

# Modeling of Electromagnetic Fields in High Speed Electronic Interconnects Using a Least Squares FD-TD Algorithm

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## ABSTRACT

A new Finite-Surface Time-Domain (FS-TD) numerical method is presented which is a generalization of the classical Finite-Difference Time-Domain (FD-TD) method of Yee. The method is divergence/charge preserving on arbitrary grids. It reverts to the standard FD-TD scheme on orthogonal meshes. The method is based on a dual mesh formalism which makes use of a staggered (in space as well as time) data storage scheme. The method conserves charge by making use of an appropriate least squares reconstruction scheme in order to recover vector fields from the dot products of the field with cell (either primary or secondary) area normal vectors. Numerical results demonstrating the accuracy and the efficacy of the new method are presented.

**Keywords:** FD-TD, Maxwell's equations, Crosstalk, Interconnects.

## 1 INTRODUCTION

As VLSI device sizes continue to shrink and frequencies continue to rise, simple models such as 1-D transmission line theory are no longer sufficient to capture all aspects of electromagnetic effects on the performance of interconnect circuits. In order to demonstrate the complexity of the fields generated in such situations Figure 1 displays instantaneous field patterns in a through hole computation where such phenomena as radiation, signal reflection, etc. are all present. Thus system designers need access to tools which can accurately model all aspects of electromagnetic phenomena in high speed, high bandwidth electronic interconnects (wire bonds, bump bonds, vias, complex traces with curves) and traces on bent flex circuit boards. Numerical methods which solve the full set of Maxwell's equations, such as Yee's classic Finite-Difference Time-Domain (FDTD) scheme, are able to simulate electrical signal propagation, crosstalk, radiation, and electromagnetic interference in high-speed interconnect circuits as well as provide important bulk performance information about the circuit such as capacitance, resistance, inductance, S-parameters, Y-parameters etc. which provide useful information to reduced order models of the interconnect systems. However Yee's original scheme was

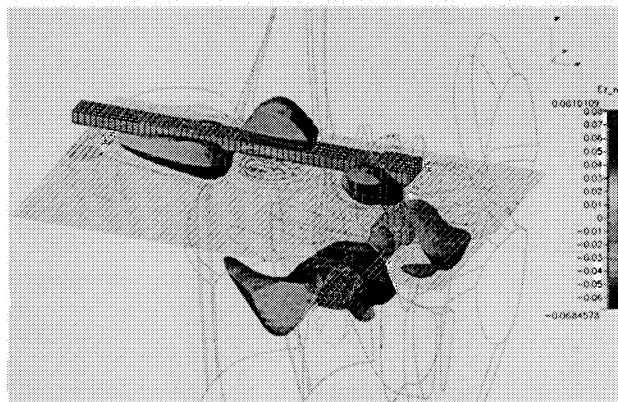


Figure 1: Field patterns in through hole computation.

limited to Cartesian grids and thus was limited in its ability to model geometrically complex objects.

This paper details the development of a new Finite-Difference Time-Domain (FD-TD) method for the solution of Maxwell's equations on arbitrary grids. The method is similar in spirit to the Finite-Surface Time-Domain (FS-TD) method developed by Vinokur and Yarrow [1] for structured grids but differs in its applicability to grids of arbitrary type. As such it has a close connection to the Discrete Surface Integration (DSI) method of Madsen [2], but differs in the approach taken to reconstructing a vector field given the components of the vector field normal to the cell faces of a grid. In this paper we first very briefly review the current status of FD-TD methods for computational electromagnetics. This is followed by a description of the new method. Finally examples are given which demonstrate the efficiency and accuracy of the new technique.

## 2 REVIEW

Finite-difference time-domain modeling of Maxwell's equations can arguably be said to have originated with Kane Yee's seminal 1966 paper [3]. Yee's scheme used centered difference in space with leap frog time differencing to numerically integrate Maxwell's equations. Yee's method is robust, accurate as well as having the desirable property of being divergence free/ charge preserving. The scheme is still of great theoretical as

well as practical interest [4]. However, Yee's scheme is limited to equally spaced Cartesian meshes. Given the simplicity and accuracy of Yee's scheme it was almost inevitable that other researchers would extend the method to non-orthogonal grid systems. In this regards, Sheen extended the FD-TD method of Yee to nonuniform orthogonal grids in [5]. Holland [6], Fusco [7], Lee et. al [8] as well as Vinokur and Yarrow all developed appropriate FD-TD schemes for general non-orthogonal hexahedral grids. Madsen [2] developed a FD-TD scheme which is capable of treating general non-orthogonal unstructured grids. Madsen's scheme posses the following desirable properties

1. No restriction on cell types.
2. Collapses to Yee's scheme on orthogonal grids.
3. Divergence Free/ Charge preserving.
4. Conditional temporal stability
5. Non-dissipative.
6. Accurate for skewed grids.

