Modeling the Effects of Joint Clearances in Planar Micromechanisms

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ABSTRACT

Surface micromachining of micro-electro-mechanical systems (MEMS) has inherent limitations that can lead to mechanisms having an undesirable amount of clearance in revolute and prismatic joints. In micro devices, where actuation of the mechanisms is often limited by the displacement capabilities of actuators, the effects of joint clearances become significant. This paper details a modeling technique that is suited to the particular sources of clearance found in surface-micromachined mechanisms. Clearance vector models are used where the clearances are represented as vectors having variable lengths and directions. The precision error of any point on any rigid link can be determined, in addition to the error in link angles, by comparison with the ideal (zero clearance) model. A bistable compliant mechanism is used as an example.

Keywords: joint clearances, precision, mechanical error.

1 INTRODUCTION

Surface micromachining of micro-electro-mechanical systems (MEMS) has inherent limitations that can lead to mechanisms having an undesirable amount of clearance in revolute (pin) joints and prismatic joints (sliders). In micro devices, where actuation of the mechanisms are often limited by the displacement capabilities of actuators, the effects of joint clearances become very significant.

Joint clearances at the micro level are limited by the processes used to separate the mating parts. In surface micro machining, the thickness of the sacrificial layer separating the mating parts determines the planar clearance when appropriate methods are used to obtain minimum clearances, such as making use of conforming layers and dimples. Otherwise, the clearance is determined by minimum space widths.

Little work has been done to model kinematic error in micromechanisms due to clearances. Causes of clearance in micro joints have been identified as being due to the limitations of the fabricating process [1],[2]. The effects of clearance on a specific planar micro-manipulator have been analyzed in [3], but the method is not suitable for more general cases.

In contrast, extensive work has been done to determine the effects of clearances in macro kinematic systems, such as robotics. Garrett and Hall [4] developed a statistical approach to determine mechanical error due to tolerances and clearances and represented the error as mobility bands. The work by Choi et al. [5] shows various improved stochastic methods applied to mechanical error. A deterministic approach was developed by Kolhatkar and Yajnik [6] in which *equivalent clearance links* were used to determine effects of joints with clearance. Ting et al. [7] used rotatability laws to develop a method for analyzing clearance in linkages based upon equivalent clearance links, and simplified the analysis by combining some of these clearance links to reduce the degrees of freedom of the mechanism. Schade and Lai [8] developed a technique in which nonlinear optimization was used to determine the maximum error in coupler-point position due to pin-joint clearances and link tolerances. The path-normal error for a straight-line mechanism was determined using this technique. Yin and Wu [9] used a very similar approach, but included some dynamic analysis in their method. Biswas and Kinzel [10] demonstrated an iterative approach for modeling the effects of clearance in revolute joints based upon quasi-static forces in the mechanism.

This paper builds on the concepts and methods in [8] and [9] to form a technique that can be applied to single or multi-loop mechanisms that have pin-joint clearances and slider clearances common in micromachined MEMS. Although the method is not statistical, it is similar to [5] in its use of clearance vectors rather than clearance links. The difference is important because the clearance link model assumes no contact loss at the joints, while in real mechanisms, joints with clearances are not always in contact [11],[12],[13]. Methods for modeling clearance in sliders are shown and a linear displacement bistable mechanism Figure 14 is used as an example.

2 MODELING CLEARANCES

2.1 Revolute Joints

The clearance (c_r) in a typical revolute joint, shown in Figure 1, is defined as the diameter of the hole, minus the

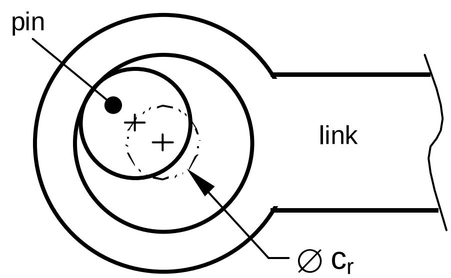

Figure 1: Revolute joint clearance.

