Viscous Dissipation Effects For Liquid Flow In Microchannels

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ABSTRACT

Different phenomena have been observed in various works indicating that the mechanisms of flow and heat transfer in microchannels are still not understood clearly. There is little experimental data and theoretical analysis in the literature to elucidate the mechanisms. It is reasonable to assume that, as the dimensions of flow channels approach the micro-level, viscous dissipation could be too significant to be neglected due to a high velocity gradient in the channel. Thus, deviations from predictions using conventional theory that neglects viscous dissipation could be expected. In this paper, the effects of viscous dissipation in microchannel flows are analyzed and examined theoretically. A criterion to draw the limit of the significance of the viscous dissipation effects in the microchannel flows is suggested based on the present analysis.

Keyword: Viscous dissipation, microfluidics

1 INTRODUCTION

With microfluidic systems becoming more attractive to biomedical and lab-on-chip systems, understanding the flow is becoming very important. Different phenomena have been observed in various works indicating that the mechanisms of flow and heat transfer in microchannels are still not understood clearly.

Choi [1] experimentally studied the friction factors, and convective heat transfer coefficients for both the laminar and turbulent flow of nitrogen gas in microtubes, for the tube diameters ranging from 3 µm to 81 µm. His experimental results indicate significant departures from the thermofluid correlations used for conventional-sized tubes. Pfahler et al. [2,3] presented an experimental investigation for gas and liquid flows in microchannels. In their studies, both gases and liquids, were used. They concluded that as the channel depth was decreased, there appeared to be a critical channel size where the general behavior of the experimental observations deviates from the predictions. Different phenomena for different fluids were also observed. For liquid flow in the smaller channels, they found that the friction constant was smaller than that predicted by the conventional theory. Peng et al. [4,5], and Wang and Peng [6] experimentally studied friction flow and heat transfer characteristics of water flowing through microchannels with different hydraulic diameters. They showed that the flow changes from the laminar to the transition regime at Reynolds numbers (Re) ranging from 200 to 700 for different channels; these values were lower than the conventional values. It was also found that for the transition to turbulent flow, the range of the transition zone, and heat transfer characteristics of both transition and laminar flow were unusual as compared to the conventional-sized flow situations.

Yu et al. [7] experimentally and theoretically investigated the fluid flow and heat transfer in microtubes with dry nitrogen gas and water as working fluids. In their work, for both fluids, a reduction in the friction factor was observed for the laminar flow and a smaller reduction was observed in the transition and turbulent regimes.

Adams et al. [8] performed an experimental investigation of single-phase forced convection in circular channels with diameter 0.76mm and 1.09mm and Re ranging from 3,200 to 23,000. Heat transfer coefficients and Nusselt numbers were found to be higher than these predicted by traditional theory and the trends were found to be in agreement with the data obtained by Yu et al. [7].

These differing phenomena observed in various research works indicate that the mechanisms of fluid flow and heat transfer through microchannels are not understood clearly. This is particularly true for liquid flows and heat transfer in microchannels. Tso et al. [9] discussed the single-phase convective heat transfer in microchannels by using the data extracted from the figures presented by Wang and Peng [6]. They explained that viscous dissipation would affect liquid flow at low Reynolds numbers in microchannels.

It seems reasonable to say that, as the dimensions of the channels approach the micro-level, viscous dissipation could be too significant to be neglected due to the existence of the high velocity gradient. Therefore in the microchannel flow prediction, the commonly neglected viscous dissipation, may be too significant to ignore. In this paper, effects of viscous dissipation in microchannel flows are analyzed and examined theoretically. A criterion to draw the boundary of the significance of the viscous dissipation effects is also suggested based on the results of the present analysis.

2 GOVERNING EQUATIONS

For the liquid flow and heat transfer in microchannels, deviations from the conventional predictions have been observed in the literature. However, a satisfying explanation has not been presented to uncover the mechanism in microchannel flows. According to Knudsen's classifications, the assumption of continuum flow still holds for liquid flow in microchannels, and the conventional theories are still applicable in describing the liquid flow. Hence, in the present analysis, the conventional theories will be employed in analyzing the flow in microchannels.

