Time Series Modelling of Surface Topography Generated by Ultrasonic Machining Process

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ABSTRACT

In these studies the surface roughness profiles were modelled and analysed for a better understanding of the mechanism of metal erosion in USM. Different materials were selected for this purpose based on their diverse properties of brittleness, hardness and amorphism. Owing to the stochastic nature of the surface roughness, time series analysis was adopted for its modelling and analysis.

Keywords: Surface roughness, USM, stochastic, time series analysis.

1 INTRODUCTION

Surface texture being the imprint left behind by the machining process, which if analysed properly can lead to a better understanding of the basic mechanism of surface generation. In those of the machining processes where the tool is fed at a specific rate relative to the work surface, the final roughness is repetitive or periodic in nature and can be easily modelled mathematically. Such surfaces are called deterministic surfaces. On the other hand some surfaces like those from Electro Discharge Machining, (EDM) and Ultrasonic Machining (USM) are generated by erosion from random attack of electrical sparks or abrasive grains respectively. Such surfaces cannot be modelled and are termed as stochastic surfaces. Their random characteristics require statistical modelling. The roughness ordinates can be estimated in a sequence by digitizing the roughness profiles and treated as a time series, which can be statistically modelled by auto regressive (AR) modelling.

In the present work the USM surfaces have been selected for time series analysis through auto regressive modelling.

Machining of brittle and atmorphous materials like ceramics and glass is not possible by conventional machining. Ultrasonic machining (USM) is the suitable process (1), which employs a slurry of abrasive grains suspended in water. A tool vibrating at ultrasonic frequency provides the desired impact through the abrasive grains to erode microchips from random locations on the work surface.

2 TIME SERIES MODELLING

A stochastic process can be modelled by treating the random data (here the roughness profile ordinate distribution) as a time series, defined as a sequence of data in a particular order of time (t-2, t-1, t, t+1, t+2.....). Time may be replaced by other variables such as space. The technique is termed as auto-regressive modelling since it expresses the dependence of a variable on its previous values. A n_{th} order model [AR(n)] gives dependence of a variable X_t on **n** previous observations [2].

$$X_t=\phi_1 X_{t-1} + \phi_2 X_{t-2} + \ldots\ldots\ldots \phi_n X_{t-n} + a_t \qquad (1)$$

$$a_t \approx NID\ (0, \sigma^2_a)$$

where ϕ is auto regressive coefficient and a_t is the disturbance or error with zero mean and σ_a^2, variance. In systems with good memory of the shocks or disturbance, a_t is dependent on a_{t-1},

a_{t-2}........ In such case the models are called Auto regressive moving average or ARMA models and have the following form.

$$\text{ARMA }(n,m): X_t=\phi X_{t-1} + \phi_2 X_{t-2} + \ldots \phi_n X_{t-n} + a_t + \theta_1 a_{t-1} + \theta_2 a_{t-2} + \ldots + \theta_m a_{t-m} \quad (2)$$

The first part of the ARMA model represents the Auto regressive component whereas the second part represents the moving average component.

3 METHODOLOGY

Four different materials namely tungstan carbide, ferrite, glass and porcelain were machined on IMECO SONIDRILL keeping the machine setting and other process variables uniform in all cases to remove the effect of all parameters except the work material. The roughness profiles were obtained from an instrument called perthometer at a high magnification.

The digitised values of roughness ordinates were treated as a Time series. Appropriate AR models were then fitted to each set of data estimating the AR parameters using the technique described in Ref.[3]. The identification of the order of AR model requires the plots of auto correlation functions (ACF) and partial auto correlation functions (PACF). The number of significant spikes in PACF denotes the order of AR model.

4 RESULTS AND DISCUSSIONS

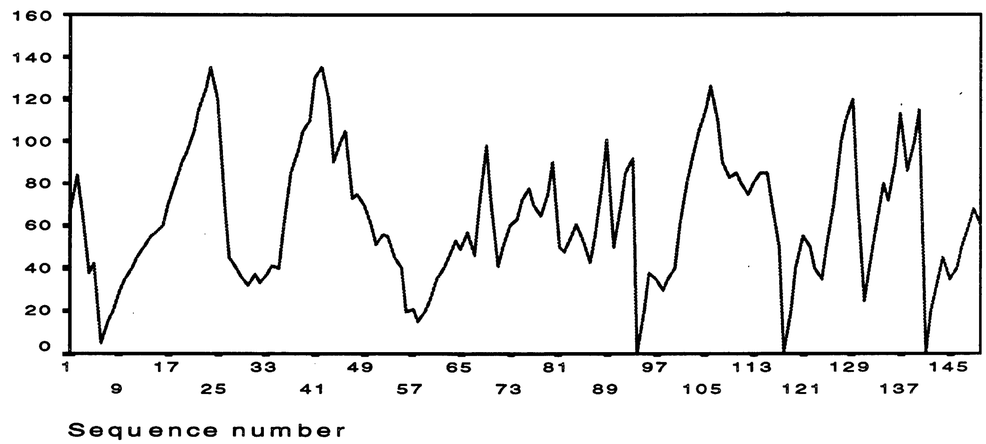
Fig.1 shows typical plots of the roughness ordinates, their ACF and PACF. Table1. lists the associated AR models. Interestingly for all the surfaces studied here low order AR models were found to be adequate and no MA component detected. This may be due to the limitations of the digitisation method adopted which was manual and very small undulations in the profile could have escaped digitisation. However this is not a limitation in any way for the analysis since in most cases of the ARMA models the MA component is only one order short than AR component [2]. A rougher surface is indicated with a higher value of regression coefficient in first order models (Glass compared to Tungsten Carbide) and with negative association of successive values (Porcelain compared to Ferrite).