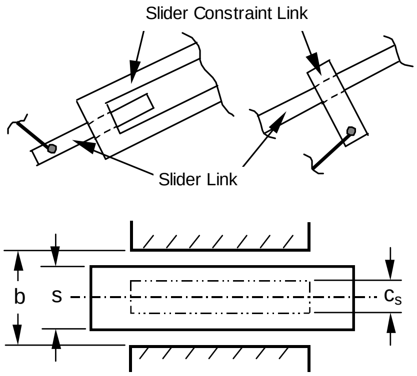

Figure 2: Prismatic joint clearance.

diameter of the pin (a negative clearance indicating an interference fit). The clearance zone can also be described as the diameter of a circular position tolerance zone (shown as a phantom line) centered at the axis of the hole, within which the axis of the pin must be located. Although in a real pin-joint, there is the potential for three-dimensional variation [15],[16], only the two-dimensional case will be covered in this paper. Although the out-of-plane movement of MEMS due to joint clearances is often a problem, it will not be addressed here. Using the method described in [5], the clearance is represented as a vector connecting the two ideal centers of the joint pair. The length of this vector is between 0 and $0.5c_r$, with $0.5c_r$ being the force-closed state. The direction of this vector is dependent upon the forces at the joint.

2.2 Prismatic Joints

Defining the clearance model for a prismatic joint is more difficult because of the many different contact modes and varieties of slider constraints. The clearance in a linear slider, shown in Figure 2, will be defined as the difference between the width of the slider, *s*, and the constraint, *b*. This clearance, c_s, can also be described as a rectangular tolerance zone (phantom line) within which the axis of the slider must be located.

It is not within the scope of this paper to present clearance vector models for all types of prismatic joints. Only the type of slider shown in Figure 2 will be modeled; however, the process can be applied to many other types of sliders. The first step is to develop vector relationships between the slider and a reference point on the slider constraint (see Figure 3). With the slider constraint reference point in the absolute center of the clearance zone, the vector $\boldsymbol{e_s}$ represents the distance from the reference point to the slider vector, measured in a direction perpendicular to the axis of the slider constraint. The second step is to form a closed vector loop that includes the slider and slider constraint. This will be shown in the example. The third step is to determine the limits of motion for the slider in terms of these vectors. The limits for this particular model are:

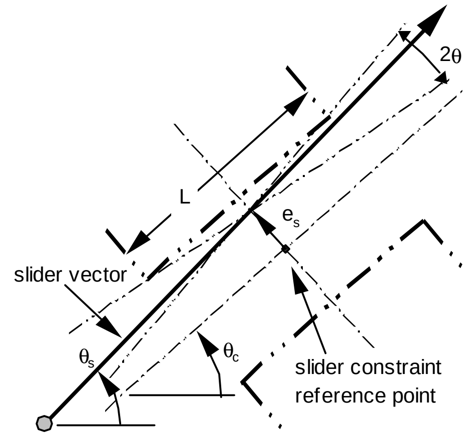

Figure 3: Prismatic clearance vector model.

$$|e_s| < 0.5c_r \text{ and } |\theta_s - \theta_c| \le \theta \text{ where } \theta = \operatorname{atan}\left(\frac{c_r - 2|e_s|}{L}\right).$$

3 EXAMPLE

A linear displacement bistable mechanism is shown in Figure 4. The zero-clearance model of one of the compliant legs is also shown including the type of slider described above. The minimum amount of y-displacement necessary to switch the mechanism is desired due to the limitations of actuator displacements. The maximum displacement of point *P*, due to joint clearances, in the negative y-direction (hereafter ΔP_y) represents the amount of displacement before the compliant segment deflects.

