Microsystem Model Evaluation Using Optimization Strategies

D. Peters, R. Laur
ITEM - Institute for Electromagnetic Theory and Microelectronics
University of Bremen, Germany
P.O. Box 330 440, 28334 Bremen, Germany
peters@item.uni-bremen.de

Abstract—**This paper presents an automatic approach for evaluating behavioral models of micro systems' components. Optimization methods are applied aiming at the specification of a model's scope of application. The method is presented by evaluating a behavioral model of a capacitive pressure sensor. The model incorporates a new approach for the calculation of the sensor's bending line. It considers small as well as large deflections.**

Keywords: behavioral modeling, model evaluation, optimization, microsystem design

I. Introduction

Microsystems usually consist of many subsytems originating from different physical domains. Due to the small dimensions the systems components strongly interact with each other. The problem is to accurately develop the subsystems while maintaining the overall system's performance. Considering each subsystem on its own, one usually applies FEM simulators. But once the whole system needs to be looked at, either simulator coupling or system simulators have to be used. Since simulator coupling not only can result in convergence problems but also entails long simulation runs a system (behavioral) simulator will be the better choice.

Thus a behavioral model has to be developed. Normally a system simulator's kernel incorporates the basic equations of one physical domain (usually the electro-magnetical equations). The physical basis of any of the other domains needs to be included into the behavioral model. In e.g. modeling a micro mechanical component not all existing micro mechanical effects can be taken care of in one model. Thus the designer has to decide for the most important ones looking at the system under investigation. The last step in developing a behavioral model should be the evaluation of the behavioral model. The usual approach is to build up a prototype, carry out measurements and compare them with the calculated data. If the calculations do not fit the measurements, a trial and error process begins with the aim to either find a better fitting set of model parameters or include some of the so far not considered effects into the model. This approach is very inefficient and its success strongly depends on the designers experience and intuition. Even if a set of parameters can be found which results in a good fit between calculation and measurement, this does not mean that the model and its parameters do have anything to do with the reality.

This paper presents an approach for evaluating behavioral models of micro systems' components via optimization strategies on the basis of reference data. These may either be taken from measurements or FEM simulations. The aim is to specify the model's scope of application.

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II. Presentation of the model

The model validated in this paper is presented in [1] and [2]. It is concerned with the modeling of a capacitive pressure sensor once small as well as large deflections of the quadratic membrane structure have to be concidered. The crucial part of this model is the calculation of the membrane's bending line, which is the solution of the following partial differential equation

$$\begin{aligned} \Delta\Delta\omega &= \frac{\delta^4\omega}{\delta x^4} + 2\frac{\delta^4\omega}{\delta x^2 \delta y^2} + \frac{\delta^4\omega}{\delta y^4} \\ &= \frac{1}{D}\left(p + N\left(\frac{\delta^2\omega}{\delta x^2} + \frac{\delta^2\omega}{\delta y^2}\right)\right). \end{aligned} \quad (1)$$

$D = \frac{Eh^3}{12(1-\nu^2)}$: stiffness of the plate
ω: plate's displacement as a function of the surface coordinates x and y, p: static surrounding pressure, N: force due to the intrinsic stress h: thickness of the membrane, E: Young's modulus, ν: poisson ratio of polysilicon

Since there is no closed solution to this equation, an approximation has to be found by applying the energy method in combination with the method of virtual displacement. Depending on the ratio of the membrane's deflection $\omega(x, y)$ to its thickness h, one differentiates between two approximate solutions: in the case of small deflections the elongation energy can be neglected compared to the bending energy as depicted in figure 1; in the case of large deflections the bending energy can be neglected compared to the elongagion energy as depicted in figure 2.

Considering the possibilities in the construction of the system under investigation neither one of these approaches is applica-

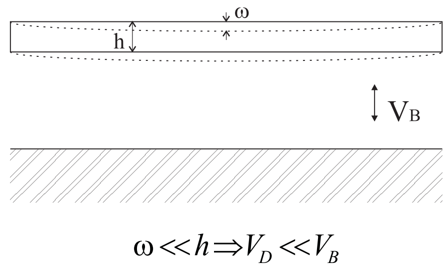

Fig. 1. Small Deflections

ble. The dimensions of h and w[1] may at least be of the same magnitude. Therefore the bending as well as the deflection energy were taken into account in the new sensor's model.

