Eigenvalue Analysis of Tunable Micro-mechanical Actuator

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ABSTRACT

An eigenvalue analysis of a tunable micro-mechanical actuator is presented. The actuator is modeled as a continuum structure. The eigenvalue modified by the tuning voltage is computed through the linearization of the relation between the electrostatic force and the displacement at the equilibrium. A staggered algorithm is employed to perform the coupled analysis of the electrostatic and elastic fields. The stiffness matrix of the actuator is modified at this equilibrium state. The displacement field is perturbed using an eigenmode profile of the actuator. The configuration change of the actuator due to perturbation modifies the electrostatic field and thus the electrostatic force. The equivalent stiffness matrix corresponding to the perturbation and the change in the electrostatic force is then added to stiffness matrix in order to explain natural frequency shifting. The numerical examples are presented and compared with the experiments in the literatures.

Keywords: Electrostatic, Frequency shifting, Numerical Method, Coupled analysis.

1 INTRODUCTION

The micro-mechanical resonators, which are driven by electrostatic force, are widely used in active micro-electromechanical systems. Electrostatic micro-actuators are usually operated at their resonant frequencies, which will give the maximum displacement amplitude [1,2]. The resonant frequency tuning of the electrostatic micro-actuator is required for post-fabrication adjustment of resolution and bandwidth. The resonant frequency of an actuator system can be tuned generally by applying DC bias voltage.

Previous works on the analysis of tunable electrostatic actuators used simplified model. Micro-mechanical actuator is modeled as a simplified mass-spring system [3,4,6]. An advanced numerical method, which can treat continuous field, is used only in calculation of the electrostatic force applied on the device [5]. In real situation, the electrostatic field and the elastic deformation field of the system are 3-dimensionally coupled. And the actuator structure assumed as rigid body in previous works is generally deformable. Therefore in order to accurately predict the behavior of the real system, the coupled analysis of the fields in 3-D and an algorithm for evaluating the effects of electrostatic field on natural frequency are essential.

In this work, an eigenvalue analysis method for tunable electrostatic actuators is proposed. BEM and FEM are used for the 3-D coupled analysis of electrostatic field and deformation of a device. In order to consider the effects of electrostatic field on natural frequency, an equivalent stiffness matrix for electrostatic tuning voltage is introduced. The equivalent stiffness due to electrostatic force is determined through perturbation of structure using concerned eigenvector and linearization of the electrostatic force variation. A tunable micromirror is analyzed as an example and the numerical analysis results are compared with experimental data.

2 COUPLED ANALYSIS

Electrostatically driven microactuator system is governed by the electrostatic force and elastic spring force. The structure deforms due to the electrostatic forces acting on and then the electrostatic fields are modified by corresponding structural deformation. A coupled analysis of the electrostatic and elastic deformation fields needs to achieve the equilibrium state. An iterative staggered algorithm is employed to perform the electrostatic-elastic coupled analysis [7]. A simple flow chart of the staggered algorithm is shown in Figure 1.

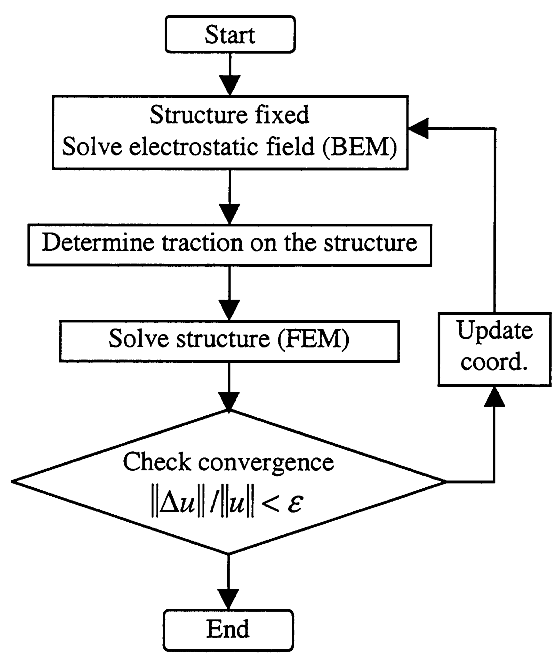

Figure 1: Flow chart of the solution procedure

The electrostatic field induced by the electric potential applied on the electrodes is analyzed by BEM. The electrostatic forces on the conductor surface can be obtained as follow:

$$f(\boldsymbol{x}) = -\frac{1}{2}\frac{q(\boldsymbol{x})^2}{\varepsilon}\boldsymbol{n}(\boldsymbol{x}) \tag{1}$$

where $q(\boldsymbol{x})$ is the charge density on the conductor surface, ε the dielectric permittivity of media and $\boldsymbol{n}(\boldsymbol{x})$ the normal vector to the inside of conductor.

