

Noise Modelling of Microwave Bipolar Transistors

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ABSTRACT

In this paper we present a stochastic approach to study the noise modelling of bipolar microwave transistors. Starting from experimental on-wafer measurements of the scattering and noise parameters of these bipolar devices we obtain the corresponding Giacoletto model with the noise internal sources. We write down then the stochastic differential equations of the Giacoletto model for a short-circuited output in order to study the noise properties of these devices. From the power spectrum of the output current we obtain the behaviour of the noise figure as a function of the operating frequency.

Our theoretical results are in good agreement with experimental ones. The present approach allows to study the statistical properties of the output current for different statistics of the internal noise sources of the bipolar transistors (BJT and HBT).

Keywords: noise modelling, stochastic differential equation, microwave transistors.

1 INTRODUCTION

The bipolar transistors have been characterised *on wafer* in terms of scattering parameters and noise figure by varying the collector current (I_c): (i) between 0.7 and 10 mA at collector-to-emitter voltage (V_{ce}) value of 1 V over 1-2 GHz frequency range for BJT, (ii) between 0.5 and 1 mA at collector-to-emitter voltage value of 2 V over 0.5-40 GHz frequency range for HBT.

The test devices and measurements have been furnished by ST-Microelectronics (BJT) and Daimler Benz (HBT). Such device is a 2-finger emitter structure, or better, the BJT tested is realised with two 1-finger devices in parallel configuration. Both devices exhibit a current gain β in excess of 100 but the BJT reach this value for low bias point (0.7 mA). Therefore the BJT has an extrinsic base resistance value larger than that of HBT. This aspect, together with an high transconductance value (g_m), causes the HBT to have a lower noise figure (NF) value than BJT for fixed collector current and collector-emitter voltage values.

To characterise the noise behaviour of transistors different noise equivalent circuits are used. In these circuits the noise properties are modelled as noise voltage and current generators in order to design microwave devices with low noise applications.

In this paper we present a new approach to study the noise modelling of bipolar microwave transistors. We consider a stochastic differential equation for the Giacoletto model of the bipolar devices obtained by experimental on-wafer measurements of the scattering and noise parameters. We obtain the probability distribution and the power spectrum of the output current of the transistors. We give an analytical temporal behaviour of the second moment of the output current, assuming particular given correlation functions between the internal noise sources. We obtain finally the behaviour of the noise figure versus the operating frequency.

The agreement between our theoretical results and experimental measurements is good.

2 GIACOLETTO MODEL

For our initial purpose we consider a very simple topology of Giacoletto model because of: (i) on wafer measurements which allow us to neglect some elements connected with the package, (ii) the very low and narrow frequency range which allow us to neglect the collector and emitter resistance especially when compared to values of the device impedance matrix. Therefore, we extract this model from scattering and DC measurements at each bias condition ($I_c=0.7$ and 1 mA). Noise generator has been added to the noise-free circuit [1]. The values of the noise generators have been fixed according to the transistor noise theory. The model network is reported in Figure 1, where:

$\langle v_b^2 \rangle$ is a source that takes into account the fluctuations due to the thermal noise of the base resistance R_{bb} ,

$$\langle v_b^2 \rangle = 4KT R_{bb} \Delta f \quad (1)$$

$\langle i_b^2 \rangle$ is the shot noise generator that takes into account of the random diffusion of holes and electrons through a potential barrier of semiconductor,

$$\langle i_b^2 \rangle = 2qI_b \Delta f \quad (2)$$

$\langle i_c^2 \rangle$ is the source that take into account the fluctuations in the collector current due to the random process of the time arrival of the diffusion and drift carriers at the collector-base junction,

$$\langle i_c^2 \rangle = 2qI_c \Delta f \quad (3)$$

Some adjustment have been requested by the complex correlation coefficient existing between emitter and collector shot noise generators for the best fitting of the measured noise figure NF.

We show the model results versus frequency and the element values of the model respectively in Figure 1 and Table 1.

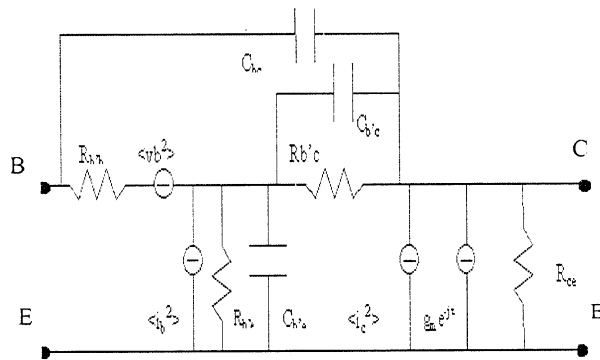


Figure1: Giacoletto Model with noise sources added.

