Vibration Analysis of Quartz Yaw-Rate Sensor to Reduce Mechanical Coupling

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ABSTRACT

For a yaw-rate sensor (gyroscope), the reduction of mechanical coupling phenomenon is a key technology for achieving high performance. We analyze the phenomenon with a combination of two models, (1) a static model of a bending sensor beam with an geometrically asymmetric cross section, and (2) a dynamic model of two degrees of freedom system. This analysis proves that an origin of the mechanical coupling is a static tiny displacement due to the asymmetry of the cross section, and the tiny displacement is amplified extremely by resonance phenomenon. The relationships between the extent of mechanical coupling, the asymmetric ratio, and a resonant frequency differential ratio, are also obtained. The relationships give a basic guideline for yaw-rate sensor design and fabrication.

Keywords: yaw-rate sensor, mechanical coupling, resonance, asymmetry, output stability at zero point

1 INTRODUCTION

1.1 Sensor Output at Zero Point

For automobile dynamic control, an accurate yaw-rate sensor is required[1]-[3]. Above all, output stability at zero point (with no yaw-rate) for temperature is highly important for the control. To stabilize the sensor output at zero point, it is very effective to decrease the magnitude of the output at zero point itself. We estimated some electrical and mechanical factors that had effects on output at zero point. In consequence, we reached the conclusion that mechanical coupling phenomenon is the most dominant factor for output at zero point. The behavior of yaw-rate sensors with mechanical coupling was reported[4]. In the article, the mechanical coupling between a driving part and a detecting part is expressed by a coupling coefficient k for a two degrees of freedom system. The origin of the coefficient k, however, is not known. In this paper, we propose a new method of finding the coupling coefficient k from the geometric asymmetry of a beam.

1.2 Quartz Yaw-Rate Sensor

Conventional metal beam fork type sensors with PZT[5] are complex and expensive. We realized a new yaw-rate sensor which is made of quartz with micromachining technology. As quartz itself has good mechanical stability for variation of temperature, the sensor has high performance and is suitable for mass production. The structure of the sensor is shown in Fig. 1. The shape of the sensor element is H. The upper fork is used for drive and the lower is used for detection by means of the piezo electric effect. With a yaw-rate Ω, a Coriolis Force that occurs at the upper fork is transmitted to the lower fork for the detection.

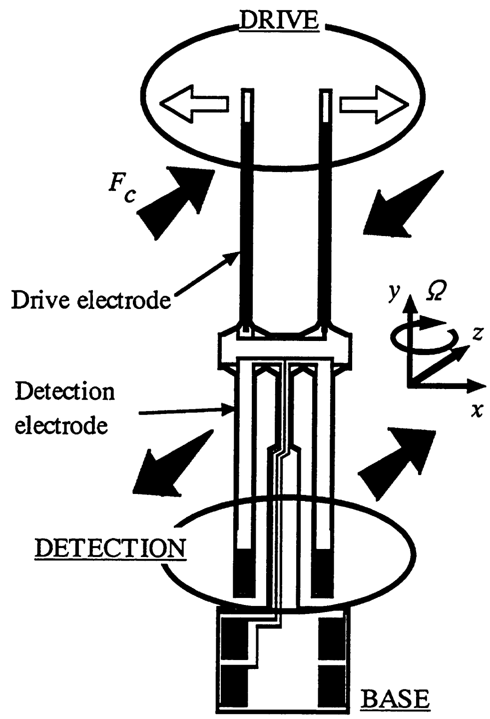

Fig. 1 Quartz Yaw-Rate Sensor

$$\vec{F}_c = 2m\vec{V} \times \vec{\Omega} \quad (1)$$

1.3 Fabrication

A close-up view of a cross-section of the quartz yaw-rate sensor beams is shown in Fig. 2. The shape of each cross section is not square, but a pentagon because each beam has a