It is known that the viscous dissipation terms in the governing energy conservation equation are commonly and conveniently neglected for describing conventional flow situations. However, in microchannel flows, the existence of a large velocity gradient, may result in significant errors by ignoring the effects of viscous dissipation. Therefore, in this analysis, the conventional theory will be employed taking into consideration the viscous dissipation effects to analyze the characteristics of microchannel flows. For simplicity, in the present study, a 2-D model for microtube is presented. The governing equations [10] used to analyze the diffusion effect in the microtube flow are given by,

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial r}+\frac{v}{r}=0 \qquad (1)$$

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial r}\right)=-\frac{\partial p}{\partial x}+\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\mu\frac{\partial u}{\partial r}\right)+\frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial x}\right)\right) \qquad (2)$$

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial r}\right)=-\frac{\partial p}{\partial r}+\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\mu\frac{\partial v}{\partial r}\right)-\frac{v}{r^2}+\frac{\partial}{\partial x}\left(\mu\frac{\partial v}{\partial x}\right)\right) \qquad (3)$$

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial r}\right)=\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{1}{r}\frac{\partial}{\partial r}\left(k\frac{\partial T}{\partial r}\right)+\mu\Phi \qquad (4)$$

where Φ is the dissipation function due to the viscous diffusion, and is given by,

$$\Phi=2\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial r}\right)^2+\left(\frac{v}{r}\right)^2\right]+\left(\frac{\partial u}{\partial r}+\frac{\partial v}{\partial x}\right)^2 \qquad (5)$$

U and v are the velocity components in the axial (x) and radial (r) directions respectively. P is the pressure, _ is the density, _ is the viscosity of the fluid, and cp is the specific heat capacity at constant pressure.

In the above governing equations, the conservation of energy equation (4) is also considered not because of the existence of any external heat source, but because energy comes from the viscous effects within the flow in the microchannel, causing changes in the fluid temperature. As a result, the characteristics of the flow change as the temperature varies. Though the adiabatic boundary conditions exist in few practical applications, the effects of viscous dissipation would be more significant in such cases than in the other situations. Therefore, the adiabatic boundary condition was applied. The boundary conditions are:

$$u,v\big|_w=0,\quad \left.\frac{\partial T}{\partial n}\right|_w=0,\quad u_{in}=U_0,\quad v_{in}=0 \qquad (6)$$

The equations were solved numerically by applying a SIMPLEC algorithm [11].

3 RESULTS AND DISCUSSIONS

The viscous dissipation could affect the flow pattern due to the high velocity gradient near the walls. One direct result of the effects is an increase in the fluid temperature due to the viscous dissipation. By employing numerical computations, the characteristics of the viscous dissipation effects are analyzed and discussed.

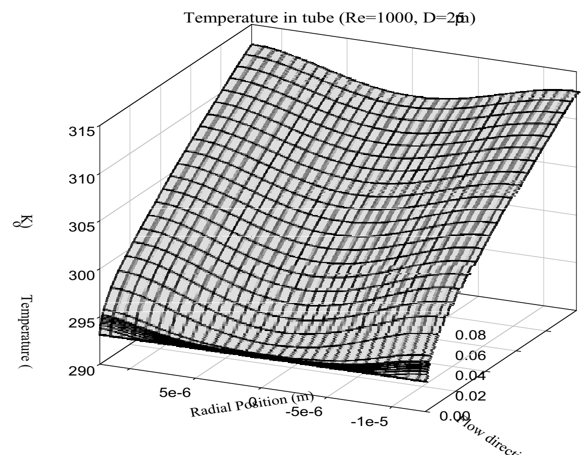

Fig. 1 Three-D profile of the temperature distribution due to viscous dissipation in microtubes

The results show that the temperature of the flowing fluid in microchannels could change due to the viscous dissipation. A 3-D temperature distribution was presented in Fig. 1 to show variations of the temperature profile resulting from the viscous dissipation heating in microchannels with an adiabatic wall. The case presented is for a water flow at Re=1000 in a microtube with diameter, D=25 μm. As shown in Fig. 1, the flow enters the tube at a uniform temperature; subsequently, the temperature of the fluid at the outer layers increases steeply over a short region immediately after the inlet. After that the temperature of the whole flow increases gradually with a lower temperature in the core regions. This phenomenon results from the high velocity gradient near the wall, which in turn gives a high rate of viscous dissipation.

Since the fluid viscosity is a function of temperature, as the temperature changes along the tube, the fluid viscosity varies along the tube and therefore the viscous shear force changes. This means the pressure distribution in the tube and the Reynolds number change also along the flow direction. Therefore, characteristics of the flow in microgeometries could be different, in terms of the friction factor and Reynolds number, from those used in the conventional macro-systems.