The deformation of the structure caused by electrostatic force is calculated with FEM. Then the electrostatic field is modified due to the structural deformation and then the electrostatic forces are updated again. The iterative process continues until the equilibrium is achieved.

3 FREQUENCY TUNING

The equation of motion for a structure can be written as follows.

$$M\ddot{U} + KU = F \tag{2}$$

where $\boldsymbol{K}$ and $\boldsymbol{M}$, respectively, the stiffness matrix and the mass matrix of the system. $\boldsymbol{K}$ and $\boldsymbol{M}$ are made by finite element approximation of the structure. Above equation of motion can be uncoupled by the linear coordinate transformation that is defined by eigenvectors [8]. Each uncoupled equation is related only one eigenmode and can be regarded as single-degree-of-freedom system. Thus after uncoupling of the motion, we can easily separate the effects of the electrostatic field on each mode.

Let us consider one resonant frequency of the structure. Only one single-degree-of freedom equation is concerned instead of whole system of motion. The equation of motion can be expressed as follow:

$$m\,\ddot{q} + k\,q = f \tag{3}$$

where q denotes the generalized coordinate, k is the lumped stiffness, m is the lumped mass, *and* f is the lumped force. Each lumped parameter is defined as follows.

$$m = Q^T M Q \tag{4}$$

$$k = Q^T K Q \tag{5}$$

$$f = Q^T F \tag{6}$$

where $\boldsymbol{Q}$ is concerned eigenvector. Simply the eigenvalue of eqn.(3) is k/m. Figure 2 shows a simple structure and its eigenmode shape of interest.

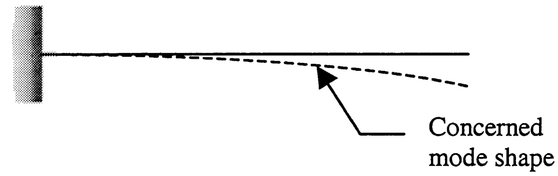

Figure 2: A simple structure and its mode shape

The generalized coordinate and the applied force can be decomposed into static equilibrium part and deviation from static equilibrium. The equation of motion can be written as follows.

$$m\,\Delta\ddot{q} + k(q_0 + \Delta q) = f_0 + \Delta f \tag{7}$$

where q_0, f_0, Δq and Δf is, respectively, equilibrium coordinate value, equilibrium force, increment of q and f deviation.

Because the electrostatic force balances the restoring force by elastic deformation at the static equilibrium, the equation of motion is described in terms of incremental variables.

$$m\,\Delta\ddot{q} + k\,\Delta q = \Delta f \tag{8}$$

The total force increment can be explained as sum of the external force, Δf_{ext}, and the electrostatic tuning force, Δf_{tuning}. The electrostatic tuning force comes from the deviation of the electrostatic field due to Δq. Δf_{tuning} can be approximated by first derivative of electrostatic force, for Δq is small.

$$\Delta f = \Delta f_{ext} - \Delta f_{tuning} = \Delta f_{ext} + \frac{\partial f_e}{\partial q}\Delta q \tag{9}$$

where f_e is electrostatic force applied to the structure. From eqn.(8) and eqn.(9), the equation of motion can be modified as follows.

$$m\,\Delta\ddot{q} + \left(k - \frac{\partial f_e}{\partial q}\right)\Delta q = \Delta f_{ext} \tag{10}$$

In above equation, stiffness is modified by ($-\,\partial f_e/\partial q$). Thus new eigenvalue can be expressed as follows.

$$\lambda = \frac{k - \partial f_e/\partial q}{m} = \frac{k - k_{tun}}{m} \tag{11}$$

