RESCUER – the New and Effective Tool for Automatic Model Reduction - Application to Electro-Thermal Problems

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ABSTRACT

This paper presents some methods of thermal model reduction and translation into hardware description languages or other ones. The very effective model reduction is achieved owing to the application of higher order approximation and joint symbolical and numerical reduction of considered problems.

Keywords: PDAE, DAE, multidomain simulations, MEMS, Smart Power, RESCUER, SPICE.

1 INTRODUCTION

The simulation of Micro-Electro-Mechanical Structures (MEMS) requires taking into account physical phenomena of different nature such as electrical, mechanical, thermal or fluidic ones. All those processes are governed by multidimensional PDAEs[1] [11]. The analytical approach is often insufficient for the description of all the necessary details of modelled devices. On the other hand, using classical FEA[2] /BEM[3] simulators (e.g. ANSYS, CFDRC ACE, CAEFEM – FEMAP), it is difficult to model multidomain phenomena in modern devices together with real electrical circuits containing different devices, such as OPAMPs, low voltage or high power devices [7]. The interfacing of these simulators to purely electrical ones requires creation of special synchronisers, but then the simulation time and memory requirements may become unacceptable. In order to simplify the modelling and simulation of recently developed multidomain silicon structures, the RESCUER programming tool has been created [12]. This tool is designed so that to represent in a simple way certain classes of second order PDE[4] and PDAE problems as models written in the developed by the authors high-level description language. The RESCUER renders possible quasi-automatic translation of the models into specified commonly used simulation environments such as ELDO HDL-A hardware description language, Mathematica language or equivalent electrical circuits in XSPACE language (ELDO SPICE, MicroSim PSPICE, ICAP4 - IsSPICE4, Berkeley SPICE and others). Both the final translated models as well as the already built-in ones can be used together in any chosen environment.

2 MULTIDOMAIN MODEL TRANSLATION

Models encoded in the proposed by the authors high abstraction level (see Figure 1) are syntactically and lexically analysed and decomposed into fundamental elements such as points, constants, variables and equations (more detailed information about the syntax and the semantic of RESCUER language is included in [12]). The decomposed models are converted into DAEs[5] using

```
-- Diffusion ver. 1.0
-- Input: TIME=t, Output: T(x,t),
-- External functions: Cos[…], Exp[…]

object 1d Diffusion1 {
  const  Pi=3.1459265358979324;
  var    T=Cos[Pi*X_0/200];                    -- Initial value

--equ    r1(T):poisson[T,diff[T,t]*Pi*Pi/20000];  -- or e.g.
  equ    r1(T):diff[T,x,x]==diff[T,t]*Pi*Pi/20000;

/* object bc1 {
            var T=Cos[Pi*X_0/200];
            equ r1(f):T==Exp[-TIME/2];
             point p1(0);
  }; */   -- or e.g
point p1(0){  -– b.c.
            var T=1;
            equ r1(T):T==Exp[-TIME/2];
};

  point P2(10);point P3(20);point P4(30);point P5(40);
  point P6(50);point P7(60);point P8(70);point P9(80);
  point P10(90);

  point p11(100){
            var T=Cos[Pi*X_0/200];
            equ r1(T):T==0;
  }  -– b.c.

}
```

Figure 1 One-dimensional diffusion problem described in the RESCUER language.

[1] PDAE – Partial Differential-Algebraic Equation
[2] FEA – Finite Element Analysis
[3] BEM – Boundary Element Method
[4] PDE – Partial Differential Equation
[5] DAE –Differential-Algebraic Equation

