Scalable Macromodels for Microelectromechanical Systems

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ABSTRACT

A fully automatic macromodeling methodology to generate scalable reduced-order models for microelectromechanical systems is presented in this paper. Krylov subspace methods are used to generate reduced-order models from detailed higher-order models of the device under consideration. The entire methodology is implemented in a symbolic computation environment to preserve dependencies on physical device parameters. The macromodeling methodology is demonstrated using the combdrive microresonator as a representative example. The results are compared with analytical models, NODAS and finite element simulations.

Keywords: reduced-order model, MEMS, macromodels, Krylov subspaces.

1 INTRODUCTION

With the rapid advances in MEMS technology, there is an increasing need for Electronic Design Automation (EDA) tools in the MEMS community. Most of the present-day MEMS CAD is done on a case-by-case basis using detailed numerical solvers [1]. Such accurate numerical solvers are both time and resource intensive, and are not suitable for system-level CAD where large design spaces typically need to be explored. Macromodeling techniques are desired in such cases in order to reduce the design cycle time and prototyping costs.

A good macromodel should capture the essential static and dynamic behavior of the device using a minimal set of equations (*small*), which are in terms of the physical design parameters and material properties (*scalable*). Previous efforts in automatic macromodel generation were primarily based on model extraction from explicit numerical simulations. Gabbay in his Ph.D. dissertation [2] presented a method for automatic generation of macromodels for nonlinear, electrostatically actuated microstructures from meshed quasistatic simulations based on modal analysis and energy methods. Commercial tools like MEMCAD (Microcosm Technologies) [3] and MEMS Modeler (MEMSCAP) use parametric curve-fitting of simulation data to obtain macromodels. The primary drawback of these methods is that they do not generate scalable macromodels.

2 MACROMODELING METHODOLOGY

A novel scheme for generating scalable macromodels of linear MEMS devices, based on Krylov subspaces is proposed in this paper. The macromodeling methodology involves the following general steps,

1. Generation of higher-order model

2. Order reduction of model obtained in the previous step to obtain macromodel

2.1 Higher-order modeling

Any linear dynamic system can be described by the following set of coupled differential algebraic equations,

$$C\dot{z} + Gz = Bu \quad y = L^T z \tag{1}$$

where z is the vector of internal states of the system, u is the vector of inputs and y is the vector of outputs.

The matrices C, G, B and L are typically generated by discretizing the constitutive partial differential equations describing the system. The most commonly used discretization techniques are the finite difference method, the finite element method, the boundary element method and the finite volume method.

2.2 Model order reduction

The reduced-order model is obtained by projecting the higher-order model onto a smaller Krylov subspace. Krylov subspace methods [4] are among the most powerful techniques to compute eigenvalues and eigenvectors for sparse systems. There has been a lot of interest recently in Krylov subspaces for obtaining reduced-order models of large linear circuits and their use in circuit simulation [5] [6]. Krylov subspace methods have also been applied to structural dynamic problems for model reduction and control [7]. More recently these techniques have been applied for reduced-order modeling and simulation of MEMS devices [8]. In the following section the application of the Krylov subspace method for model-order reduction of linear dynamic systems is discussed.

3 KRYLOV SUBSPACES

A Krylov subspace is mathematically defined as

$$\kappa_m(A, r) = span\{r, Ar, A^2r, \ldots, A^{m-1}r\} \quad (2)$$

In linear algebra terms, Krylov subspace methods approximate the solution of $Ax = b$ by $p(A)b$, where $p(A)$ is a polynomial in A. The two most popular methods for generating Krylov subspaces are the Arnoldi [9] [5] and the Lanczos [5] techniques. The Lanczos method is in general more accurate and uses a shorter recurrence formula, which implies faster computation, as compared to the Arnoldi method. Hence the Lanczos method is used to generate the basis vectors of the Krylov subspaces in this paper.

3.1 Lanczos Biorthogonalization Algorithm

The Lanczos process (as applied to matrix A, with right starting vector r and left starting vector l) produces the following sequences of vectors,

$$V_n = [v_1, v_2, \ldots, v_n] \qquad W_n = [w_1, w_2, \ldots, w_n] \quad (3)$$

The Lanczos vectors W and V span the Krylov subspaces $\kappa_n(A, r)$ and $\kappa_n(A^T, l)$ and are constructed to be biorthogonal.

$$W_n^T V_n = diag(\delta_1, \delta_2, \ldots, \delta_n) \quad (4)$$

The Lanczos algorithm to generate W_n and V_n from A, r and l is given in [5].

