Boundary Independent Exact Thermal Model for Electronic Systems

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ABSTRACT

A compact thermal model is presented, which describes the hot spot (junction) temperatures and contact heat flows of electronic packages or systems in the stationary state. The model is exact provided that the underlying heat conduction equation is linear (i.e. no temperature dependence of thermal conductivities is assumed) and the thermal contact areas to the environment have uniform temperature distribution. The model leads to a systematic method to construct thermal resistor networks. The number of model parameters for *n* contact areas and *m* independent heat sources is ½ (*n*-1) (*n* + 4 *m*) + m^2 . They are determined by successive linear fits to simulated and measured temperatures and heat flows of the system. The method is demonstrated by application to IC packages and compared to a description with seven-resistor networks. The accuracy is improved considerably, however at the expense of an increase of the number of model parameters to 26 for a package with 6 thermal contact areas.

Keywords: Compact thermal model, boundary condition independence, thermal resistor networks, electronic packages.

1 INTRODUCTION

Extensive work has been done in the past in developing compact thermal models of different electronic packages for use in conduction cooled applications such as printed circuit boards [1, 2, 3]. Also for multichip modules - e.g. in power electronic applications - such models are needed in order to predict maximum (hot spot or "junction") temperatures of the semiconductor devices in the system by simple means using e.g. circuit simulators instead of finite element (FE) methods. In addition also the heat flow through the sides or thermal contacts to the environment has to be reproduced correctly. If the number of model parameters is very much lower than the number of parameters describing a "detailed model" as, for instance, a finite element model (fig.1), the model is called a compact model.

In the European projects DELPHI and SEED [4, 5] a suitable set of boundary conditions for the generation of compact thermal models has been defined as test condition. Thermal resistor networks were generated intuitively for different packages and the resistances (model parameters) fitted to give as close as possible the junction temperatures and heat flows for the set of boundary conditions [1,2,3]. The model parameters obtained by the fit stay constant for all boundary conditions (boundary condition independent model).

The exact model to be presented in this work leads to a systematic method to construct thermal resistor networks and is derived from the properties of the general heat conduction equation for the temperature field T

$$\rho(\vec{x})\, c(\vec{x})\, \frac{\partial T(\vec{x},t)}{\partial t} = H(\vec{x},t) + \nabla_{\vec{x}} \cdot \left(\lambda(\vec{x})\, \nabla_{\vec{x}} T(\vec{x},t)\right) \qquad (1)$$

The mass density ρ, the specific heat c, and the thermal conductivity λ depend only on the position x , but not on T itself . H denotes the heat generation density. Since eq. (1) is linear in T , solutions for different boundary conditions and heat sources can be superposed (added) to give new solutions for the added boundary conditions and heat sources [6, 7, 8]. In the following the case of stationary temperature distribution is considered, i.e. the left hand side of (1) is zero.

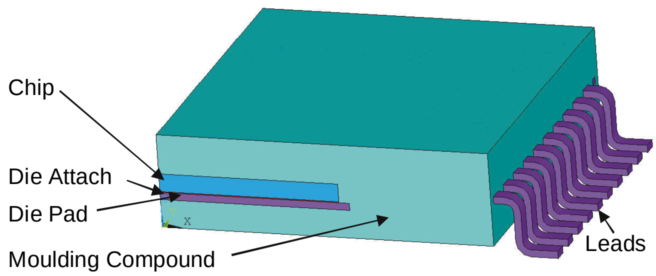

Figure 1: ¼ Finite Element model for IC-package

2 MODEL EQUATIONS

The package may have generally n thermal contact areas with temperatures T_1 , T_2 , ... , T_n . It is assumed that the temperature along each contact area is uniform. This can be achieved by subdividing contact areas in smaller ones, if necessary. It is also assumed that the junction temperature *Tjunc* of the device is defined at a fixed hot spot location (usually the centre of the semiconductor chip within the package). The other surfaces not attributed to contact areas have adiabatic (von Neumann) boundary conditions $\partial T/\partial n = 0$, which means that there is no heat flow across these surfaces. Then it follows from the linearity of (1) that *Tjunc* depends linearly on the boundary temperatures