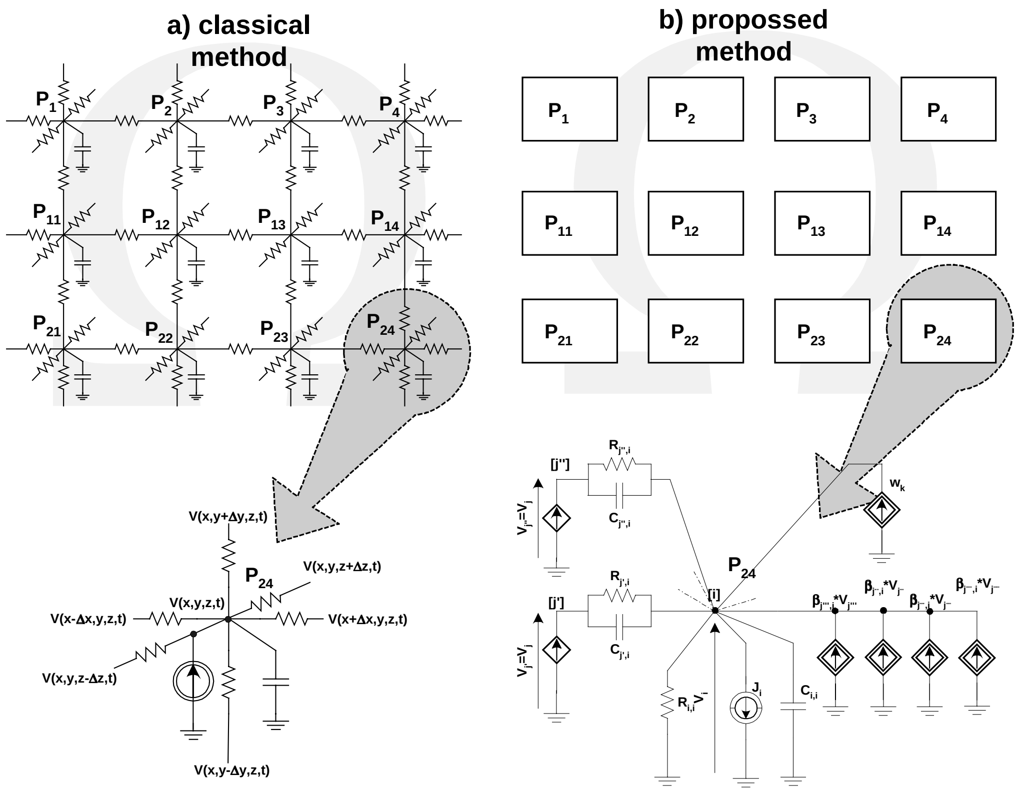

Figure 2 Outline of the transformation methods of the differential equations to the electric equivalent circuit.

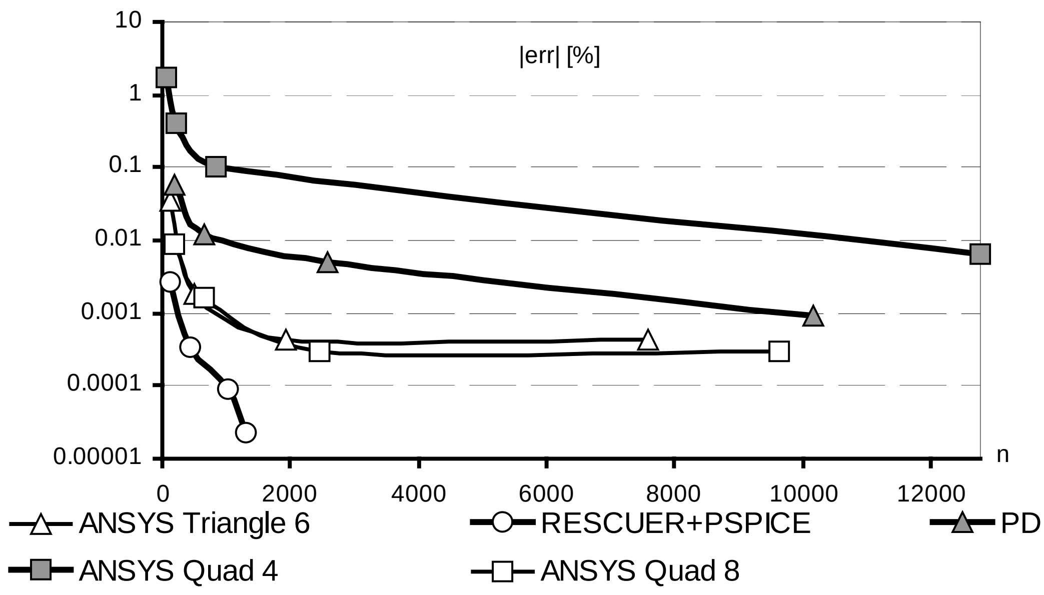

Figure 3 Local accuracy in the point (5,5) vs. number of equations (area discretisation) of the Dirichlet boundary problem (no additional numerical reduction is used).

no-mesh approximation of first and second order partial differential operators based on the Taylor expansion in one-, two-, three- and four-dimensional spaces [14][12][3][4]. In the next stage, the models are symbolically simplified and numerically reduced using the algorithm proposed by the authors. The final approximate models are mapped into structures available in a destination environment. In the case of electrical simulators, the DAEs are converted into equivalent electrical circuit using the method presented in Figure 2b. As opposed to the traditional method (proposed by the Силич in 1957 [5]) based on the equivalence of the diffusion equation in thermal and electrical domains (see Figure 2a) or others (see [6] and [2]), the conversion of wide-class of non-linear PDAEs is possible. The exemplary results for the classical Poisson benchmark problem are presented in Figure 3.

