# A New Material Interpolation Scheme for the Topology Optimization of Thermally Actuated Compliant Micromechanisms

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## ABSTRACT

Thermally actuated compliant micromechanisms are selectively heated elastic continua that take advantage of nonuniform thermal expansion and mechanical leveraging to amplify displacements and forces. These can be designed systematically, as shown in the earlier work by others and us, by using *material distribution* type topology optimization techniques. By noting that using more than one material helps in enhancing the performance of these mechanisms, we show in this paper how only one fictitious density variable is enough to handle not just two but multiple materials, and in fact, multiple properties of multiple materials. This has a clear practical advantage in reducing the size of the optimization problem and a functional advantage in being able to handle multiple properties. This is achieved with a new material interpolation scheme that we call a *peak function*. We also show how convection, from top, bottom, and side surfaces, can also be included in topology optimization. Illustrative examples are included.

***Keywords***: Topology optimization, thermal actuation, multi-material design.

## 1 INTRODUCTION

Material and shape are two important choices a designer has to make. Recent advances in topology optimization allow the designer to achieve the optimum shape(s) of a chosen material or a set of materials for a given task. It should be noted, however, that all shapes produced by the topology optimization method may not be easily manufacturable. Thus, it is important to realize that manufacturing controls what materials can be processed into what shapes. Fortunately, in the case of micro-scale devices, the micromanufacturing techniques based on photolithography and others allow almost any shape or topology in 2-D. Thus, systematic design of mechanical components of Microsystems (or MEMS) using topology optimization has practical applicability. In this paper, we present a novel technique for topology optimization with multiple materials. Thermally actuated compliant micromechanisms are chosen to exemplify this technique.

It is well known in the MEMS literature that thermal forces are much larger than other types of actuation. Bonding two materials of dissimilar thermal expansion coefficients is one way to achieve this. It can also be achieved with any single conducting material as explained next. A nonuniformly heated, mechanically constrained elastic continuum will deform to achieve thermoelastic equilibrium. This is used as the principle of embedded actuation for micro compliant mechanisms to perform mechanical tasks [1-3].

The design problem here is to obtain a topology and shape for a specified behavior. Three such situations are shown in Figures 1a-1c. In each case, the distribution of material within the design domain (rectangular in this case) is to be determined. This is done in topology optimization by assigning a fictitious density variable at each point and letting the optimization algorithm find the optimal density. In [3], it was shown that using multiple materials helps in improving the performance. Then, for each point two density variables are needed in that work. Here, we present a new technique in which only one variable is enough.

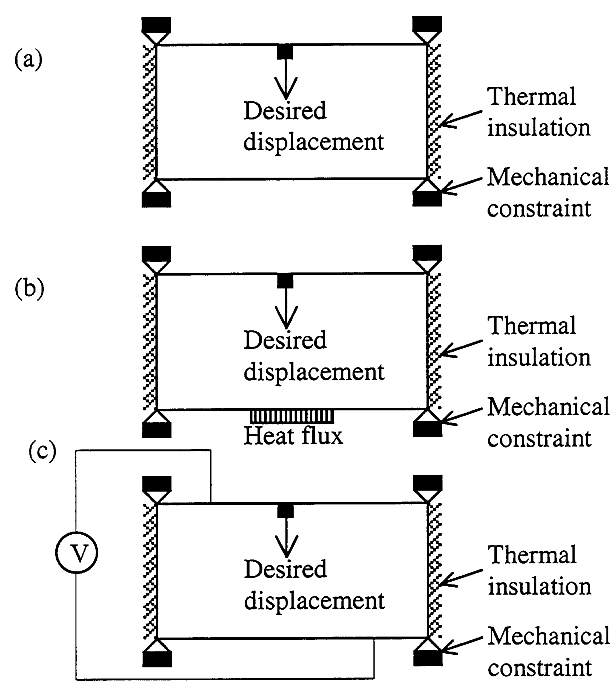



Figure 1: Three types of embedded thermal actuation (a) uniform heating (b) input heat flux on a portion of the boundary (c) electro-thermal Joule heating with voltage application between two points on the boundary

## 2 THE TOPOLOGY DESIGN PROBLEM

As shown in Figure 1, the material should be distributed within the permissible geometry so that the designated point moves in the desired direction upon heating or voltage application. If the resulting device is used as an actuator or a manipulator, the objective will be to maximize this displacement. There can also be constraints on the amount of material used, an upper bound on the ensuing maximum temperature or current, etc. Furthermore, as is usual in compliant mechanism design problems [4], a stiffness measure (strain energy) should also be included.