3.2 Application to Linear Dynamic Systems

The linear dynamic system represented by Equation 1 can be rewritten for single-input single-output (SISO) as

$$C\dot{z} + Gz = bu \quad y = l^T z \quad (5)$$

where b and l are column vectors.

3.2.1 Reduced-order models based on projection

A reduced-order model of order n for the system described by the Equation 5 is then obtained by the following steps,

1. Define
$$A = G^{-1}C \; and \; r = G^{-1}b \quad (6)$$

2. Compute V_n and W_n using the Lanczos method such that
$$span\{V_n\} = \kappa_n(A, r) \text{ and} \quad (7)$$
$$span\{W_n\} = \kappa_n(A^T, l) \quad (8)$$

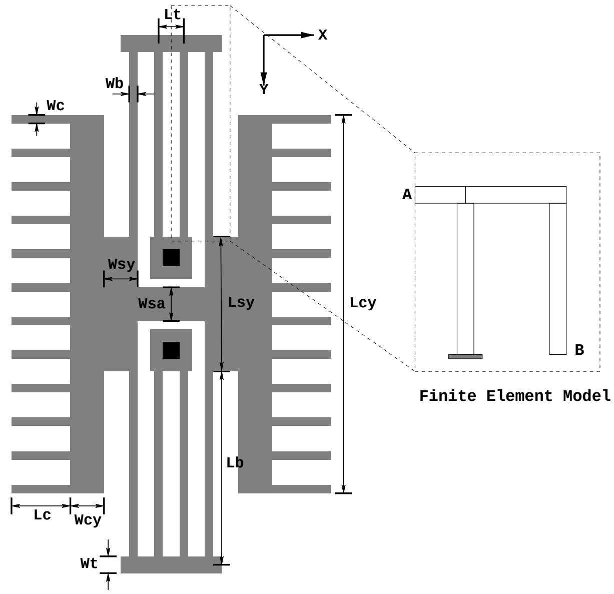

Figure 1: Comb-drive resonator

3. Compute the reduced matrices C_n, G_n, b_n and l_n by

   Double-sided projection
$$C_n = V_n^T C W_n, \quad G_n = V_n^T G W_n \quad (9)$$
$$b_n = W_n^T b, \quad l_n = V_n^T l \quad (10)$$

   Single-sided projection
$$C_n = V_n^T C V_n, \quad G_n = V_n^T G V_n \quad (11)$$
$$b_n = V_n^T b, \quad l_n = V_n^T l \quad (12)$$

The double-sided projection [6] formula does not always guarantee a stable reduced-order model, except for certain trivial cases like RC networks. On the other hand the single-sided projection onto V_n guarantees an unconditionally stable reduced-order model [10]. The reduced-order model however is generally less accurate than the one obtained by the double-sided projection. For the special case of LC circuits (undamped systems) the single-sided projection has the same accuracy as the double-sided projection.

4 MICRORESONATOR MACROMODEL

The comb-drive resonator, shown in Figure 1, is a very popular device in the MEMS community and has been well characterized, making it an ideal test-bench for the reduced-order modeling methodology. The design specifications of the resonator and the simulation results for comparison were obtained from the High Q Resonator Canonical Design Problem web page [11]. The design parameters for the microresonator are listed in Table 1.

4.1 Higher-order model

The finite element method (FEM) is the most widely used discretization technique to solve structural mechanics prob-

Table 1: Design parameters for Microresonator

f_X nom. [kHz]	3	10	30	100	300
Wb [μm]	2	2	2	2	2
Lb [μm]	300	300	153.7	67.87	30.79
Lt [μm]	17.8	17.8	17.8	17.8	17.8
Wt [μm]	4	4	4	4.587	10.5
Wsa [μm]	11	11	11	11	11
Wsy [μm]	11	11	11	11	11
Lsy [μm]	49	49	49	49	49
Lcy [μm]	650.5	340.4	296.9	328.3	413
Wcy [μm]	69.23	11	11	11	11
Lc [μm]	11.3	11.3	11.3	11.3	11.3
Wc [μm]	2	2	2	2	2
N	82	43	37	41	52

lems. Since the primary component of the combdrive microresonator is a mechanical microstructure, finite element techniques are the most appropriate discretization technique to create a higher-order model for the device. The finite element routines are implemented in MATLAB using the Symbolic Toolbox [12] to preserve physical parameter information. The FEM routine generates the mass matrix, stiffness matrix and loading vector for the microstructure for various loading and boundary conditions. The finite element model of the folded-flexure spring structure is shown in Figure 1. The boundary condition at end A is a rolling-pin condition ($y = 0$) and at end B the boundary condition is a guided-end condition ($y = 0,\ \theta = 0$).