$$Tjunc = RthJ0 \cdot P + \sum_{i=1}^{n} a_i \, T_i \tag{2}$$

and the total power dissipated by the distributed heat sources, $P = \int H(x)\, d^3x$. If, for instance, we set all $T_i = 0$, it follows immediately from (1) that *Tjunc* is proportional to *P*: $Tjunc = RthJ0 \cdot P$. On the other hand, if we set $P = 0$ (i.e $H = 0$ in (1)) and $T_i \neq 0$ for one *i* and all other $T_k = 0$ ($k \neq i$), then $Tjunc = a_i \cdot T_i$. Now the solutions of (1) with $P = 0$, $T_i \neq 0$ and $P \neq 0$, $T_i = 0$ can be superposed to give the hot spot temperature *Tjunc* for general P, T_1, .., T_n according to (2). *RthJ0* and a_i are model parameters. Since the addition of a constant T_0 to the temperature field and boundary temperatures gives again a solution of (1), we have in case of $T_i = T_0$ = const.: $Tjunc - T_0 = RthJ0 \cdot P$. This is only compatible with (2), if the a_i satisfy the constraint

$$\sum_{i=1}^{n} a_i = 1 \tag{3}$$

Generally the temperature field within a volume - and thus also *Tjunc* - is determined completely by eq.(1) and the knowledge of the boundary conditions T_i and $\partial T/\partial n = 0$ on the volume surface. With these data, also the heat flows on the surface are computable. However, this necessitates the knowledge of the continuous material parameters ρ, c and λ over the whole volume or - in lumped form - the knowledge of the topology and values of the equivalent thermal resistor network. In other words, "the detailed model" has to be known in order to predict the heat flows. Many different detailed models satisfy the same relation (2) but may have different heat flows at the contact areas. Therefore, to complete the compact model a relation is needed that gives the heat flows as a function of P, T_1, .., T_n .

In the case $T_i = 0$ for all i , the heat flow portion of P at the thermal contact area k is given by $P \cdot q_k$ with the additional model parameters q_k satisfying

$$\sum_{i=1}^{n} q_i = 1 \tag{4}$$

In case of $P = 0$, $T_i \neq 0$ for one i and for all $k \neq i$ $T_k = 0$, a heat flow $J_{i,k}$ through the external contact k is caused by the excitation T_i at contact i according to $J_{i,k} = T_i / R_{i,k}$ with symmetrical resistor matrix $R_{i,k}$. Again, the solutions of (1) with $P = 0$, $T_i \neq 0$ and $P \neq 0$, $T_i = 0$ can be superposed to give the total heat flow J_k at contact k for general P , T_i :

$$J_k = P \cdot q_k + \sum_{i=1}^{n} T_i / R_{i,k} \tag{5}$$

Because of heat flow (energy) conservation

$$\sum_{k=1}^{n} J_{i,k} = 0 \;\Rightarrow\; \sum_{k=1}^{n} 1/R_{i,k} = 0 \tag{6}$$

Eq. (2) and (5) constitute the compact thermal model with the model parameters *RthJ0* , a_i , q_k and $R_{i,k}$. These $(n+1)^2$ parameters are reduced by the constraints (3), (4), (6) and by the symmetry of $R_{i,k}$ to ½ (n-1) (n + 4) + 1 independent parameters. The model is exact under the stated suppositions of isothermal contact areas and linearity of (1).

3 APPLICATION EXAMPLE

The standard boundary conditions defined in ref.[1, 2] are applied by specifying the thermal contact resistors in distributed form (inverse heat transfer coefficients) between the contact areas of an IC package and zero ambient temperature. Extensive and elaborate thermal measurements (fig.2) for a variety of IC packages have been performed [1] in order to check the accuracy of detailed model FE calculations [9] for a subset of the boundary conditions. The verified FE calculations, in turn, were used to adjust a seven resistor network to the full set of 38 boundary conditions with the help of non-linear optimisation. The accuracy of *Tjunc* and the contact heat flows are displayes in fig.3 for an IC-package of type P-TQFP-144-2 with six thermal contact areas, which are referred to in fig.3 as top inner, top outer, bottom inner, bottom outer, sides and leads. Fig. 4 shows the result for the new model (2), (5). Obviously the accuracy has improved considerably. The remaining discrepancies are attributed to non-isothermal contact areas and inaccuracies of the data basis.