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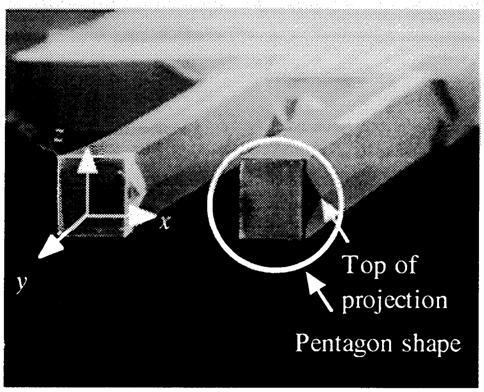

Fig. 2 Close-up view of cross section of the quartz yaw-rate sensor beam

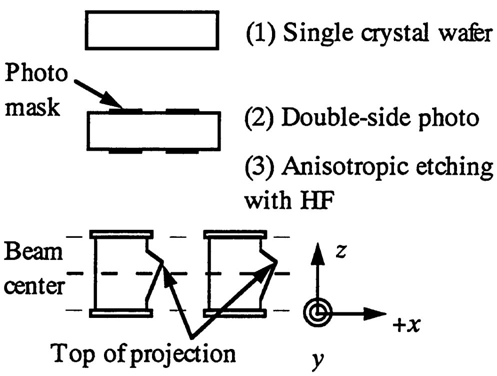

Fig. 3 Quartz beam etching process

projection on the right side. This projection is produced by anisotropic etching with HF solution, as shown as Fig. 3, because quartz has a trigonal system. As the difference of etching speed between both wafer faces, the top of the projection deflects from a beam center, and the deflection to the beam center makes the cross section asymmetric. Although we discuss here the quartz sensor, even a silicon type sensor with D-RIE (Deep Reactive Ion Etching) process, has asymmetry existing on the cross section.

2 MODELING

First we came up with a static model of the pentagon cross-section that has the projection deflected from the beam center. Second we combined the static result with a dynamic model of two degrees of freedom for the yaw-rate sensor.

2.1 Static Model of Asymmetric Cross Section

Suppose there is a bending beam that has two axes of symmetry with a driving force. One must note carefully that if one of the symmetric axes and the direction of the driving force are not equal, the direction of bending and that of the driving force will be different[6] .
In Fig. 4, the cross section of the pentagon is expressed by the combination of a square A and a triangle B. The triangle