2.1 Reduction of one-dimensional diffusion models

In the case of one-dimensional diffusion problems, it is possible to approximate the proposed model using the spectral decoupling. Let us consider the following exemplary problem

$x \in \Omega, \Omega = \langle 0,100 \rangle, t \geq 0$

$$\frac{\partial^2 T(x,t)}{\partial x^2} = \frac{\pi^2}{20000} \cdot \frac{\partial T(x,t)}{\partial t} \qquad (1)$$

with initial and boundary conditions

$$\left. \begin{aligned} T(x,t)\big|_{t=0} &= \cos\left(\frac{\pi \cdot x}{200}\right) \text{ for } x \in \Omega \\ T(x,t)\big|_{x=0} &= \exp\left(-\frac{t}{2}\right) \text{ for } t \geq 0 \\ T(x,t)\big|_{x=100} &= 0 \qquad \text{ for } t \geq 0 \end{aligned} \right\} \qquad (2)$$

where *T(x,t)* is the unknown solution. The above problem can be rewritten using more general forms (3) which can be decoupled using the decomposition presented in Equation (4).

$$\left. \begin{aligned} \frac{d\hat{\mathbf{T}}(t)}{dt} &= \mathbf{A} \cdot \hat{\mathbf{T}}(t) + \mathbf{B} \cdot \exp\left(-\frac{t}{2}\right) \text{ for } t \geq 0 \\ \hat{\mathbf{T}}(t) &= \mathbf{C} \qquad \text{ for } t = 0 \end{aligned} \right\} \qquad (3)$$

where: **A**, **B**, **C** –matrix and vectors of real and finite elements.

$$\mathbf{A} = \mathbf{D}^T \cdot \Lambda \cdot \mathbf{D}, \ \mathbf{D}^T = \mathbf{D}^{-1} \qquad (4)$$

$$\left. \begin{aligned} \frac{d\mathbf{U}(x,t)}{dt} &= \Lambda \cdot \mathbf{U}(x,t) + \mathbf{H}(t) \text{ for } t \geq 0 \\ \mathbf{U}(t) &= \mathbf{G} \qquad \text{ for } t = 0 \end{aligned} \right\} \qquad (5)$$

$$\left. \begin{aligned} \hat{\mathbf{T}}(t) &= \mathbf{D}^T \cdot \mathbf{U}(t) \\ \mathbf{U}(t) &= \left(\mathbf{G} + \Lambda^{-1} \cdot \mathbf{H}(t)\right) \cdot \exp(\Lambda t) - \Lambda^{-1} \cdot \mathbf{H}(t) \end{aligned} \right\} \qquad (6)$$

where: Λ - eigenvalues of **A** (in this case $\Lambda = \{\lambda_1 \ \cdots \ \lambda_n\}, \lambda_1 > \ldots > \lambda_n$), $\mathbf{U}(t) = [u_1(t) \ \cdots \ u_n(t)]^T$ - the unknown solution and $\mathbf{H}(t) = \mathbf{D} \cdot \mathbf{B} \cdot \exp(-t/2)$ $\mathbf{G} = \mathbf{D} \cdot \mathbf{C}$.

Analysing the decoupled solution (see Figure 4), it is possible to find that most of the equations can be reduced for a specified accuracy (see Figure 5).

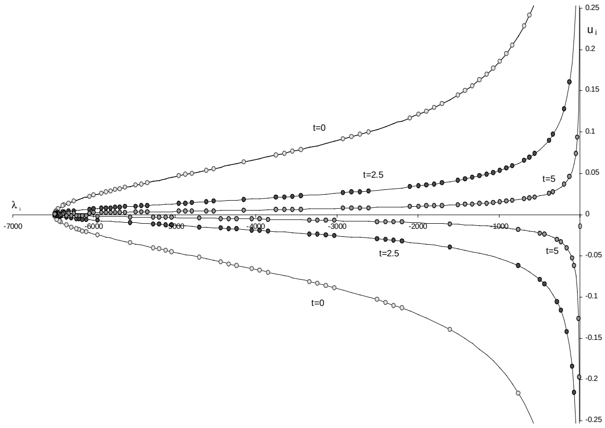

Figure 4 The solution of decoupled one-dimensional diffusion problem for 101 discretisation points (99 equations) and t=0, 2.5, 5 [s].