A very fast procedure to determine the 26 parameters of the new model consists in performing successive linear fits, thus circumventing the problems of non-linear or genetic algorithms with numerous local minima. At first eq.(2) is adjusted to the measured and simulated *Tjunc* values by a standard linear fit routine to obtain the values of *RthJ0* and a_i . Then the n = 6 equations (5) are fitted one after the other for each contact k to the heat currents J_k and heat flow portions q_k of the data basis in order to obtain the inverse resistor matrix $1/R_{i,k}$. The resistor matrix obtained in this way satisfies the constraint (6), but is not completely symmetric. In order to enforce the symmetry constraint in the fitting-process, the full equation system (5) should be adjusted simultaneously instead successively to prevent an unbalanced distribution of the heat flow error bars among the contacts.

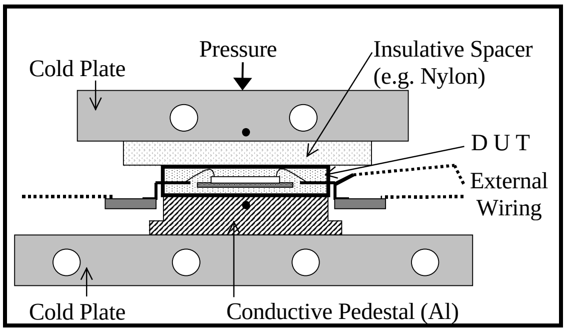

Figure 2: Thermal measurement set-up for special boundary conditions (example from ref.[1])

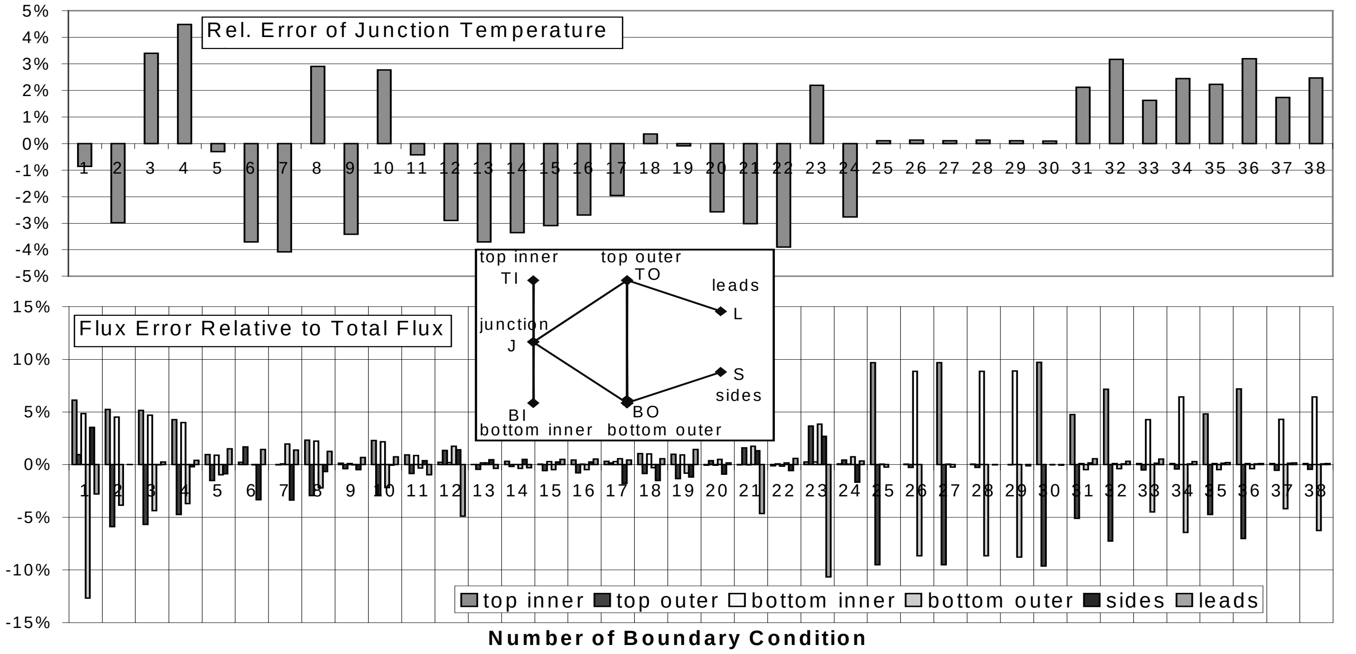

Figure 3: Junction temperature and heat flow errors at 6 thermal contacts for seven resistor network (centre diagram) compared to FEM and measurement. Test for IC package P-TQFP-144-2 for standard set of 38 boundary conditions [1].