Transistor	BJT	HBT
Bias Point	$V_{ce}=1\text{ V } I_c=1\text{ mA}$	$V_{ce}=2\text{ V } I_c=1\text{ mA}$
$R_{be}(\Omega)$	104	24
$R_{bc}(\Omega)$	3000	1260
$R_{ce}(\text{K}\Omega)$	18	20
$R_{ce}(\text{K}\Omega)$	25	30
$g_m(\text{mS})$	26	30
$C_{be}(\text{pF})$	0.0075	0.0015
$C_{bc}(\text{pF})$	0.0077	0.024
$C_{ce}(\text{pF})$	0.1809	0.1264
Coef _{BEA}	0.7213	0.6131
Coef _{mm}	0.7213	0.6942

Table 1. Element values for the Model

Starting from the Giacoletto model obtained with MMICAD software and applying Kirchhoff's laws to the equivalent circuit (Figure 1), we obtain the expression of the output current as a function of the circuit parameters and the noise sources, with $R_L=0$ and $Y_s=G_s$. To do this, we use the Laplace transform method [2]. We get finally an integro-stochastic equation for the output current:

$$i_{out}(t) = a i_{sn}(t) + b v_{bn}(t) + c i_{cn}(t) + \int_0^t [i_{sn}(\tau) a(t-\tau) + v_{bn}(\tau) b(t-\tau) + i_{cn}(\tau) c(t-\tau)] d\tau \quad (4)$$

where the coefficients a , b and c and the functions $a(t-\tau)$, $b(t-\tau)$ and $c(t-\tau)$ have different expressions depending on the source admittance.

3 LANGEVIN APPROACH

From equation (4), by assuming some given correlation function between the internal noise sources, it is possible to calculate all the moments of the output current. The correlation functions are directly related to the physical pictures of the microscopic internal structures of the bipolar transistors.

To calculate the second moment of the stochastic process of the output current we consider:

- all the noise sources expressed in terms of the Wiener process;
- no correlation between the external noise sources and the internal ones;
- the internal noise sources are δ -correlated:

$$\begin{aligned} \langle v_{bn}(\tau) i_{bn}(\tau') \rangle &= \sigma_{vb} \sigma_{ib} \delta(\tau - \tau') \\ \langle v_{bn}(\tau) i_{cn}(\tau') \rangle &= \sigma_{vb} \sigma_{ic} \delta(\tau - \tau') \\ \langle i_{cn}(\tau) i_{bn}(\tau') \rangle &= \sigma_{ic} \sigma_{ib} \delta(\tau - \tau') \\ \langle i_s(\tau) v_{bn}(\tau') \rangle &= 0 \\ \langle i_s(\tau) i_{bn}(\tau') \rangle &= 0 \\ \langle i_s(\tau) i_{cn}(\tau') \rangle &= 0 \end{aligned} \quad (5)$$

By using the statistical properties of the Wiener process we obtain the second moment of the output current

$$\begin{aligned} \langle i_o(t)^2 \rangle &= \sigma_{is}^2 + b^2 \sigma_{vb}^2 + c^2 \sigma_{ib}^2 + \sigma_{ic}^2 + \\ &+ \int_0^t [\sigma_{is}^2 a^2(t-\tau) + \sigma_{vb}^2 b^2(t-\tau) + \sigma_{ib}^2 c^2(t-\tau) + 2\sigma_{vb} \sigma_{ic} b(t-\tau) c(t-\tau)] d\tau + \\ &+ 2b \langle v_b(t) i_b(t) \rangle - 2b \langle v_b(t) i_c(t) \rangle - 2c \langle i_b(t) i_c(t) \rangle + \\ &+ 2\sigma_{is}^2 a(t) + 2b [\sigma_{vb}^2 b(t) + \sigma_{vb} \sigma_{ic} c(t)] + 2c [\sigma_{ib} \sigma_{vb} b(t) + \sigma_{vb} \sigma_{ic} c(t)] - 2[\sigma_{vb} \sigma_{ic} b(t) + \sigma_{ic}^2 c(t)]. \end{aligned} \quad (6)$$

Where we use a short-circuited output and the following correlation functions between the different internal noise sources:

$$\begin{aligned} \langle i_{b1c} \rangle &= \sigma_{ib} \sigma_{ic} e^{-t/\tau_{bc}} \\ \langle v_{b1c} \rangle &= \sigma_{vb} \sigma_{ic} e^{-t/\tau_{bc}} \\ \langle v_{b1b} \rangle &= \sigma_{vb} \sigma_{ib} \end{aligned} \quad (7)$$

where τ_{bc} is the carriers transit time from base region to collector region.