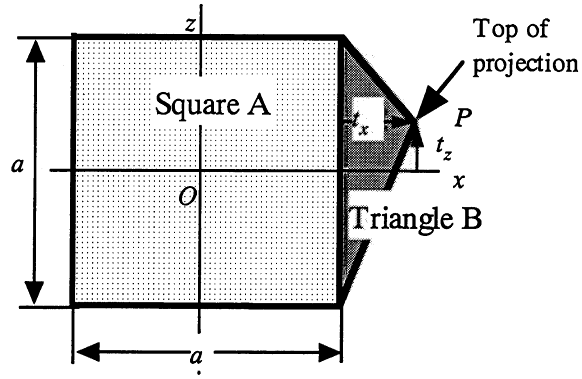

Fig. 4 Modeling of the asymmetric beam cross section

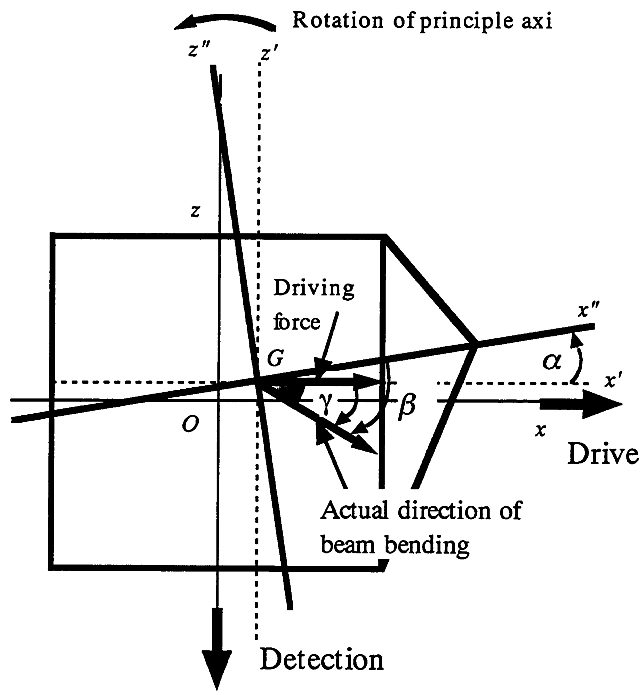

Fig. 5 Transfer and rotation of principal axes by the geometric asymmetry

B means the projection that deflects from the beam center. The top position P is away from square A by t_x and from x axis by t_z. Only with the symmetrical square A, the principal axes of the cross section are x and z in coordinates o-x-z. On the other hand, with the asymmetrical (square A + triangle B), the principal axes move to coordinates G-x'-z' and rotate at angle α to coordinates G-x''-z'' in Fig. 5. When a driving force is applied to the beam parallel to the x axis, the pentagon beam bends in the rotated direction at angle β from the x'' axis, because the direction of the principal axis x'' differs from that of the driving force. The α is given by the "second moment of area" $I_{x'}$, $I_{z'}$ and the "product of inertia of area" $I_{x'z'}$ in coordinate G-x'-z' by Eq. 2. The β is given by the "second moment of area" $I_{x''}$, $I_{z''}$ in coordinate G-x''-z'' and the α by Eq. 3. In consequence, the beam bends

$$\alpha = \frac{1}{2}\tan^{-1}\left[\frac{2I_{x'z'}}{I_{z'} - I_{x'}}\right] \tag{2}$$

$$\beta = \tan^{-1}\left[\frac{I_{x''}}{I_{z''}}\tan(-\alpha)\right] \tag{3}$$

$$\gamma = \alpha + \beta \tag{4}$$

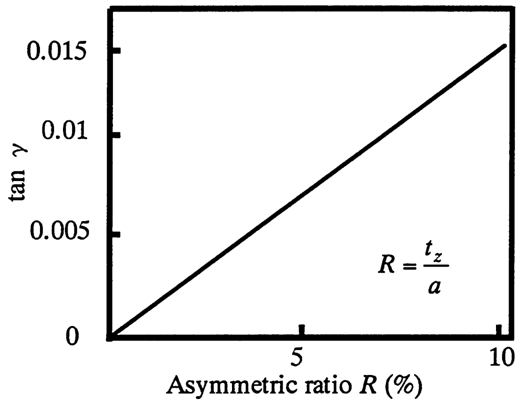

Fig. 6 Relationship between asymmetric ratio R and tan γ (t_x is 15%)

in the rotated direction at angle γ from the x' axis by Eq. 4. Fig. 6 shows that tan γ is proportional to an asymmetric ratio R. R is defined as t_z/a. In this way, the static tiny displacement toward z axis occurs. Therefore, a component of the displacement in the direction of the detection has arisen.

2.2 Dynamic Model of Two Degrees of Freedom System

For the yaw-rate sensor, a driving and detection resonant frequency are very close to each other so as to be sensitive to the yaw-rate, and the static tiny displacement to z axis is amplified extremely. A vibration model of two degrees of freedom for the yaw-rate sensor is defined in Fig. 7. In the figure, k_x and k_z *are* stiffnesses to x and z axis. Coupling coefficient k_{xz} means the coupling between a driving part and a detecting part[4]. c_x and c_z are dumping coefficients of each part. m is equivalent mass of the beam. A_x and A_z are the driving and the detecting amplitudes.