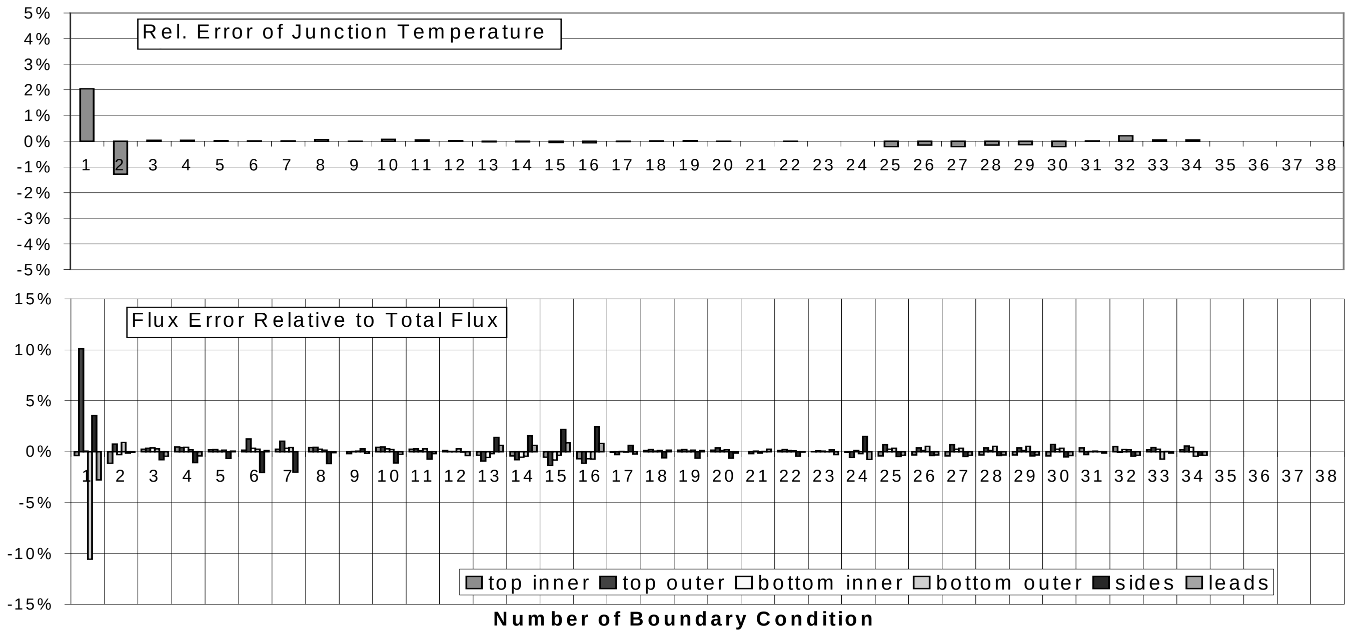

Figure 4: Junction temperature and heat flow errors for new compact model (eq. (2), (5)) for the same IC-package and FEM and measurement data used in fig.3.

4 THERMAL RESISTOR NETWORKS

It is possible to represent the exact model (2), (5) by a thermal resistor network. Fig.5 shows the network for $n = 3$ thermal contacts. The construction principle is simple: Connect the contact areas with each other by a direct thermal resistor connection. Then introduce one junction node and one additional node, which is connected to the heat current source P, and connect all nodes (contacts, junction, heat source) with direct resistor links. The network obtained in this way has ½ (n-1) (n + 4) + 3 thermal resistors, i.e. two parameters more than the compact model of section 2. This does not mean that two of the resistors, e.g. the one between junction and heat source node, can be omitted, since without that resistor *Tjunc* would be zero for all P with $T_i = 0$. The model parameters can be represented by analytical expressions of the resistors, which are too lengthy to be presented here.