4 NOISE FIGURE

In order to obtain the noise figure at 50 Ω from temporal series of the output current, we consider the noise figure definition

$$NF = \frac{\frac{S_{IN}}{N_{IN}}}{\frac{S_{OUT}}{N_{OUT}}} \quad (8)$$

where S_{IN} and N_{IN} are signal and noise source power and S_{OUT} and N_{OUT} are signal and noise output power. After straightforward calculations we obtain, in decibels, the following expression:

$$NF = 10 \log \frac{N_{OUT}}{G_a N_{IN}} = 10 * \log \frac{N_0}{N_s} \quad (9)$$

where G_a is the maximum available gain which is independent of the load. Therefore from output current temporal series with and without internal noise sources, we obtain respectively N_0 and N_s and for each frequency we calculate the noise figure (equation 9). The noise behaviour of the bipolar transistor versus bias point show that the NF increases with collector current increase.

5 RESULTS

In Figure 2 we plot the probability distribution of the output current, which is Gaussian as we expected from equation (4). In order to calculate the noise figure by FFT method we calculate the output current power spectrum with and without internal noise sources.

For each frequency value, we calculate the output power level, obtaining the noise figure as a function of the operating frequency. In the same figure we report the noise figure behaviour obtained from experimental measurements with bias points $I_c=1$ and 0.7 mA for the BJT (see, Figure 3), and $I_c=1$ mA for the HBT (see, Figure 4).

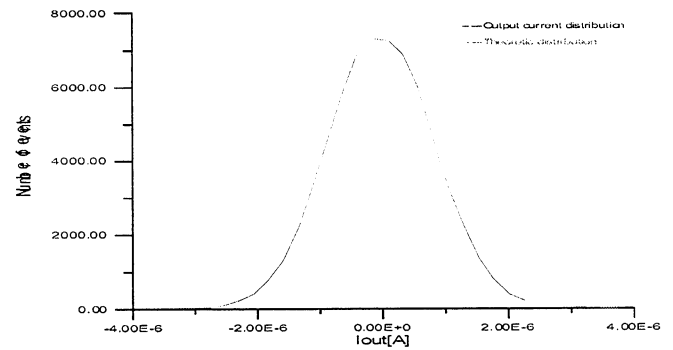


Figure 2: Probability distribution of the output current.

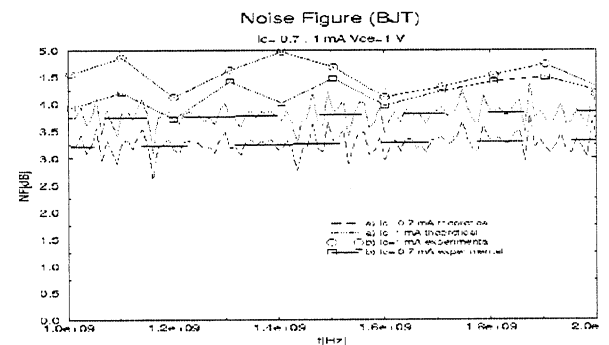


Figure 3: Noise Figure at 0.7 and 1 mA: a) theoretical, b) experimental.

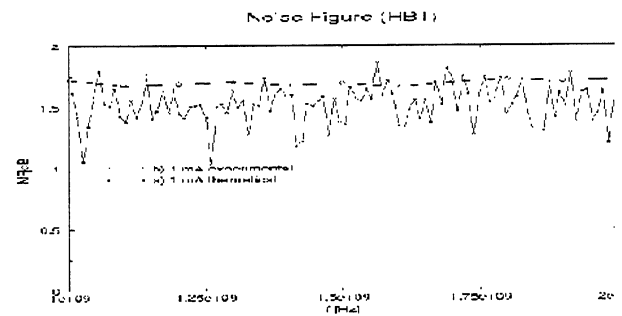


Figure 4: HBT Noise Figure at 1 mA: a) theoretical, b) experimental.