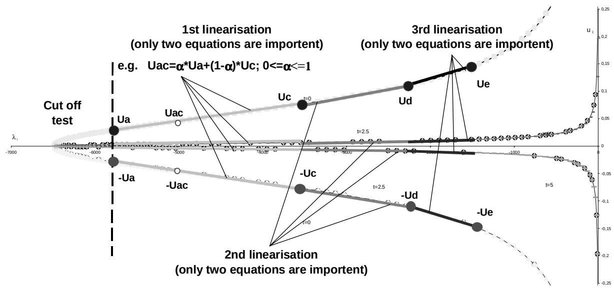

Figure 5 The principles of the refined compact model generation (compare with Figure 4).

2.2 Reduction of multi-dimensional diffusion problems.

The method presented in the previous section is not effective for multi-dimensional diffusion problems with idempotential material parameters. In this case, the partitioning of the distributed model into several connected sub-models and the subsequent application of the Discrete Spectral Transformation to the particular sub-problems is more effective. Let us consider the following example:

$$\left. \begin{aligned} \dot{\mathbf{T}}(t) &= \mathbf{A} \cdot \mathbf{T}(t) + \mathbf{B} \cdot \mathbf{f}(t) \text{ for } t \geq 0 \\ \mathbf{T}(t) &= \mathbf{C} \qquad \text{ for } t = 0 \end{aligned} \right\} \qquad (7)$$

where: **A**, **B**, **C** - matrix and vectors of real and finite elements, ***T****(t)* - unknown solution, **f**(t) – external excitation.

Using the Jordan decomposition $\mathbf{A} = \mathbf{Q} \cdot \mathbf{\Lambda} \cdot \mathbf{Q}^{-1}$, equations (7) can be decoupled and written in the following form:

$$\left.\begin{aligned} &\dot{\mathbf{U}}(t) = \mathbf{\Lambda} \cdot \mathbf{U}(t) + \mathbf{Q}^{-1} \cdot \mathbf{B} \cdot \mathbf{f}(t) && \text{for } t \geq 0 \\ &\mathbf{U}(t) = \mathbf{Q}^{-1} \cdot \mathbf{C} && \text{for } t = 0 \\ &\mathbf{X} = \mathbf{Q}^{-1} \cdot \mathbf{U}(t) && \end{aligned}\right\} \quad (8)$$

where $\mathbf{\Lambda}$ - Jordan matrix (eigenvalues of matrix **A**).

It is obvious that the less significant equations can be found and eliminated from Equation (8) using the following formula $|\varepsilon_i| < \varepsilon$; where ε_i is determined from Equation (9) and ε is the error of arithmetic precision of a destination simulator.

$$\mathbf{Q}^{-1} \cdot [1 \cdots 1]^T = [\varepsilon_1 \quad \cdots \quad \varepsilon_n]^T \quad (9)$$

An example of the application of the above method for the two-dimensional diffusion problem with Dirichlet boundary condition is presented in Figure 6. The application of this method gives possibility of automatic model reduction from 49 ordinary differential equation to 16 dominant equation with neglegible approximation error.

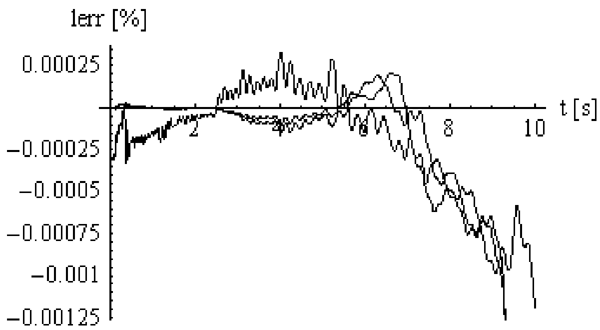

Figure 6 The relative maximal error between the models containing 49 and 16 equations.

2.3 Conclusions

The presented in this paper methods and their implementation in the RESCUER allow for effective conversion of wide class of ordinary differential-algebraic equations, for an arbitrary discretised area, into most commonly used simulation environments. The approximated models can be symbolically and numerically reduced using a method based on the Discrete Spectral Transformation.

It should be underlined that not only the program flexibility and its model reduction capability is important. The RESCUER should be also treated as the first of new generation of compilers, which can translate and optimise specified distributed problems into a destination language, similarly to FORTRAN[6] for regular algorithms.

[6] FORTAN – FORmula TRANslation

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