The local friction factor may not be useful in micro-scale flow since the information about the local flow parameters are not available in experimental measurements of the microfluidics, and the average friction factor (apparent friction factor) should be used for experimental presentations. In Fig. 2, the predicted average friction factor was plotted as a function of Reynolds number. In this figure, a case of flow in a tube with D=25 μm, L=0.1m was studied as an example. Since Re changes along the flow direction in the tube, the trend of friction factor versus Re were discussed based on inlet Re, outlet Re, and average Re. The average Re is defined as, $\mathrm{Re}_a = \rho u_m D / \mu_a$, where μ_a is the fluid viscosity at the average temperature, $T_a = (T_{in} + T_{out})/2$. As shown in Fig. 2, at the same value of Re, the friction factor of the conventional prediction without considering the viscous dissipation is higher than the case with dissipation, based on the inlet Re. It is lower than the one based on the outlet Re, and it is close to the one which is based on the average Re. As Re increases, the deviation from the conventional prediction without considering viscous dissipation becomes significant.

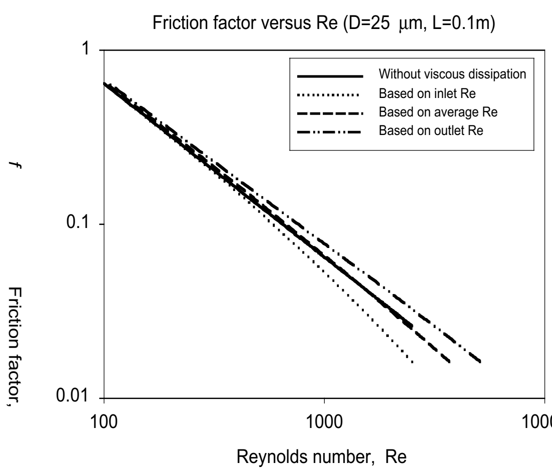

Fig. 2 Variation of friction factor versus Reynolds number

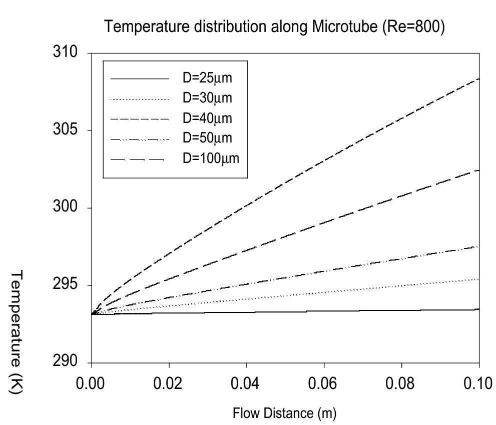

Fig. 3 Average temperature along the flow direction considering the viscous dissipation effect

As mentioned, due to high velocity gradients in microchannel flows, viscous dissipation could induce an increase of the fluid temperature in the flow, and therefore causes variations in the characteristics of flows. In Fig. 3, the average temperature variation along the flow direction due to viscous dissipation is presented to show the effects of the dimension. The cases presented are compared at the same length, L=0.1m, and Reynolds number, Re=800. As shown in the figure, the local average temperature increases along the flow direction, and the temperature difference between the inlet and the outlet increases as the diameter of the flow channel decreases.

It is clear that the effects of the viscous dissipation are more significant if the temperature difference between the inlet and the outlet is larger. Therefore, the significance of the viscous dissipation in flows can be analyzed based on the temperature rise from the inlet and the outlet as a result of the viscous dissipation. The parameters affecting the viscous dissipation in microtubes include of D, L, ρ, c_p, k, μ, U_m, and ΔT. From the Buckingham Pi theorem, for a case with eight parameters and four primary dimensions, there exist four independent dimensionless groups, Π_1, Π_2, Π_3, and Π_4, in descriptions of the relationships of these eight parameters. Therefore, there is,

$$G(\Pi_1, \Pi_2, \Pi_3, \Pi_4) = 0 \tag{7}$$

Adopting the standard approach in dimensional analysis [12], and choosing ρ, c_p, U_m, and D as the repeating parameters, the remaining four parameters may be expressed in terms of the repeating parameters as (here [ ] denotes "dimensions of "), $[k]$=ML/t^{3}T=$[\rho U_m c_p D]$, $[\mu]$=M/Lt=$[\rho U_m D]$, $[\Delta T]$=T=$[U_m^2/c_p]$, $[L]$=L=$[D]$. Thus, the four dimensionless groups may be recapitulated as:

$$\begin{aligned} &\Pi_1 = \rho \cdot U_m \cdot c_p \cdot D / k, \quad \Pi_2 = \rho \cdot U_m \cdot D / \mu \\ &\Pi_3 = c_p \cdot \Delta T / U_m^2, \quad \Pi_4 = L / D \end{aligned} \tag{8}$$