The model in Figure 4 is a zero-degree of freedom mechanism (when the compliant segment is not deflected). The dimensions and angles are as follows:

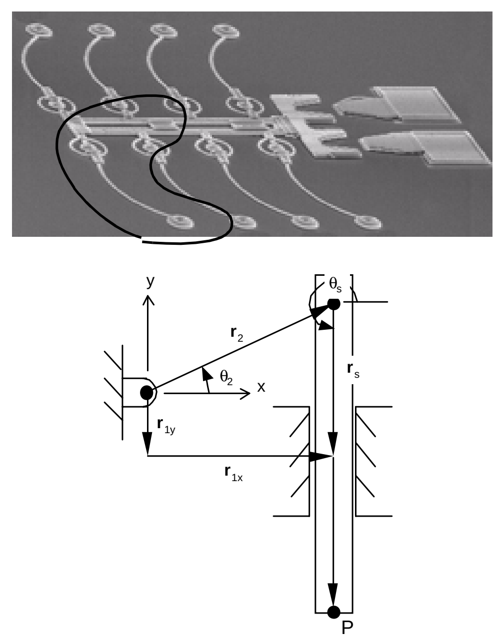

Figure 4: Linear displacement bistable mechanism.

$r_2 = 217.1\mu m \qquad \theta_2 = 14.6936°$

$r_{1x} = 210\mu m \qquad r_s = 85.0673\mu m$

$r_{1y} = 140\mu m \qquad \theta_s = 270°$

$P(x, y) = (210\mu m, -245\mu m)$

3.1 Clearance Model

Figure 5 shows the clearance vector model corresponding to Figure 4. The clearance vectors increase the degrees of freedom in the mechanism by 6 (4 for the two pin-joints and 2 for the prismatic joint), but r_{1x} , $\boldsymbol{r_s}$, and θ_2 are also allowed to vary from the zero-clearance model due to these added degrees of freedom. The values for these variables in the clearance model will be indicated with a hat (^).

The typical minimum clearances associated with the Multi User MEMS Processes (MUMPs™) used to create this mechanism are 1.5µ*m*, so we set $c_r = c_s = 1.5\mu m$. The length of the slider constraint is $L = 40\mu m$. The vector loop equation is $\boldsymbol{c_1} + \hat{\boldsymbol{r_2}} + \boldsymbol{c_2} + \hat{\boldsymbol{r_s}} - \hat{\boldsymbol{r_{1x}}} - \boldsymbol{r_{1y}} = \boldsymbol{0}$. After solving the zero-clearance model, the setup for the optimization of the clearance model is defined as follows:

Maximize: $\Delta P_y = (P_y - \hat{P}_y)$

By changing: $\hat{\theta}_2, \hat{\theta}_s, \hat{r}_s, c_{1x}, c_{1y}, c_{2x}, c_{2y}, e_s$

Subject to constraints: $c_{ix} + c_{iy} \le c_{ir}$

and $|e_s| < 0.5c_r$

and $|\theta_s - 270°| \le \operatorname{atan}\left(\frac{c_r - 2|e_s|}{L}\right)$

and $c_{1x} + r_2\cos\hat{\theta}_2 + c_{2x} + \hat{r}_s\cos\hat{\theta}_s - \hat{r}_{1x} = 0$

and $c_{1y} + r_2\sin\hat{\theta}_2 + c_{2y} + \hat{r}_s\sin\hat{\theta}_s - r_{1y} = 0$

where c_{ir} is the clearance in the i^{th} pin-joint, $\hat{r}_{1x} = r_{1x} + e_s$, and c_{1x} and c_{1y} are the x and y components of the clearance vector.

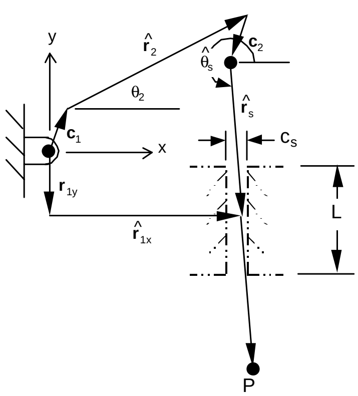

Figure 5: Clearance vector model.

This optimization method has been found to successfully determine the maximum ΔP_y; however, care must be taken to ensure that a global optimum is found. Changing initial values for the design variables or using simulated annealing is a way of addressing this challenge. The most consistent results were found in this study by setting the initial directions of the clearances to correspond to the reaction forces at the joints that would be expected if a load were applied in the direction of ΔP_y. This leads to an interesting theory discussed in section 4.

3.2 Results

The results of the example problem have been divided into three cases corresponding to the initial and subsequent designs.