In what follows, we use a multi-criteria objective function that combines the flexibility (maximum output displacement), minimizing maximum temperature, and maximizing stiffness (minimize strain energy) requirements. The thermal and elastic equilibrium governing equations are included in their weak form as constraints to the optimization problems. $\rho(x)$ is the fictitious design variable at every point $x$ and is the optimization variable. The formulation is written in such a way that multiple materials can be distributed within the domain. We explain how multiple materials are handled in topology optimization in Section 3.

### 2.1 Uniform heating

For given uniform temperature rise, $\phi = T - T_0$,

Minimize $-u_{out}$ (i.e, maximize output displacement)

Subject to

$$\int_\Omega \boldsymbol{\varepsilon}(\mathbf{u}) : \mathbf{E} : \boldsymbol{\varepsilon}(\mathbf{w}_u) d\Omega = \int_\Omega \boldsymbol{\beta} : \boldsymbol{\varepsilon}(\mathbf{w}_u) \phi d\Omega \qquad (1)$$

for all $w_u \in U$ (kinematic admissibility)

$$\begin{Bmatrix} E_{ijkl}(x) \\ \beta_{ij}(x) \end{Bmatrix} = \sum_{m=1}^{n} \rho_m(x) \begin{Bmatrix} E_{ijkl}^m \\ \beta_{ij}^m \end{Bmatrix} \qquad (2)$$

$$\rho_m(x) = \begin{cases} 1 & \text{if } x \in \Omega^m \\ 0 & \text{if } x \in \Omega^m / \Omega \end{cases} \qquad (3)$$

$$\int_\Omega \sum_{m=1}^{n} \delta_m(x) d\Omega \le V \qquad (4)$$

where

$\mathbf{u}$ = displacement; $\boldsymbol{\varepsilon}(\mathbf{u})$ = strain; $\mathbf{w}_u$ = virtual displacement (or test function for $\mathbf{u}$); $\boldsymbol{\varepsilon}(\mathbf{w}_u)$ = virtual strain; $\mathbf{E}$ = constitutive (stress-strain) property tensor; $\beta_{ij} = E_{ijkl}\alpha_{kl} = E_{ijkl}\alpha\delta_{kl} = E_{ijkk}\alpha$ ; $\alpha$ =thermal expansion coefficient; $V$ = upper bound on the volume of material; $\Omega$ = design domain; $\Omega_m$ = design domain occupied by material $m$; $n$ = number of materials to be distributed

Equation (1) is the thermo-elastic equilibrium governing equation in the weak form, Equations (2) and (3) are multi-material selection (to be explained further in Section 3), and Equation (4) is the volume constraint.

### 2.2 Non-uniform heating with input heat flux

$$\text{Minimize } -\left\{ \frac{\text{output displacement}}{\text{"heat work"} + \text{strain energy}} \right\} =$$

$$-\frac{u_{out}}{\int_\Omega \nabla^T \phi k \nabla \phi d\Omega + \frac{1}{2}\int_\Omega \boldsymbol{\varepsilon}(\mathbf{u}) : \mathbf{E} : \boldsymbol{\varepsilon}(\mathbf{u}) d\Omega}$$

Subject to

$$\int_\Omega \nabla^T \psi k \nabla \phi d\Omega - \int_\Omega \psi Q d\Omega - \{\psi\}^T \{q\} = 0 \qquad (5)$$

$$\int_\Omega \boldsymbol{\varepsilon}(\mathbf{u}) : \mathbf{E} : \boldsymbol{\varepsilon}(\mathbf{w}_u) d\Omega = \int_\Omega \boldsymbol{\beta} : \boldsymbol{\varepsilon}(\mathbf{w}_u) \phi d\Omega \qquad (6)$$

$$\begin{Bmatrix} k(\mathbf{x}) \\ E_{ijkl}(\mathbf{x}) \\ \alpha_{ij}(\mathbf{x}) \end{Bmatrix} = \sum_{m=1}^{n} \rho_m(\mathbf{x}) \begin{Bmatrix} k^m \\ E_{ijkl}^m \\ \alpha_{ij}^m \end{Bmatrix} \qquad (7)$$

where

$k$ = thermal conductivity; $\psi$ = test function for $\phi$

$q$ = specified input heat flux

Equation (5) is the thermal equilibrium governing equation in the weak form. Note that since the objective function includes self-conflicting criteria, a volume constraint is not necessary in this case. It can be included if desired.