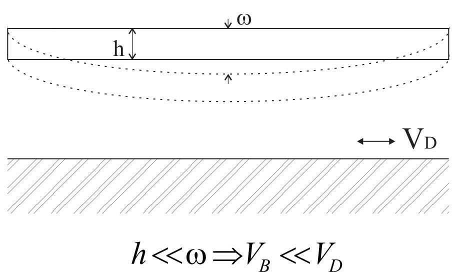

Fig. 2. Large Deflections

III. Model evaluation

A. Basis of comparison

The new model's quality is assessed in comparison to an earlier solution of the bending line calculation. The old approach derives the calculation of the maximum deflection ω_0 from a simple solution for the bending line

$$\omega(x, y) = \omega_0 \cdot cos(\frac{\pi x}{a}) \cdot cos(\frac{\pi y}{a}), \qquad (2)$$

while the behavioral model uses the more elaborate approach according to [3] to calculate the deflection of each single partial plate

$$\begin{aligned} \omega(x, y) \quad = \quad & \omega_0 \cdot [1 + 1.604\tfrac{x^2+y^2}{a^2} + 18.58\tfrac{x^2 \cdot y^2}{a^4}] \\ & \cdot cos(\tfrac{\pi x}{a}) \cdot cos(\tfrac{\pi y}{a}). \end{aligned} \qquad (3)$$

The new model exclusively depends on equation 3.

B. Method of Model Evaluation

In order to make the process of model evaluation more efficient and less dependent on the designer's experience and intuition a new method based on optimization strategies is introduced. Superficially it can be described as the inversion of the well known parameter extraction process. Parameter extraction aims at the determination of unknown parameter values on the basis of a verified model. The model evaluation process aims at the verification of the model on the basis of curve fittings to some kind of reference data and parameter extraction. The reference data may either be taken from FEM-simulations or from measurements. If the curve fitting process results in a good match between the model and the reference data and if the thereby extracted parameters lie within physically validated ranges, the model reproduces the dependence between the characteristic curve and the considered parameters correctly. Otherwise it does not and the model would have to be enhanced.

In practice the proposed method could replace costly, time-consuming and sometimes also error prone measurements.

C. First Evaluation Approach

In order to evaluate the new model's quality in comparison to the old one, fittings between calculated and measured[2] $C(p)$-curves[3] are performed. These fittings aim at the extraction of some of the process parameters in combination with the geometric parameters of the sensor and the heuristic model parameter c (table I). The parameter c is solely needed for modeling the sensor's behavior in touch-mode [2]. Since none of the studied sensors were operated in touch-mode, it should not have any influence on the optimization result.

TABLE I

Parameters chosen for Curve Fitting and Extraction

parameter name	token
Young's modulus [Pa]	E
poisson ratio	ν
oxide thickness [m]	d_{ox}
dielectric constant of oxide	ε_{ox}
intrinsic stress [Pa]	σ
edge length [m]	a
thickness of the membrane [m]	h
height of pressure chamber [m]	w
heuristic model parameter	c

The fittings were carried out using some of the optimization strategies implemented into the model based optimization system MODOS [4]. The quality of a set of parameters is judged by the so called *quality function* $\mathrm{qf}(C_{calc})$ (eq. (4)). Once every measured $C_{mess_i}(p)$ is exactly matched by the corresponding calculated $C_{calc_i}(p)$, the optimal value of $\mathrm{qf}(C_{calc})$ is reached.

[1] w is the height of the pressure chamber.

[2] The measurements were carried out at the IMSAS by Mr. Dipl.-Ing. T. Eggers (University of Bremen).

[3] The $C(p)$-curve describes the dependency of the sensor's capacity onto the extern pressure p.

$$\mathrm{qf}(C_{calc}) = \left| \sum_{i=1}^{m} (C_{mess_i} - C_{calc_i}) \right| \quad (4)$$

On the basis of this function the applied optimization strategies decide, whether a chosen search direction tends to deliver better results or not. Performing such calculations it has to be kept in mind that process as well as geometric parameters vary across a wafer. In order to get mean parameter values instead of values being special to one sensor only, the measured $C(p)$-curves of 29 sensors placed at different spots on a wafer were used.