Hardness, brittleness and amorphism improve machinability in USM [5,6] the basic nature of ultrasonic suggests repetitive impacts of abrasive grains therefore fatigue is a significant factor in material erosion which may be considered to occur in three stages of crack initiation, propagation and conjugation. This results in an incubation period before rupture [4]. For brittle non-crystalline materials like glass the incubation period is smallest and the crack initiation and propagation is fastest. In the absence of plastic flow and grain boundary resistance, the cracks run deeper. Stock removal is thus by impact crushing. The machining rate is high and so is the roughness. Hard and brittle materials but crystalline in nature, have slightly higher incubation period and the failure mode is cleavage fracture. The crack front propagation is resisted by grain boundaries. Repeated impacts lead to fatigue failure. Tough materials have highest incubation period and exhibit signs of plastic deformation and dents. Repeated hammering can lead to strain hardening and fatigue crack formation [5]. Individual cracks on coalescene lead to rupture by dimple formation.

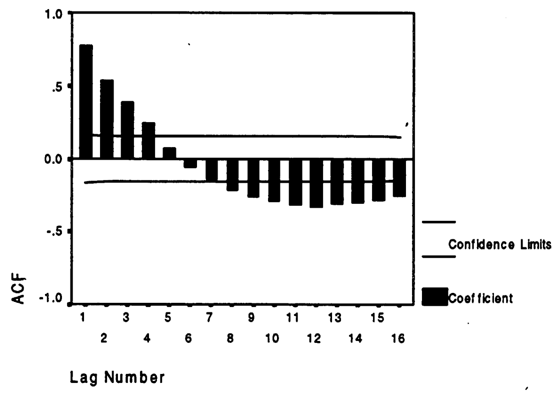
The complex nature of crack propagation in glass and porcelain lead to a highly non-deterministic nature of surface roughness. Ferrite and tungsten carbide are brittle but crystalline and manufactured from powder metallurgy method. As such they have well defined grain boundaries to guide the crack propagation. The eroded debris are smaller leading to lower roughness.

These AR models have the following form

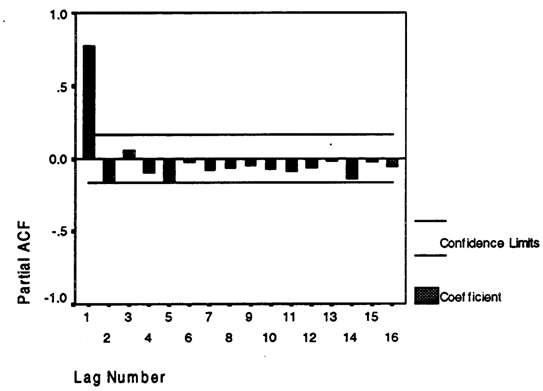
1. $X_t = 0.56563\, x_{t-1} + a_t$;
   $a_t \cong \text{NID}(0; 0.36)$
2. $X_t = 0.89788\, x_{t-1} + 0.15813\, x\, t\text{-}2 + a_t$;
   $a_t \cong \text{NID}(0; 0.27)$
3. $X_t = 0.78877\, x_{t-1} + a_t$; $a_t \cong \text{NID}(0; 0.40)$
4. $X_t = 0.91010\, x_{t-1} - 0.12633\, x_{t-2} + a_t$;
   $a_t \cong \text{NID}(0; 0.50)$



(a)



(b)



(c)

Fig.1 The roughness ordinate distribution (a), its ACF (b) and partial ACF (c) of Ferrite surface after USM.

Table 1. AR models of surface roughness profiles of different work materials

S.No.	Work Material	AR Model	AR Parameters	Ra Value (μm)	σ_a^2
1	Tungsten Carbide	AR (1)	ϕ_1=0.56563	0.56	0.36
2	Ferrite	AR (2)	ϕ_1=0.89788 ϕ_2=0.15813	0.91	0.27
3	Glass	AR (1)	ϕ_1=0.78877	1.56	0.40
4	Porcelain	AR (2)	ϕ_1=0.91010 ϕ_2= -0.12633	1.96	0.50

5 CONCLUSIONS

1. Low order AR models were found to be adequate for the surfaces produced by ultrasonic machining process.
2. Rougher surfaces are indicated by a higher value of regression coefficient and negative association of successive coefficients.
3. Amorphous materials have higher roughness compared to crystalline materials.

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