The electrostatic tuning stiffness, k_{tun}, can be calculated by finite difference method. At first, system configuration is modified by $\Delta \boldsymbol{U} = \Delta q \boldsymbol{Q}$. Figure 3 shows the equilibrium states and a small perturbation of the system. Then the variation of electrostatic forces is evaluated at the perturbed configuration. Finally, the approximate tuning stiffness is calculated as follows.

$$k_{tun} \cong -\frac{\Delta f_e}{\Delta q} = -\frac{Q^T \Delta F_e}{\Delta q} \tag{12}$$

where $\Delta \boldsymbol{F}_e$ is electrostatic force increment vector. Using eqn.(11) and eqn.(12), the eigenvalue that is tuned by electrostatic field is given as follows.

$$\lambda_{tun} = \frac{k - k_{tun}}{m} = \left(k - \frac{Q^T \Delta F_e}{\Delta q} \right) / m \tag{10}$$

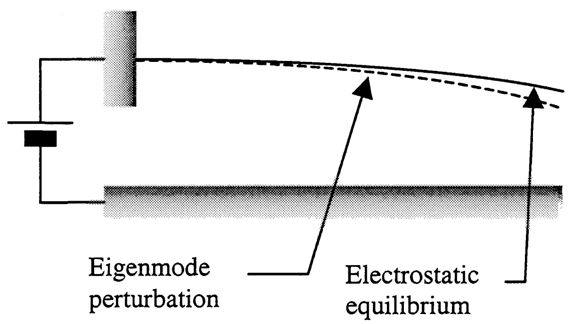

Figure 3: Equilibrium state and perturbation

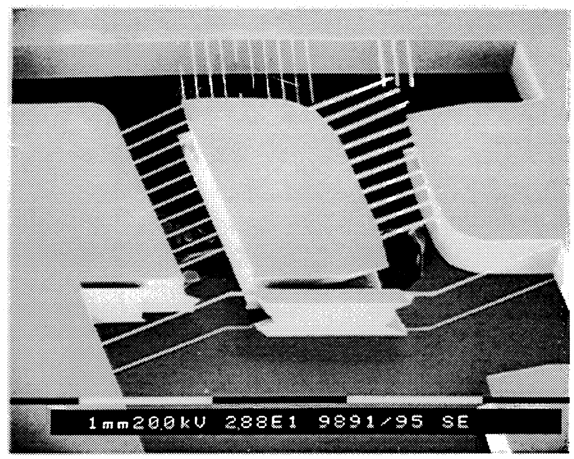

Figure 4: Photograph of micromirror structure

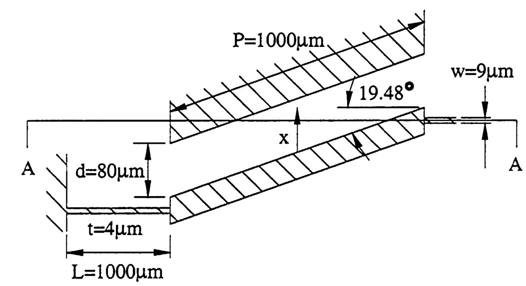

Figure 5: Top view of the silicon micromirror with a counter electrode.

4 APPLICATION

4.1 Tunable micromirror

The electrostatic tunable micromirror is applicable to a wide variety of static and dynamic optomechanical microdevices, including optical microswitches, optical shutters, laser beam choppers, optical filters and optical couplers. In figure 4, a SEM photograph of the micromirror fabricated by an anisotropic etching of (110) silicon wafer is presented [4]. Two pairs of boron-diffused microbeams suspend a bulk-micromachined electrostatic micromirror.

The vertical micromirror can be driven parallel to the silicon substrate by the electrostatic actuating force. The electric potential difference between two electrodes generates the electrostatic field and then the corresponding electrostatic forces are applied to the vertical micromirror. The net force on the micromirror is attractive in the x-direction, i.e. parallel to the substrate. Figure 5 shows the schematic diagram of structure with a counter electrode. At a large voltage, the mirror will be unstable and be stuck to the fixed conductor. In a static performance test, stable operation of micromirror has been observed up to the threshold voltage of 330V.

