 105–113, February 2000.

[5] G. Casinovi, "Efficient simulation of MEMS using element stamps," in *Proceedings of the Fourth International Conference on Modeling and Simulation of Microsystems*, (Hilton Head, SC), pp. 112–115, Mar. 2001.

[6] N. Zhou, J. V. Clark, and K. S. J. Pister, "Nodal analysis for MEMS design using SUGAR v0.5," in *Proceedings of the International Conference on Modeling and Simulation of Microsystems*, (Santa Clara, CA), pp. 308–313, Apr. 1998.

[7] T. Mukherjee and G. K. Fedder, "Hierarchical mixed-domain circuit simulation, synthesis and extraction methodology for MEMS," *Journal of VLSI Signal Processing*, vol. 21, pp. 233–249, July 1999.

[8] W. J. McCalla, *Fundamentals of Computer-Aided Circuit Simulation*. Norwell, MA: Kluwer Academic Publishers, 1988.

[9] R. Mahadevan, "Analytical modelling of electrostatic structures," in *Proceedings of IEEE Micro Electro Mechanical Systems*, (Napa Valley, CA), pp. 120–127, Feb. 1990.