### 2.3 Modeling convection in the topology optimization

It was shown in [5] that convection can have significant effect on thermally actuated micro devices. Therefore, it should be included in the model in design to produce practically useful designs. Modeling convection in the topology optimization is not straightforward because we do not know *a priori* where the material and holes are going to be. We solve this problem by using the fictitious density to modify the convection heat transfer coefficient appropriately depending on whether there is material or not. We consider convection from the top and bottom surfaces as well as the sides. Convection from the sides poses additional problem because we now need to pay attention to the surrounding points to see if the current point is on the boundary of a hole created by the topology optimization. This can be done with the help of discretized model. With convection included, the thermal equilibrium governing equations will be modified as shown below in Equation (8). Note this equation also includes the heat generation term. When the third type of the problem (Figure 1c) is considered, this will depend on the design variable. Due to limited space, this problem is not described here.

$$\int_\Omega \nabla^T \psi k \nabla \phi d\Omega + \int_\Omega 2\psi \hat{h} \phi d\Omega - \sum_e \left( \sum_{k=1}^{s} \int_{l_k} \psi \hat{h} \phi dl^k \right) + \int_\Omega \psi Q d\Omega - \{\psi\}^T \{q\} = 0 \tag{8}$$

where

$$\hat{h} = \begin{cases} 0 & \text{if there is material} \\ h & \text{if there is a void} \end{cases}$$

$h$ is the effective heat transfer coefficient for natural convection to the environment

$Q$ is the heat generation term

The third time is applicable to the discretized model. Therefore, $s$ is the number of neighbors for an element.

## 3 THE PEAK FUNCTION

In [3], when two materials are to be distributed optimally two fictitious density variables per point are required. When there are three materials, three are required and so on. Furthermore, these density variables need to be restricted to the range [0,1], which introduces added complexity in the numerical solution step. To address this problem, we present here a new material interpolation technique wherein multiple properties of multiple materials can be interpolated with just one fictitious density variable $\rho$ which can take any value from $-\infty$ to $\infty$. We do this by multiplying the property to be interpolated by a *peak function* which is simply a weighted linear combination of several Gaussian normal distribution functions. For example, if the Young's modulus of two materials and void need to be interpolated, we use the following.

$$E = E_1 e^{-\frac{(\rho-\mu_1)^2}{2\sigma_1^2}} + E_1 e^{-\frac{(\rho-\mu_2)^2}{2\sigma_2^2}} + E_{void} \tag{9}$$

This is illustrated in Figure 2. The "mean" values of $\mu_1$ and $\mu_2$ are chosen as desired. The "standard deviation" values $\sigma_1$ and $\sigma_2$ decide how smoothly the property is interpolated. When $\sigma_1$ and $\sigma_2$ are almost zero, we get peaks at locations where the density variable $\rho$ is equal to $\mu_1$ or $\mu_2$ thus giving material 1 or material 2. Any other value then will lead to a void. So, in optimization we begin with large values of and gradually decrease them to almost zero. As can be imagined, any number of materials and any number of properties (including heat transfer coefficients) can be interpolated in this manner. We can now write the general material interpolation function as shown in Equations 10 and 11. Note that a void term is included in these equations. This is common in the topology optimization to make sure that the interpolated properties do not go zero but to a numerically convenient very small value.

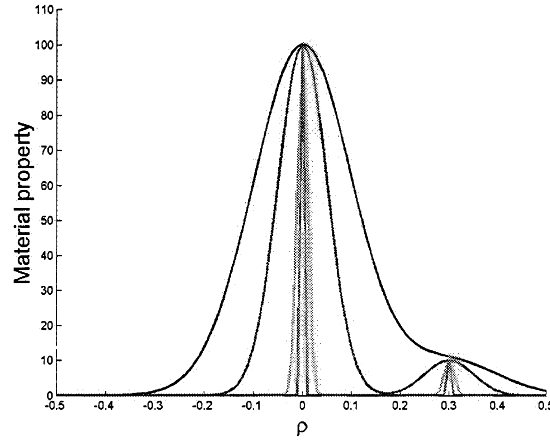



Figure 2: A peak function interpolating a material property for a two material and void case (different curves were drawn for different values of $\sigma_1$ and $\sigma_2$; $\rho$ is the design variable. $\mu_1$=0; $\mu_2$=0.3; $E_1$=100; $E_2$=10)

For material properties:

$$\begin{Bmatrix} k \\ E_{ijkl} \\ \alpha_{ij} \end{Bmatrix} = \sum_{m=1}^{n} \exp\left( -\frac{(\rho-\mu_m)^2}{2\sigma_m^2} \right) \begin{Bmatrix} k^m \\ E_{ijkl}^m \\ \alpha_{ij}^m \end{Bmatrix} + \begin{Bmatrix} k^{void} \\ E_{ijkl}^{void} \\ \alpha_{ij}^{void} \end{Bmatrix} \tag{10}$$