Like Madsen's scheme our newly developed least squares finite surface scheme also shares all the above properties. The distinctions between the two schemes are spelled out in the following section.

### 3 Least Squares Reconstruction Finite Surface Method

Before deriving the least squares reconstruction finite surface method we first review Yee's original scheme in the context of finite surface methods. Yee's original scheme can be viewed (for purposes of comparison with later schemes) as solving Maxwell's equations on an uniform Cartesian mesh and a mesh dual to this mesh. The dual mesh is formed by connecting the cell barycenters of the original mesh with straight lines. In this dual grid system there is a one-to-one correspondence between the nodes, edges, faces and cells of the original mesh with the cells, faces, edges and nodes of the secondary mesh. For example, each edge of the original (or primary) mesh corresponds to a unique face of the secondary mesh, etc. Yee's original scheme uses a staggered data storage scheme in which the the normal components of the magnetic flux density vector  $\mathbf{B}$  are stored at the cell face centers of the primary mesh at half-integer time steps while the normal components of the electric field vector  $\mathbf{E}$  are stored at the cell face centers of the secondary mesh at integer time steps.

In order to understand how Yee used the dual grid system to solve Maxwell's equations we start by writing Maxwell's equations in coordinate independent vector notation:

$$\frac{\partial}{\partial t} \mathbf{B} = - \nabla \times \mathbf{E}, \quad (1)$$

$$\frac{\partial}{\partial t} \mathbf{E} = \frac{1}{\epsilon} \nabla \times \frac{\mathbf{B}}{\mu}. \quad (2)$$

Yee's original scheme is rederived by first integrating Eqn. 1 over cell face  $F_i$  of the primary grid at time level  $t_n$  to obtain

$$\int \int_{F_i} \frac{\partial}{\partial t} \mathbf{B}(\mathbf{X}, t_n) \cdot \mathbf{n}_i dS = - \oint_{F_i} \mathbf{E}(\mathbf{X}, t_n) \cdot d\mathbf{l}. \quad (3)$$

Following Yee, the time derivative is replaced using a centered difference approximation, while mid-point formulas are used to numerically evaluate the surface and line integrals, such that the following update equation for the normal components of  $\mathbf{B}$  is obtained

$$A_i \left( \frac{B_i^{n+\frac{1}{2}} - B_i^{n-\frac{1}{2}}}{\Delta t} \right) = - \sum_{j=1}^{n_{ei}} \mathbf{E}(\mathbf{L}_j, t_n) \cdot \mathbf{l}_j. \quad (4)$$

It is at this point in the derivation that the data storage as related to the dual mesh scheme is invoked. Recall that for the dual mesh system, every edge of the primary mesh is uniquely related to a face in the secondary mesh. In addition, for Cartesian meshes the secondary face normal vectors are parallel to the primary mesh edge vectors, i. e.,  $\tilde{\mathbf{n}}_j \parallel \mathbf{l}_j$ . Also for Cartesian meshes  $\mathbf{L}_j = \tilde{\mathbf{X}}_j$  Therefore Eqn. 4 can be written

$$A_i \left( \frac{B_i^{n+\frac{1}{2}} - B_i^{n-\frac{1}{2}}}{\Delta t} \right) = - \sum_{j=1}^{n_{ei}} \pm E_j^n \|\mathbf{l}_j\|. \quad (5)$$

Following the same line of reasoning, it follows that the  $\mathbf{E}$  field normal component equation is given as

$$\tilde{A}_j \left( \frac{E_j^{n+1} - E_j^{n-1}}{\Delta t} \right) = \frac{1}{\epsilon} \sum_{i=1}^{n_{ej}} \pm \left( \frac{B_i^n}{\mu} \right) \|\tilde{\mathbf{l}}_i\|. \quad (6)$$

Equations 5 and 6 are completely equivalent to Yee's original central difference scheme except that they are cast in the language of finite surface schemes [1] rather than that of finite differences schemes.

However, as soon as any cell in the grid loses orthogonality the above equations are no longer valid. This is because  $\tilde{\mathbf{n}}_j$  is no longer parallel to  $\mathbf{l}_j$ , etc. In this situation the normal component of  $\mathbf{E}$  is no longer sufficient to evaluate the dot products

$$\mathbf{E}(\mathbf{L}_j, t_n) \cdot \mathbf{l}_j$$

which appear in Eqn. 4. All three components of  $\mathbf{E}$  are in general necessary to evaluate the dot products. In this case Eqn. 5 is replaced by Eqn. 4 while Eqn. 6 is replaced with the following equation

$$\tilde{A}_j \left( \frac{E_j^{n+1} - E_j^{n-1}}{\Delta t} \right) = \frac{1}{\epsilon} \sum_{i=1}^{n_{ej}} \left( \frac{\mathbf{B}(\tilde{\mathbf{L}}_i, t_n)}{\mu} \right) \cdot \tilde{\mathbf{l}}_i \quad (7)$$

It is the manner in which the full vectors  $\mathbf{E}(t_n)$  and  $\mathbf{B}(t_{n+\frac{1}{2}})$  are reconstructed at the secondary and primary cell faces which distinguishes our current method from Madsen's original scheme.