Case One: Initial design

The initial design, using the values given above, results in a displacement error of $\Delta P_y = 48.24\mu m$. For a zero-clearance bistable mechanism that has a travel of 55µm to its toggle position (the unstable equilibrium position), this error is very significant (about 88% of the required displacement). In this case, the magnitudes of the pin-joint clearance were both binding, in addition to the angular variation constraint for the slider. Due to process limitations, these clearances cannot be reduced, but the design of the mechanism can be changed to try to reduce the effects of the clearances as discussed below.

Case Two: Changing the slider parameters

First, it should be mentioned that treating the slider as ideal (zero clearance), while including the pin-joint clearances in the model yields $\Delta P_y = 6.25\mu m$. This shows that for this particular mechanism the slider clearance contributes more to the error than the pin-joint clearances. Further analysis showed that the value of r_{1y} was significant. By evaluating the optimum for different values of r_{1y}, the sensitivity of the optimum (f^*) with respect to r_{1y} was:

$$\frac{df^*}{dr_{1y}} = \begin{cases} -0.0432 & r_{1y} \le -49.38\mu m \\ 0.0382 & r_{1y} \ge -49.38\mu m \end{cases}$$

This means that holding all other parameters constant, the ideal value for r_{1y} is -49.38µ*m* for minimizing ΔP_y .

Second, the constraint on the angular variation in the slider can be tightened by increasing the length of the slider constraint (L). The displacement error was found to decrease exponentially with an increase in L. (The mechanism in Figure 4 actually has a slider constraint of length $L = 160\mu m$).

Using $L = 160\mu m$ and including more legs, one of which has r_{1y} equal to -49.38µ*m*, leads to a displacement error of $\Delta P_y = 6.26\mu m$, which is very close to the ideal slider case (only about 11% of the required displacement).

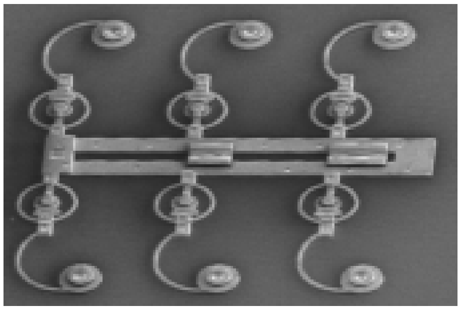

Figure 6: A new configuration for the mechanism.

Case Three: Using a different initial configuration

The effects of the pin-joint clearances can often be reduced by changing the initial configuration of the mechanism. In this example, the overall displacement necessary to flip the switch can be reduced by changing $\boldsymbol{r_2}$ and correspondingly r_{1x}. In an effort to minimize the necessary displacement, a new mechanism considering both joint clearances and a minimum compression of the compliant segment was developed (see Figure 6). Compared to case two, the displacement to the toggle position was reduced from 55μ*m* to 35μ*m*, while ΔP_y was reduced from 6.26 to 4.96μ*m*.

4 ERROR DUE TO APPLIED FORCE

Most of the analysis techniques for determining the effects of clearances on position focus either on worst-case or statistical scenarios. However, often the goal is to determine the maximum backlash in the mechanism due to a certain applied load (similar to the idea of determining the amount of backlash in a gear train). This would force the mechanism into a position that might not represent the maximum error case, and certainly would not be statistical (the clearance would no longer be taken up in a random manner). Vocaturo [17] used the method of applying an arbitrary force in a given direction in order to determine the position error in that direction. This was based on the virtual work principle, where the maximum error due to an applied force will be that which minimizes the potential energy in the mechanism. Using a reverse technique and the method in [8] for determining the error of a point in a given direction could lead to a way of determining the backlash in a mechanism due to an applied load. In every case discussed in section 3, the direction of the clearances corresponded with that expected of a force-closed loop. This is a concept worthy of further research.

5 CONCLUSION

The approach described in this paper for modeling joints with clearances, combined with optimization, was found to be an effective method for determining maximum position error. It provides a way to include prismatic joint clearances in the analysis of position error, and may lead to a general method for determining the maximum backlash in a mechanism due to an applied load.

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