Π_2 can be Re. Π_1 and Π_2 may be re-grouped to form Pr (=Π_1/Π_2). For convenience, introduce a reference temperature T_{ref} into the function G; accordingly, a reference dimensionless group is also introduced into Eq.(7), this is $\Pi_5 = c_p \cdot T_{ref} / U_m^2$. Note that Π_3 and Π_5 may be regrouped to form a dimensionless temperature, ΔT^* =$\Delta T/T_{ref}$, namely dimensionless temperature rise. Π_4, Π_5 and Π_2 may be regrouped to form a new dimensionless number, which can be defined as,

$$Vi = \frac{\Pi_4}{\Pi_2 \Pi_5} = \frac{\mu U_m^2 L}{\rho U_m c_p T_{ref} D^2} \tag{9}$$

Vi may be referred to as viscous number and it measures the viscous dissipation energy relative to fluid energy rise. If a unit temperature is chose as T_{ref}, that is, T_{ref}=1K, Eq.(7) may be re-written as a correlation in the form of,

$$\Delta T^* = A \cdot (Vi)^a \cdot (\mathrm{Re})^b \cdot (\mathrm{Pr})^c \cdot (L/D)^d \tag{10}$$

Based on numerical results, for different cases, if each dimensionless number is processed and organized in the form of Eq.(10), the trend of the dimensionless total temperature increase between the inlet and the outlet can be described as a function of $Vi \cdot \mathrm{Pr}^{-0.1}$ in Fig. 4. It is clear that ΔT^* is a monotonically incresing function of $Vi \cdot \mathrm{Pr}^{-0.1}$. Further correlating the predicted data results in the following simplified relationship,

$$\Delta T^* = \frac{93.419 Vi\,\mathrm{Pr}^{-0.1}}{5.2086 + Vi\,\mathrm{Pr}^{-0.1}} \qquad (11)$$

It can be shown in Fig. 4 that this equation agrees well with the numerical results.

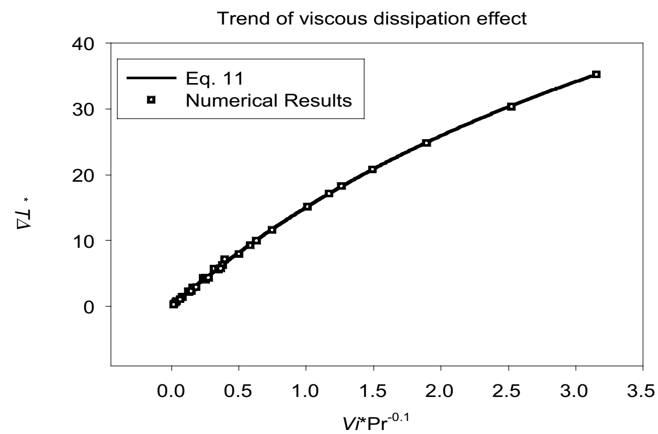

Fig. 4 Total temperature increase due to viscous dissipation

It is obvious that the value of ΔT^* can be used to indicate the significance of the viscous dissipation in flows. The value of $Vi \cdot \mathrm{Pr}^{-0.1}$ can also act as an indicator of significance of the viscous dissipation. For water, the change of viscosity is less than 3% when temperature increases one Kelvin degree. Therefore it is reasonable to let $\Delta T^*=1$ as the limit of the viscous dissipation effects on flows. Hence, the criterion of the significance of the viscous dissipation can be defined as,

$$\begin{cases} Vi\,\mathrm{Pr}^{-0.1} \le 0.056, & \text{no viscous dissipation effects} \\ Vi\,\mathrm{Pr}^{-0.1} \ge 0.056, & \text{singanificant dissipation effects} \end{cases} \qquad (12)$$

4 CONCLUSIONS

Analyzing the effects of the viscous dissipation on the characteristics of the liquid flow in microchannels, it was found that the effects become significant and influence the temperature, pressure and velocity distributions in the flow. Therefore, relationships between the average friction factor and the Reynolds number change when the hydraulic diameter of the microchannel is very small. The viscous dissipation effects are brought about by rises in the velocity gradient as hydraulic diameter reduces for constant Reynolds number.

By choosing water as an example, the criterion for the significance of the viscous dissipation effects on flows has been established. It is given in Eq. (13).

This criterion could be used to evaluate the possibilities of the viscous dissipation effects on flows in microgeometries. The method employed can also be used to discuss the effects of the viscous dissipation for other fluids flowing in microgeometries.

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