In Madsen's approach at each cell face  $F_i$  (which without loss of generality will be assumed to be a primary cell face) a list is formed of every node  $j$  which is a member of this face. Next one uses the values of the components of  $\mathbf{B}$  normal to the primary cell faces to form and solve the following system of equations at each node  $j$  for the full vector  $\mathbf{B}_{i,j}$

$$\begin{aligned} \mathbf{B}_{i,j} \cdot \mathbf{n}_i &= B_i \\ \mathbf{B}_{i,j} \cdot \mathbf{n}_2 &= B_2 \\ \mathbf{B}_{i,j} \cdot \mathbf{n}_3 &= B_3 \end{aligned} \quad (8)$$

where faces  $F_i$ ,  $F_2$ , and  $F_3$  are three faces which intersect node  $j$  ( $F_i$  is always a common member of the triad). These equations are solved at node  $j$  for every unique combination of faces which intersect  $F_i$  at node  $j$ . For a warped hexahedral grid, such linear systems are solved eight times at each face. After these linear equations have all been inverted, the resulting full vectors  $\mathbf{B}_{i,j}$  are weighted to  $\tilde{\mathbf{L}}_i$  and dotted with  $\tilde{\mathbf{l}}_i$  where they are used in conjunction with Eqn. 7 to advance the normal components of  $\mathbf{E}$ .

In contrast to Madsen's method, our method attempts to find the best average  $\mathbf{B}$  in the region surrounding the face in question. The least squares reconstruction method starts by forming a list of all  $m$  faces  $F_j$  which bound either of the primary cells which share face  $F_i$ . Next we form the following over-determined system of equations given the values of  $\mathbf{B}_j$  at the various faces.

$$\begin{pmatrix} n_{ix} & n_{iy} & n_{iz} \\ \vdots & \vdots & \vdots \\ n_{jx} & n_{jy} & n_{jz} \\ \vdots & \vdots & \vdots \\ n_{mx} & n_{my} & n_{mz} \end{pmatrix} \begin{pmatrix} (\mathbf{B} \cdot \hat{\mathbf{x}})_i \\ (\mathbf{B} \cdot \hat{\mathbf{y}})_i \\ (\mathbf{B} \cdot \hat{\mathbf{z}})_i \end{pmatrix} = \begin{pmatrix} B_i \\ B_j \\ \vdots \\ B_m \end{pmatrix}. \quad (9)$$

In principle one could solve Eqn. 9 using either the normal equations or a QR factorization [9] and use  $\mathbf{B}_i$  in conjunction with Eqn. 7 to advance the face-normal components of  $\mathbf{E}$ . However the resulting numerical method would no longer be charge preserving as in general

$$\mathbf{B} \cdot \mathbf{n}_i \neq B_i \quad (10)$$

which is a sufficient condition to ensure that the computed fields remain divergence free under time evolution [2], [4]. However, divergence free time evolution of the computed fields can be ensured by using Eqn. 10 to eliminate one of the unknowns. At face  $F_i$  assume without loss of generality that the  $|n_{ix}| \geq \max(|n_{iy}|, |n_{iz}|)$ , then we set

$$(\mathbf{B} \cdot \hat{\mathbf{x}})_i = \frac{B_i - n_{iy} * (\mathbf{B} \cdot \hat{\mathbf{y}})_i - n_{iz} * (\mathbf{B} \cdot \hat{\mathbf{z}})_i}{n_{ix}}. \quad (11)$$

Next, we use  $\mathbf{B} \cdot \hat{\mathbf{x}}$  as computed in Eqn. 11 in order to reduce the number of unknowns from 3 to 2. The resulting reduced order system of equations is then solved

(in a least squares sense) by using the normal equation approach [9]. A similar technique is used to reconstruct  $\mathbf{E}$  at the secondary cell faces.