The starting point was set to the measured nominal parameter values. The lower (lower b.) and upper bounds (upper b.) (table II) correspond with the results according to [5] and [6].

TABLE II

Starting Values and Bounds for the first Evaluation Approach

parameter	start value	lower b.	upper b.
E	1.69E11	1.50E11	1.90E11
ν	0.26	0.25	0.27
d_{ox}	2.40E-08	5.00E-09	1.50E-7
ε_{ox}	3.9	2.5	4.5
σ	3.0E7	1.2E7	4.8E7
a	9.0E-5	5.0E-5	2.0E-4
h	9.7E-7	5.0E-7	2.0E-6
w	1.05E-6	5.00E-7	2.00E-6
c	1.0	0.0	-

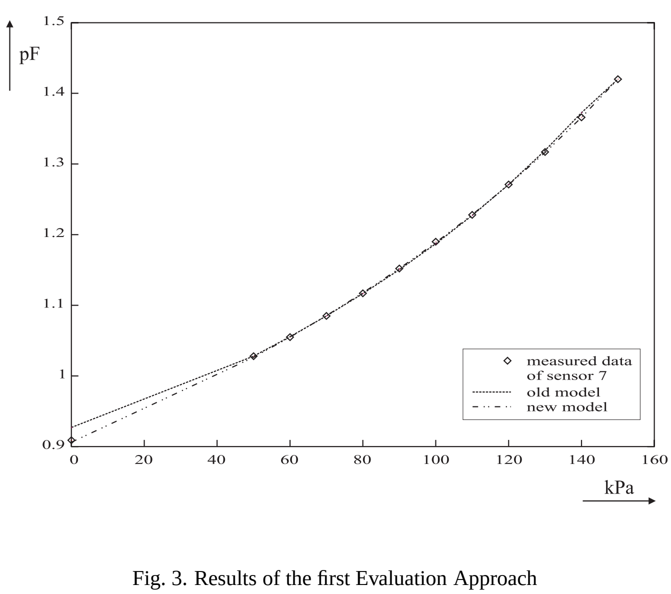

Fig. 3. Results of the first Evaluation Approach

The achieved fits are presented in Fig. 3. Only a small deviation between old and new model can be observed for low pressures. But the mismatch between the old model and the measured data is neglectable. Thus if the evaluation would solely be based on these fitting results, hardly any improvement would be achieved by the new modeling approach. But once the numerical values of the optimized parameters are taken into account, an interesting distinction can be observed (table III).

TABLE III

Results of the first Evaluation Approach

parameter	old model	new model
E	1.5E11	1.52E11
ν	0.251	0.253
d_{ox}	6.5E-8	4.1E-8
ε_{ox}	3.35	2.95
σ	1.96E7	2.56E7
a	1.00E-4	8.98E-5
h	7.45E-7	9.00E-7
w	1.12E-6	1.02E-6
c	5.5E1	1.0E0

The heuristic parameter c, introduced to describe the sensor's behavior in touch-mode, is varied once the old model is used and has no influence on the results obtained with the new model. Besides the oxide thickness d_{ox} and the dielectric constant ε_{ox} were not considered anymore. According to [5] they only vary with 1% around their nominal values. Hence the valid ranges for the oxide thickness would be: $23.76nm < d_{ox} < 24.24nm$. The dielectric constant could vary between 3.861 and 3.939. Neither the results obtained with the old model nor the results achieved by applying the new approach were in these ranges. For those reasons a second evaluation approach was chosen, which does not consider c, d_{ox} or ε_{ox}. They were all set to their nominal values.

D. *Second Evaluation Approach*

In the second evaluation approach the parameters c, d_{ox} and ε_{ox} were set to their nominal values. Furthermore the parameter ranges were chosen in conformity with the results of [5].