For convection heat transfer coefficients:

$$\hat{h} = h^{void} - \sum_{m=1}^{n} \left[ \exp\left( -\frac{(\rho-\mu_m)^2}{2\sigma_m^2} \right) \right] h^m \tag{11}$$

## 4 SOLUTION PROCEDURE

From the optimization problem statement, we write the Lagrangian. The first variation of the Lagrangian is set equal to zero as the first order necessary condition for a minimum. The resulting equation is used to derive an *optimality criteria* type design variable update formula. The Langrage multipliers corresponding to the constraints are updated in each iteration in an inner loop. See [6] for a general description of this technique.

## 5 EXAMPLES AND DISCUSSION

The specifications for Example 1 are shown in Figure 3a. A square design domain is mechanically anchored on three sides as shown and is heated uniformly throughout. A downward displacement is expected at the middle of the top edge. A solution for two-phase, i.e., a single material and void is shown in Figure 3b. Since the problem is linear in terms of elastic behavior, nominal material properties of a reference material were used to get the topology shown. It should be noted that only the top portion of the square design domain is utilized here.

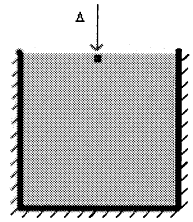
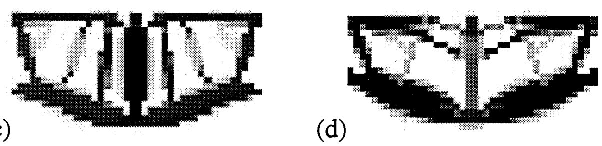



Figure 3a: Specification for the uniform heating example

Figure 3b: Optimal topology with a single material

Next, the same problem was solved with two materials. First with $E_1 = 5E_2$ and $\alpha_1 = 2\alpha_2$ were used to get the topology shown in Figure 3c. The (red) lighter color represents stronger material (i.e., material 1) and (blue) darker color represents the flexible material. Next with $E_1 = 5E_2$ and $\alpha_1 = \alpha_2 / 2$, the topology shown in Figure 3d was obtained. It can be seen that the material with smaller $\alpha$ was put in the central vertical bar because the objective is to maximize the downward displacement. So, it helps if this bar expands less.



(c) (d)

Figure 3c and 3d: Two-material designs for uniform heating

In the second example, a rectangular domain anchored at the four corners is given a heat flux on a small portion of the bottom edge as shown in Figure 4a. Three solutions for with and without convection are shown in Figures 4b-4d. This example was solved only with one material. In Figure 4b, the intermediate density exists even though the material interpolation was made as sharp as possible. That is if $\mu = 0$ and $\sigma$ was made very small so as to get a peak, some points continued to have $\rho$ equal to a 0.01 or 0.001 so that the interpolated material property was half or one-third the nominal value. Such an intermediate density is to be expected in body-force problems because putting material increases the force. Comparing Figures 4c and 4d, we can see that the hole has become smaller when convection from the side surfaces is included.

## 6 CONCLUSIONS

We have presented here a novel material interpolation technique for topology optimization with multiple materials. The technique was illustrated using thermally actuated micro compliant mechanisms. Another new contribution is including convection from top and bottom surfaces as well as from the sides of the holes created during optimization.

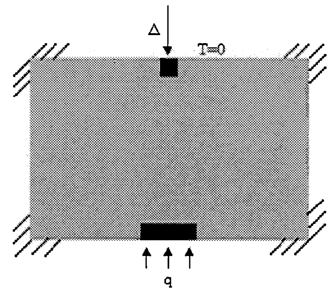



Figure 4a: Specifications for the input heat flux example

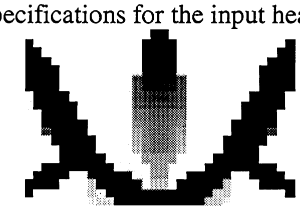


Figure 4b: Solution for Figure 4a without convection

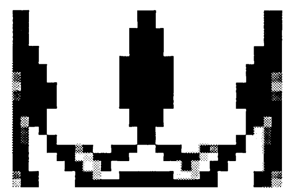


Figure 4c: Solution for Figure 4a with convection from top and bottom surface only

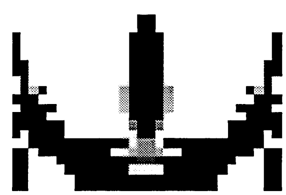


Figure 4d: Solution for Figure 4a with convection from top, bottom, and side surfaces

## ACKNOWLEDGMENTS

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