## 4 Computational Results

In order to demonstrate the efficacy of the new code for the modeling of electromagnetic fields in high speed interconnect circuits we have applied the code to a through hole (or via hole) configuration which corresponds to a case considered by Maeda, *et al.* [10]. The geometry of the situation is shown in Figure 2. The width of each stripline is  $3.3\text{mm}$  while the diameter of the lands (pads) is  $3.9\text{mm}$ . The diameter of the connector rod is  $0.7\text{mm}$  while the diameter of the clearance hole in the ground plane between the two levels is  $3.9\text{mm}$ . The space between the two lines is filled with a dielectric substrate whose relative permittivity is  $\epsilon_r = 3.4$ . In this computation all conductors are assumed to be perfect and have zero thickness (i. e.  $\mathbf{E} \cdot \hat{\mathbf{t}} = 0$  where  $\hat{\mathbf{t}}$  is any vector tangent to the conductor. In addition an outline of the grid used in the computation is shown on the surface of the conductors in Figure 2 to illustrate the geometric flexibility of the new scheme. As can be seen, skewed hexahedral cells are used to model most of the geometry while prismatic cells are used to handle the lands, clearance rods, etc. exactly without the need to invoke any geometrical approximations such as stair stepping. The stripline is excited applying an uniform electric field in the  $yz$  plane under the upper strip line as follows:

$$\begin{aligned} E_z &= 1 - \cos(2\pi f_{band} t) : 0 \leq t \leq \frac{1}{f_{band}} \\ E_z &= 0 : \frac{1}{f_{band}} \leq t \\ f_{band} &= 12.0(\text{GHz}). \end{aligned} \quad (12)$$

The transmitted voltage as computed by the least squares FD-TD code (CFD-MAXWELL) is shown in Figure 3 along with results (computations as well as experiments) taken from Maeda, *et al.* [10]. As can be seen the least squares FD-TD code results agree well with the previous results. In addition, the code has been used to compute cross-talk in several situations which are not well suited for traditional FD-TD methods. Results taken from a computation involving a curved flex board are shown in Figure 4 while those taken from a bent stripline computation are shown in Figure 5

## 5 Acknowledgements

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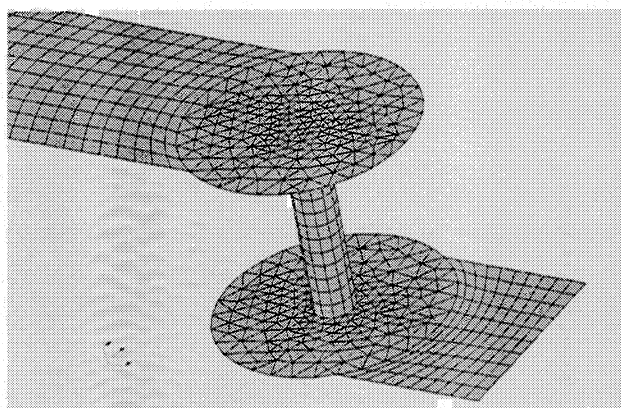


Figure 2: Mixed grid system used in through hole computation

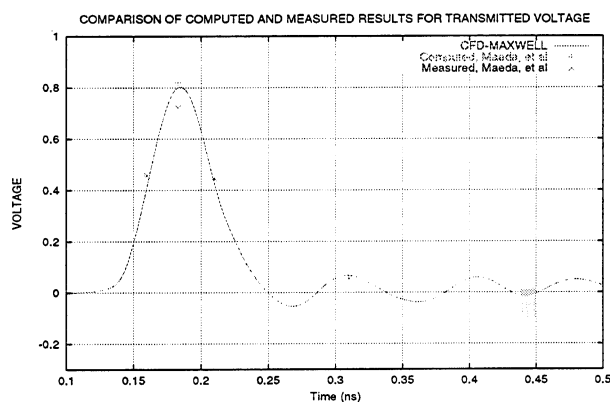


Figure 3: Comparison of measured and computed transmitted voltages.

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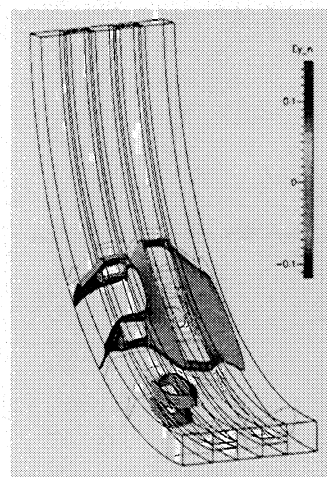


Figure 4: Crosstalk computation on curved flex board interconnects.

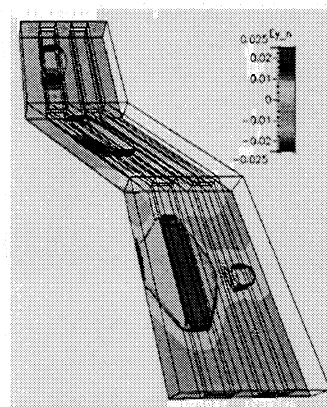


Figure 5: Crosstalk computation on bent interconnect striplines.

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