A Fluid Dynamics Model for a Plasma Ion Accleration System

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ABSTRACT

We present preliminary results of some simulations for a plasma ion acceleration system, which we plan to construct. This system tries to model the complex interactions between the multiple species of ions (about 5) used as the medium for particle acceleration. By modeling the ion species as fluids with the use of the well known magnetohydrodynamics (MHD) equations, Mac Cormack's method, a numerical technique commonly used in fluid dynamics simulations can be utilized.

Keywords: plasma physics, magnetohydrodynamics

1 INTRODUCTION

It was previously suggested that plasma is an ideal medium for particle acceleration because very strong transient electric fields (with energies in the order of a few GeV) can be generated within it by simply inducing the appropriate periodic disturbances to the uniform number density of a certain particle specie [1]. Methods for inducing this disturbance rely on either direct introduction of particle streams [2] or optical excitation [3] [4]. The quasi-neutrality of plasma also shields the environment from the strong electric fields that would otherwise tear apart the system's supporting structures.

2 NUMERICAL MODEL

In this paper, we present an idealized model of the system. The idealized model consists of 2 particle species, the electrons and the positive ions. The initial distribution of the particles are such that the net charge density of any point in space is zero. Each particle specie is represented by a 1 dimensional fluid, whose behavior is described by the 3 magnetohydrodynamic (MHD) equations [2] given as:

$$\frac{\partial n}{\partial t} + \frac{\partial (nv)}{\partial x} = 0 \tag{1}$$

$$m\left(\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x}\right) = E - \frac{1}{n}\frac{\partial (nT)}{\partial x} - R \tag{2}$$

$$\frac{3}{2}\left(\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial x}\right) + T\frac{\partial v}{\partial x} = qEv + \frac{1}{n}\left(\chi\frac{\partial T}{\partial x}\right) - Q \tag{3}$$

,where each of the variables n, T, and v refers to the number density, temperature and the average velocity respectively of a fluid representing a particular particle specie. Also note that m and q refers to mass and charge of a particle in the particular specie, while E refers to electric field, and χ is the thermal conductivity of the plasma. The terms Q and R refers to the energy and momentum loss due to particle collisions.

A time dependent disturbance, $n_1 = S(t)$, to the number density of the electron is introduced at $x = x_i$. The problem of the time evolution of the electric field is approached by taking advantage of the fact that it is described by the equation:

$$\frac{\partial E}{\partial x} = 4\pi\rho \tag{4}$$

.

Note that ρ is the linear charge density which is given by:

$$\rho = \sum_s q_s n_s \tag{5}$$

,where the summation over s is about the 2 particle species.

The time evolution of the 3 variables n, v, and T are solved using MacCormack's method [5] commonly used in fluid dynamics calculations, which computes the value of a space discretized variable for a particular time.

3 RESULTS

The figures below show the results of the simulation method for 6 different situations where periodic boundary conditions were imposed. Note that in all the cases considered below, a step size of 10^{-5} was used to satisfy the usual stability constraints for MacCormack's method. Also note that all the quantities are depicted in arbitrary units.

Figure 1 depicts the time evolution of the magnitude of the electric field, E, with respect to position, x, for the trivial case of n_1 being zero for all values of x and t. We use this set of parameters only as a test case of the simulation, since it is a well known result that a neutral plasma will stay neutral without the presence of any disturbance to the charge configuration. The flat profile of the surface in fig. 1 agrees with this expected result.

The case of an instantaneous disturbance present only at time $t = 0$ and at the position $x = 128$ is tackled next. This set of parameters corresponds to a physical system wherein the amount of charge in the system is conserved for time $t >= 0$. With this system, one would only expect a redistribution of charges towards a neutral configuration. Because of the small step size used, fig. 2 is not able to show the point at which the charge configuration becomes neutral. But note that the evidence of such a redistribution is already apparent.

Figures 3 to 6 depicts the variation of the electric field for cases wherein the charge disturbance is in the form a temporal square wave, present only at a single point in space. Figure 3 shows the results for a square wave which oscillates between a positive and a negative value. This corresponds to a system wherein at $x = 0$ a particle source and a particle sink appears alternately at different times. Note that these strength of these sinks and sources were chosen to depend on the charge density at $x = 0$ at that particular time. It is seen in fig. 3 that this type of disturbance produces a transient increase in the electric field magnitude around its vicinity.

Figure 4 shows the results for a system almost identical to that considered in fig. 3 except that the square wave oscillates between a positive value and zero. This depicts a physical system wherein a charge density dependent particle source appears and disappears during the duration of the simulation. It is seen in fig. 4 that despite the absence of particle sinks redistribution of charges still occur. It is also observed that the electric field magnitude is oscillating but markedly decaying.

Figure 5 shows the results for a system almost identical to that considered in fig. 4 except that the particle sources are not anymore dependent on the charge density. Figure 6 also depicts a similar system to fig. 5 except for that the disturbance is now placed at $x = 128$ instead of being at $x = 0$. Note that a similar, localized, oscillating peak is observed in both figures.

4 CONCLUSIONS

It was shown in this paper, that using a method commonly used in fluid dynamic simulations, we were able to get the expected results for an ideal plasma system whose behavior is described using the MHD equations. The method has the advantage of being straightforward to implement and being relatively efficient (order n, where n is the number of time steps). Extension to higher dimensions and more realistic models needed for our actual system is also not a big problem.