From a static even balance of force, Eq. 5 is given by

$$\begin{pmatrix} F_x \\ F_z \end{pmatrix} = \begin{pmatrix} k_x + k_{xz} & -k_{xz} \\ -k_{xz} & k_z + k_{xz} \end{pmatrix}\begin{pmatrix} A_x \\ A_z \end{pmatrix} \tag{5}$$

$$k_{xz} = k_z \frac{\tan\gamma}{1 - \tan\gamma} \tag{6}$$

where F_z=0, Eq. 5 is transformed as Eq. 6 with γ. Eq. 6 means the coupling coefficient k_{xz} is a function of tan γ. Since the direction of the drive and the detection is perpendicular, the sensor vibration model can be defined as a two degree of freedom system connected by the coupling coefficient k_{xz}. The Motion equation is derived as Eq. 7.

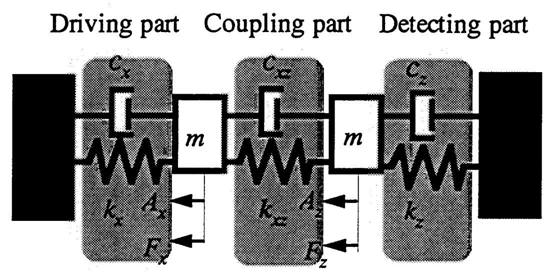

Fig. 7 Vibration model of two degrees of freedom system

To solve Eq. 7 as to the driving amplitude A_x and the detecting amplitude A_z. It is assumed that the resonant frequency differential ratio S is defined as $(f_x - f_z)/f_x$ with the resonant frequency of drive f_x and that of the detection f_z. The k_x and the k_z is given by Eq. 8.

$$k_x = m(2\pi f_x)^2$$
$$k_z = m(2\pi f_z)^2 \tag{8}$$

Damping coefficients c_x and the c_z are defined by quality factor Q_x and Q_z The c_{xz} is assumed to be zero because of the high quality factor Q_{xz} of quartzy crystal.

3 Results

Transfer functions were calculated with asymmetric ratio R of 5% as shown in Fig. 8. We show the mechanical coupling phenomenon by the transfer function. The ratio A_z/A_x at the driving frequency f_x (circled in Fig. 8) is the so called 'output at zero point'. It means that some of the vibrational energy transfers from the driving to the detection as a leakage vibration .

$$\begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix}\begin{pmatrix} \ddot{A}_x \\ \ddot{A}_z \end{pmatrix} + \begin{pmatrix} c_x + c_{xz} & -c_{xz} \\ -c_{xz} & c_z + c_{xz} \end{pmatrix}\begin{pmatrix} \dot{A}_x \\ \dot{A}_z \end{pmatrix} + \begin{pmatrix} k_x + k_{xz} & -k_{xz} \\ -k_{xz} & k_z + k_{xz} \end{pmatrix}\begin{pmatrix} A_x \\ A_z \end{pmatrix} = \begin{pmatrix} F_x \cos\omega t \\ 0 \end{pmatrix} \tag{7}$$

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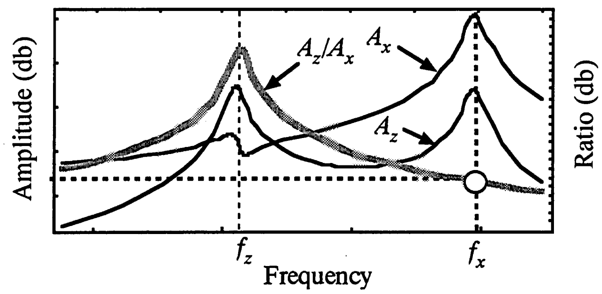
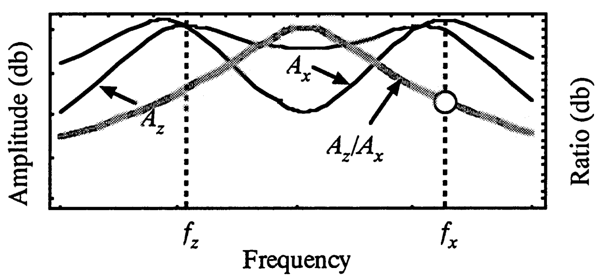


(a) f_z is far from f_x (S = 5%)



(b) f_z is near to f_x (S = 0.8%)

Fig. 8 Transfer function between the drive and the detection

The ratio A_z/A_x increases in inverse proportion to the resonant frequency differential ratio S. When f_z is near to f_x, A_z can be equivalent to A_x at f_x. In such a case, as f_x and f_z are dependent on each other, the mechanical coupling is too huge to reduce the sensor output at zero point.