4.2 Reduced-order model

The finite element routine generates the mass matrix, stiffness matrix and the loading vector for the microstructure of interest. The equations of motion can then be written as,

$$M\ddot{x} + D\dot{x} + Kx = Pu \qquad \text{and} \tag{13}$$

$$y = Q^T x + R^T \dot{x} \tag{14}$$

where M is the mass matrix, D is the damping matrix, K is the stiffness matrix, P is the loading vector, u is the input vector, y is the output vector and x is the state vector. Q and R are chosen depending on the output variable of interest.

Equations 13 and 14 can be rewritten in the form of Equation 5 by defining

$$C = \begin{bmatrix} D & M \\ M & 0 \end{bmatrix} \qquad G = \begin{bmatrix} K & 0 \\ 0 & -M \end{bmatrix}$$

$$b = \begin{bmatrix} P \\ 0 \end{bmatrix} \qquad l = \begin{bmatrix} Q \\ R \end{bmatrix}$$

and

$$z = \begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix}$$

The reduced mass and stiffness matrices of the microresonator can now be obtained by projecting the G and C matrices onto a smaller Krylov subspace, generated by the Lanczos method. The Lanczos algorithm is also implemented in MATLAB using the Symbolic Toolbox. The order of the subspace is chosen according to the frequency range where matching is required. For matching of q resonant peaks n has to be at least $2q$. Therefore to obtain the effective mass and stiffness of the resonator (for matching the fundamental resonant frequency) along the X-axis, a value of $n = 2$ must be used. The expression of the effective stiffness in the X-direction is shown in Table 2. The expression for the effective mass is very long and is not shown.

4.3 Comparison with FEM, Analytical solution and NODAS

The effective stiffness obtained using the Krylov based reduced-order model was compared with the results from FEM, analytical expressions and NODAS. The results are shown in Table 2. The analytical expressions in this case were derived using the Castigliano's theorem. [13]

From Table 2 we see that the Krylov subspace based reduced-order model matches reasonably well with the FEM results, and very well with the analytical and NODAS [14] results. This is because the simplifying assumptions in the analytical method, NODAS and the Krylov subspace method are the same. The NODAS model actually corresponds to the original finite element model used in the Krylov subspace method.

5 CONCLUSIONS

An fully automatic method to generate scalable macromodels for microelectromechanical systems was demonstrated in this paper. The method also does not require any numerical simulations to obtain the reduced-order model. Most of the existing macromodel generation techniques do not generate scalable macromodels, and require repeated simulations of the finite element model. Very good frequency response matching is also exhibited in the frequency range of interest. Scalable/parameterizable models are be obtained by implementing the reduced-order modeling algorithm in MATLAB's Symbolic Toolbox. The drawback of using symbolic computation is that the computation time increases very rapidly with increasing complexity of the higher-order model.

The reduced-order modeling technique presented in this thesis can be easily extended to make it more useful and more applicable to composite microsystems. Some of these extensions are fairly straightforward and the theory already exists. The technique is currently limited to linear SISO systems can be extended to linear MIMO systems quite easily. Non-linear

Table 2: Comparison of Krylov reduced-order model with FEM, analytical solutions and NODAS

f_X nom. [kHz]	Stiffness in X direction = k_X [N/m]				Resonant frequency = f_X [kHz]			
	FEM	Anal.	NODAS	Krylov	FEM	Anal.	NODAS	Krylov
3	0.193	0.194	0.194	0.1956	3.288	3.27	3.28	3.31
10	0.193	0.194	0.194	0.1956	9.063	8.56	9.05	9.18
30	1.4108	1.44	1.43	1.4540	26.75	25.99	27.8	27.34
100	15.82	16.6	16.5	16.8762	87.77	88.45	89.5	91.35
300	162.4	180	180	180.7653	245.3	267.1	271.8	271.72

$$\mathrm{Eff.\ stiffness\ along\ X} = k_X = \frac{E\,T\,Wb^3}{Lb^3}\,\frac{Wb^6\,Lt^6 + 32\,Wb^3\,Wt^3\,Lb^3\,Lt^3 + 108\,Wt^6\,Lb^6}{2\,Wb^6\,Lt^6 + 43\,Wb^3\,Wt^3\,Lb^3\,Lt^3 + 54\,Wt^6\,Lb^6}$$

$$\mathrm{Eff.\ mass\ along\ X} = m_X\ (\text{not shown , very long expression})$$

$$\text{First resonant mode in X direction} = f_X = \frac{1}{2\pi}\sqrt{\frac{k_X}{m_X}}$$

macromodeling presents yet another level of complexity and is an active research topic. Techniques to speed up symbolic computation also need to be investigated and implemented.

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