We can see that our theoretical results are in good agreement with experimental ones. Finally, we consider correlation between collector and base noise sources and observe that our fitting of the noise figure improves as module of correlation coefficient increases.

In figure 5 we show the BJT noise figure behaviour for different values of the correlation coefficient.

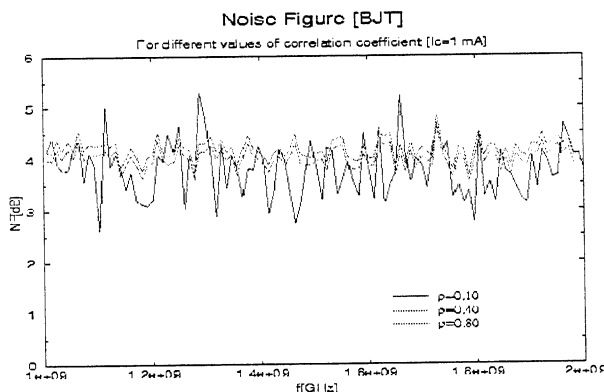


Figure 5: BJT Noise Figure at 1 mA for different correlation coefficient.

6 CONCLUSIONS

This approach, after few adjustments and improvements, will permit to predict noise figure device from only scattering parameter measurements and to avoid the main problems connected to perform additional measurements [3,4,5]. On the other hand we will obtain also more information about the statistical properties of internal noise sources of Bipolar transistors.

Conventional approaches analyse all noise sources into an equivalent noise voltage and assume Gaussian white noise, limiting therefore the analysis only to the knowledge of the first and the second moment of the stochastic processes associated to the noise sources.

As an alternative approach we derive a generalised Langevin equation, or more exactly a stochastic differential equation, for the Giacoletto model of bipolar transistors. The output current is a stochastic process as a consequence of the fluctuations due to the different internal noise sources of the transistor.

Applying the Langevin approach to the Giacoletto model we can know the statistical properties of the output current from the knowledge of the probability distribution of the noise sources and the correlation functions between the internal sources of the bipolar transistors.

Concerning the effect of the correlation between the noise sources we study only the case of a correlation between collector and base noise sources and we find that a greater correlation coefficient between these noise sources improves the noise figure of our device, as it can see from the last figure 5.

We can improve our noise figure fitting with the experimental measurements, by changing the probability distribution of the noise sources and testing the effect of the correlation functions between all the noise sources of the microwave transistors considered.

Finally we can apply our method for a wide variation of the bias point and, after changing our simple circuit, to different devices and for a wider frequency range (Figure 5).

Although the study in this paper has been restricted to bipolar transistors, the described methodology should be applicable to a wider class of microwave devices.

ACKNOWLEDGEMENTS

We wish to thank the ST-Microelectronics for hospitality at the Laboratory of Catania (Italy) to do measurements on devices useful for this study. We are grateful to Prof. Sannino and Prof. A. Caddemi for useful discussions. This work is supported in part by the MURST and the INFM.

REFERENCES

- [1] A. Caddemi, F. Patti, M. Sannino "Bias Dependence of the Noise performances in Si/SiGe HBT's at Microwave Frequencies", in *Electronic Devices for Microwave and Optoelectronic Applications*, ed. Ali A. Rezazadeh IEEE Microwave Engineering **98 TH**, 57-61, 1998.
- [2] V. Miceli, B. Spagnolo "Langevin Approach to study the Statistical properties of the Giacoletto Model of the Bipolar device," *Tech. Rep. University of Palermo*, April 1998.
- [3] H. Ghaddab, M. Gannouchi, F. Choubani and Boulegue "Small-Signal Modelling of HBT's Using a Hybrid Optimization/Statistical Technique," *IEEE Trans. Microwave Theory Tech*, vol 46, 292-298, 1998.
- [4] F. Patti, V. Miceli and B. Spagnolo, "Stochastic Approach to Noise Modelling of Bipolar Microwave Transistors," *Electronic Devices for Microwave and Optoelectronic Applications*, ed. Ali A. Rezazadeh IEEE Microwave Engineering **99 TH**, 236-241, 1999.
- [5] F. Patti, V. Miceli and B. Spagnolo, "Langevin Approach to Noise Modelling of Bipolar Microwave Transistors," *Nuclear and Condensed Matter Physics*, A. Messina ed. American Institute of Physics **513**, Melville, NY, USA, 1999, 241, 2000.