However, its inherent disadvantage is its instability for larger time steps (larger than the one used in this paper). This limitation reduces the period of time which can be covered by the simulations, since the smaller time steps means more iterations needed to finish a certain run. This limitation could drive us to look for other more stable methods, probably other ones also used in fluid dynamic simulations.

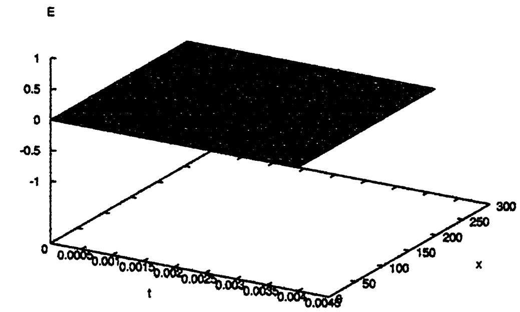

Figure 1: E vs. x and t, for the case without any disturbance in the charge density.

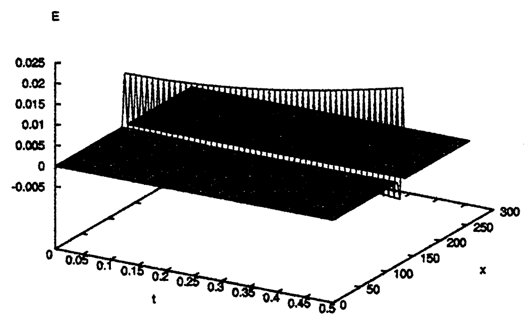

Figure 2: E vs. x and t, for the case of an instantaneous disturbance present only at $t = 0$ at $x = 128$.

REFERENCES

[1] T.Tajima and J.Dawson, Phys. Rev. Lett. 43, 267-70 (1979)

[2] C.S. Liu and V.K. Triapathi, "Interaction of Electromagnetic Waves with Electron Beams and Plasma", (World Scientific, Singapore 1994).

[3] C. Joshi, W.B. Mori, T. Katsouleas, J. Dawson, and D.Forslund, Nature 311, 525-9 (1984).

[4] G.Shivets, J. Fisch, A. Pukhov, and M. Vehn, Phys. Rev. E 60 (1999).

[5] J. Wendt *ed.*, "Computational Fluid Dynamics", (Springer, New York 1996).

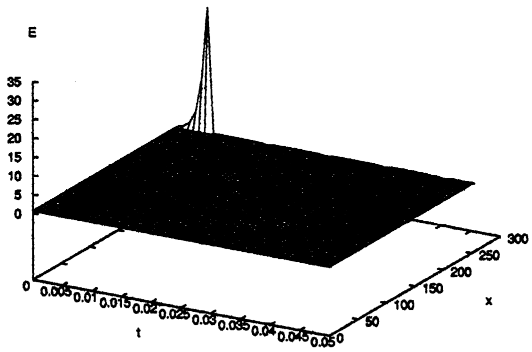

Figure 3: E vs. x and t, for the case of a temporal square wave periodic disturbance both in the form density dependent particle sources and sinks present at $x = 0$.

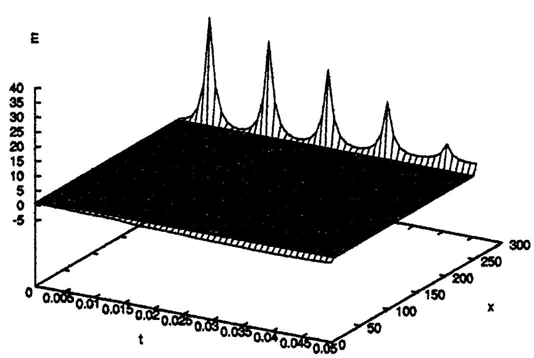

Figure 4: E vs. x and t, for the case of a temporal square wave periodic disturbance in the form of a density dependent particle source present at $x = 0$.

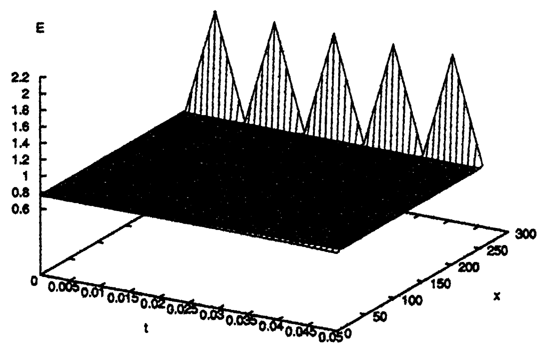

Figure 5: E vs. x and t, for the case of a temporal square wave periodic disturbance in the form of a particle source present at $x = 0$.

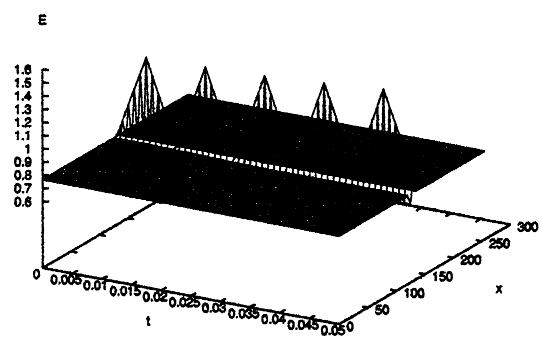

Figure 6: E vs. x and t, for the case of a temporal square wave periodic disturbance in the form of a particle source present at $x = 128$.