A similar network has been suggested in ref.[3] (shunted network with one additional floating node). It is important to note that in our case the floating node describes the junction temperature *Tjunc* and the additional node is used for connecting the heat current source P . This accounts for the fact, that the heat generation density H generally is not a point source at the hot spot location of *Tjunc*. Otherwise P would have to be connected with the *Tjunc* node as in ref.[3] and other previous work. It can be shown that in this case *Tjunc* and J_k in eq (2), (5) can be described by a shunted network and that the additional floating node is superfluous. The parameters of the compact model then satisfy the additional equation $a_i = q_i$. This relation also holds for networks whose junction node is connected directly to the heat source P and which have one additional floating node. It is an unsettled question whether this is true also for two or more additional floating nodes.

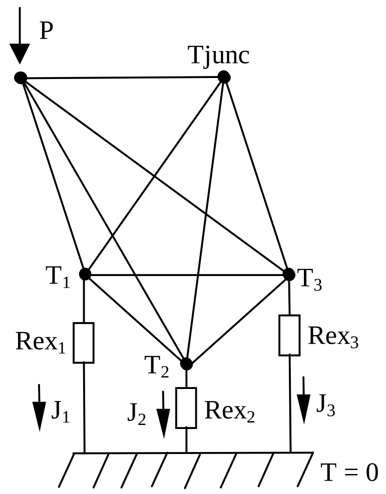

Figure 5: Thermal resistor network for exact compact model (n = 3 contact areas). Each straight line connecting nodes represents one thermal model resistor. The Rex characterise the heat transfer (boundary conditions) from the thermal contact areas to the ambient.

5 SEVERAL HEAT SOURCES

In many practical cases several independent heat sources $P_1 , ... , P_m$ have to be considered as, for instance, in multichip modules. Every chip l dissipates the heat P_l and has a junction (hot spot) temperature $Tjunc_l$. Again, making use of the superposition principle of the heat conduction equation (1) the straightforward generalisation of the model equations of section 2 leads to:

$$Tjunc_j = \sum_{l=1}^{m} RthJ0_{j\,l} \cdot P_l + \sum_{i=1}^{n} a_{j\,i}\, T_i \qquad (7)$$

where the matrix $RthJ0_{j\,l}$ describes the thermal interaction of the different chips [7, 8]. For the same reason as for eq.(3), the $a_{j\,i}$ always satisfy the relation

$$\sum_{i=1}^{n} a_{j\,i} = 1 \qquad (8)$$

for all $j = 1, ..., m$. In analogy to (4) the heat flow contributed by P_l at the thermal contact area k of the system is given by $P_l \cdot q_{l\,k}$ with the model parameters $q_{l\,k}$ satisfying

$$\sum_{k=1}^{n} q_{l\,k} = 1 \qquad (9)$$

With this eq.(5) for the heat flows at the contacts k =1, .. , n can be written as

$$J_k = \sum_{l=1}^{m} P_l \cdot q_{l\,k} + \sum_{i=1}^{n} T_i / R_{i,k} \qquad (10)$$

with the symmetric resistor matrix $R_{i,\,k}$ fulfilling the constraint (6). Eq. (7) and (10) constitute the compact thermal model for m independent heat sources and n thermal contact areas with the model parameters $RthJ0_{j\,l}$, $a_{l\,i}$, $q_{l\,i}$, and $R_{i,k}$. By the constraints (8), (9), (6) and the symmetry of $R_{i,k}$ the number of independent model parameters is ½ (n-1) (n + 4 m) + m^2 .

A thermal resistor network representing these model equations can be constructed as follows: In the same way as in section 4 connect the n contact areas with each other by thermal resistors. Then introduce m junction nodes for $Tjunc_j$ and m additional nodes to be connected to the heat sources P_l . All nodes are connected with each other by resistors with the exception of the nodes P_l and $Tjunc_j$ for $j \neq l$. This gives altogether ½ (n-1) (n + 4 m) + m^2 + 2m thermal resistors, 2m more than independent model parameters. The circuit constructed this way can be used alternatively to the model equations.

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