4.2 Frequency results

The resonant frequency of the micromirror can be adjusted by the tuning voltage, while the movable part is actuated by the driving voltage. In experiment, the resonant frequency of the fabricated micromirror can be found as the frequency at which the amplitude of displacement has the maximum value for various actuating frequencies. In numerical analysis, the natural frequency can be calculated by eqn.(10). Figure 6 shows the concerned mode (2nd mode) shape of the micromirror, calculated by FEM. In this system, the first mode is in the vertical direction. The resonant frequency tuning is performed with this concerned mode shape profile. The 0.1% perturbation of the maximum displacement is used in the calculation.

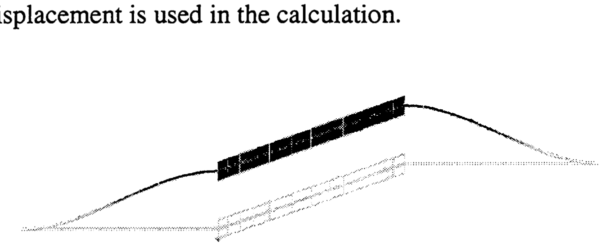

Figure 6: Concerned mode shape of the micromirror

Table 1: Material Properties.

Property	Value
Density	2330 kg /m3
Elastic modulus	130 GPa

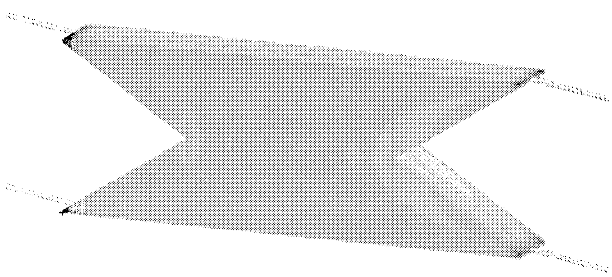

Figure 7: Charge density on mirror surface at 150V

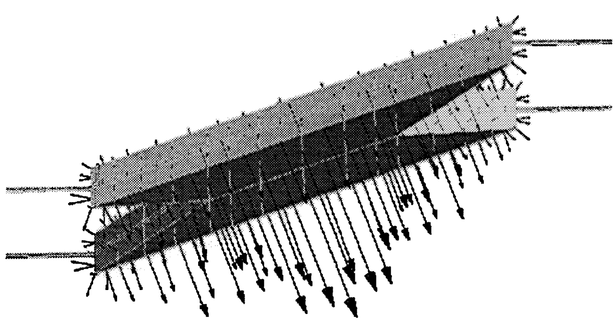

Figure 8: electrostatic force on mirror at V150

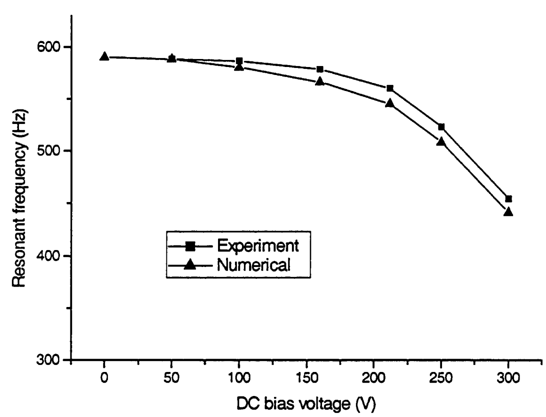

Figure 9: Resonant frequency of the micromirror

The mechanical properties of silicon are presented in Table 1. Figure 7 and Figure 8, respectively, show the charge density and electrostatic force on the micromirror surface when the system is in the equilibrium for the tuning voltage of 150V.

Figure 9 compares the estimated and measured frequency of the frequency tunable micromirror for the varying tuning voltage. From figure 7, 23% reduction of the resonant frequency is measured, and 25.3% reduction of resonant frequency is calculated for the tuning voltage increase of 300V. Considering the fact that the geometrical irregularities in the fabricated structure are not exactly reflected in the computational model, the differences seem to be moderated. It is noted that the numerical results are very close to the experimental results and the proposed numerical scheme is reasonable.

5 CONCLUSIONS

An eigenvalue analysis method for tunable electrostatic actuators is proposed. By the use of BEM and FEM, the 3-D coupled analysis of electrostatic field and deformation of a device are performed accurately. And the natural frequency tuning from electrostatic field is efficiently estimated using small perturbations with concerned mode profile. A tunable micromirror is analyzed as an example and the numerical analysis results are compared with experimental data. From the results, the result of the proposed numerical method is close to the experiments.

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