TABLE IV

Second Parameter Set

parameter	start value	lower b.	upper b.
E	1.69E11	1.6731E11	1.7069E11
ν	0.25	0.2475	0.2525
σ	3.0E7	1.2E7	4.8E7
a	9.0E-5	8.9E-5	9.1E-5
h	9.7E-7	9.2150E-7	1.0185E-6
w	1.05E-6	9.9700E-7	1.1025E-6

This is necessary because the optimization problem is a multi modal one. One physical reason for this is that larger deflections may either be obtained by increasing the intrinsic stress or by making the membranes longer and thinner. The aim of model evaluation is to find out whether any physically relevant parameter combination results in a good fitting between measured and calculated data. As figure 4 and table V point out the new modelling approach meets theses requirements while the old one does not.

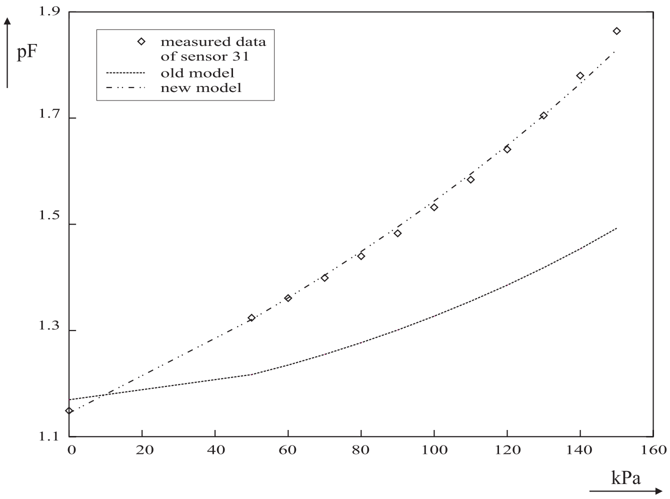

Fig. 4. Results of the second Evaluation Approach

Three out of the six varried parameters lie directly on or very close to their lower boundary once the old model is applied (table V). Using the new approach, none of the optimal values is chosen equal to a boundary Thus the quality of the new model is much higher than the quality of the old model.

TABLE V

Results of the second Evaluation Approach

parameter	old model	new model
E	1.673E11	1.681E11
ν	0.2476	0.2496
σ	1.2E7	2.0E7
a	9.081E-5	8.923E-5
h	9.2152E-7	9.3910E-7
w	9.9987E-7	1.0233E-6

E. *Third Evaluation Approach*

The third evaluation approach will explore whether the new model is capable of delivering good fitting results if in addition to the parameters of the second evaluation approach d_{ox} and ε_{ox} are taken into account within their phyiscal ranges. According to figure 5 the new model is even capable of discribing the oxide layer correctly.

F. *Conclusions*

Final result of the model evaluation process is that the new model represents the dependency of all major design parameters correctly. The old model runs into difficulties once the heuristic model parameter c is not considered. Therefore design optimizations can be performed on the basis of the new model. If the sensor is to be fabricated with another process and if some of the process parameters are unknown, this model may also be used for parameter extractions.

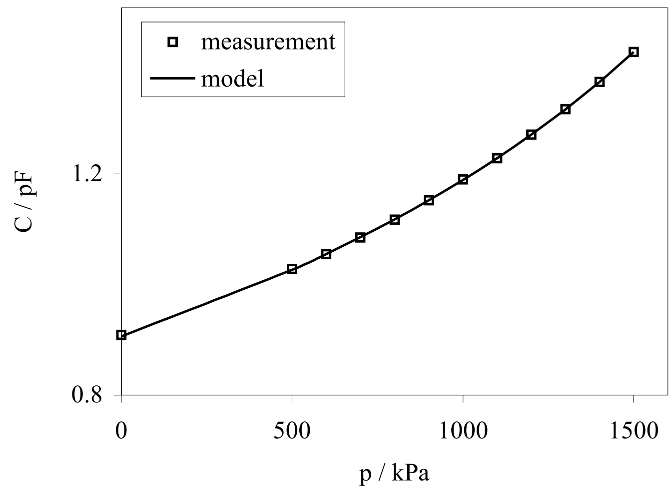

Fig. 5. Results of the third Evaluation Approach

IV. Summary

The paper presents a very efficient and reliable method for model evaluation using optimization strategies. In practice this method could replace costly, time-consuming and sometimes also error prone trial and error processes. The reliability of a new behavioral model for capacitive pressure sensors using quadratic membrane structures could be demonstrated.

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