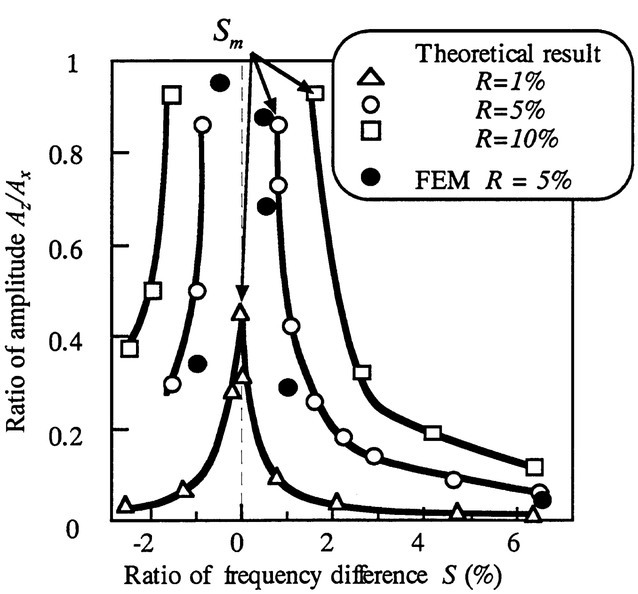


Fig. 9 Relationship between ratio of amplitude A_z/A_x and asymmetrical ratio R, and ratio of frequency difference S

The relationships between the ratio of leakage vibration amplitude A_z to the driving amplitude A_x, and the resonant frequency differential ratio S, are shown in Fig. 9. To realize A_z/A_x = 0.1 at S = 1%, the asymmetric ratio R should be below 1%. In Fig. 9, FEM results of R = 5% are also shown. These are well matched to the results given by the theoretical calculations.

The minimum S is indicated as S_m in Fig. 9. S should be small to achieve a high sensitivity, but S is limited to S_m because of the existence of the large mechanical coupling.
To reduce the mechanical coupling, we carried out the following methods: (1) improvement of the etching process to decrease the height t_x and the deflection t_z of the triangle B, (2) mechanical modification of the rotation angel α of principle axes after the etching process, and (3) setting S above 1%. With these methods, the mechanical coupling decreases enough to become practicable.

4 CONCLUSIONS

We clarified the origin of the mechanical coupling in yaw-rate sensor by analysis of a static model and a vibration model. The mechanical coupling occurs by the combination of the asymmetry of the cross-section of the sensor beam and the resonance phenomenon.
The theory was applied to a quartz yaw-rate sensor design and fabrication, and enabled reduced output at zero point of the sensor, which we have developed for automobile use. This idea can be applied not only to the quartz yaw-rate sensor, but also to all kinds of vibrating type sensors and actuators.

REFERENCES

[1]A Reppich, Yaw Rate Sensor for Vehicle Dynamics Control System, SAE paper 950537 (1995)
[2] Y Tomioka, S Sugawara, M Konno, Characteristics of a Quartz Crystal Tuning Fork by Finite Element Analysis The Transactions of the IECE of JAPAN, Vol. E63, No.3, pp. 177-183 (1980)
[3] N Wakatsuki, M Ono, S Yamada, LiTaO3 Crystal Fork Vibratory Gyroscope, Proceedings of the IEEE Ultrasonics Symposium pp. 581-584 (1994)
[4] Jan Soderkvist, Dissertations of UPPSALA Univ., pp. 78-79 (1990)
[5] T Ichinose, Angular Rate Sensor for Automotive Application, SAE paper 950535 (1995)
[6]K Yuasa, Strength of material, pp. 126-131 (1963)