Closed Loop Micromachined Inertial Sensors with Higher Order ΣΔ-Modulators

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ABSTRACT

Micromachined inertial sensors are often incorporated in closed loop force feedback structures; a particularly advantageous approach is based upon the inclusion of the sensing element in a sigma-delta modulator (ΣΔM) type control structure. The order of the ΣΔM, and hence the noise shaping properties, is usually limited by the dynamics of the mechanical sensing element and may be insufficient for high performance applications.

This paper presents a novel approach suggesting the use of a sigma-delta modulator with an order higher than the mechanical sensing element. This is achieved by a modified cascaded (or MASH) ΣΔM loop architecture. A model of the electromechanical MASH is presented and the quantisation noise analysis is derived. Simulation results indicate nearly 30 dB increase in signal to quantisation noise ratio.

Keywords: Inertial sensor, ΣΔ Modulator, accelerometer, noise shaping.

1 INTRODUCTION

High performance micromachined inertial sensors usually consist of a proof mass suspended between two electrodes forming a capacitive half-bridge. The differential change in capacitance provides a precise measure of the position of the mass. To further improve the performance, typically electrostatic force feedback is used which increases the bandwidth, linearity and dynamic range of the sensor. In recent years a closed loop control strategy based upon the incorporation of the mechanical sensing element in a ΣΔM feedback loop has become increasingly popular. This approach has been used both for micromachined accelerometers [1,2] and – more recently – vibratory rate gyroscopes [3]. A block diagram of such an accelerometer is shown in fig. 1. A major advantage over analogue force feedback control strategies is that the sensor produces a direct digital output signal in form of a pulse density modulated bitstream which is suitable to be processed by a standard DSP. Furthermore, in analogue force feedback systems a bias voltage is required which linearises the quadratic force–voltage relationship but introduces an electrostatic pull-in problem if the proof mass is displaced appreciably away from its rest position. In a ΣΔM control system only the electrode further away from the proof mass is energized, the other being grounded; consequently only

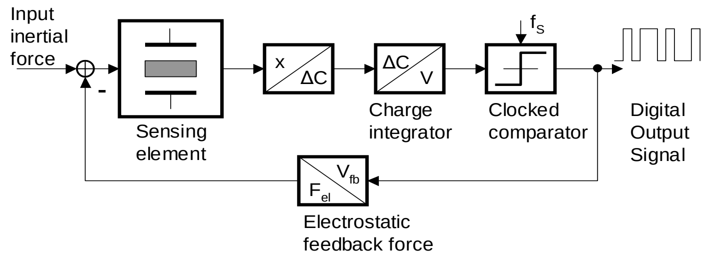


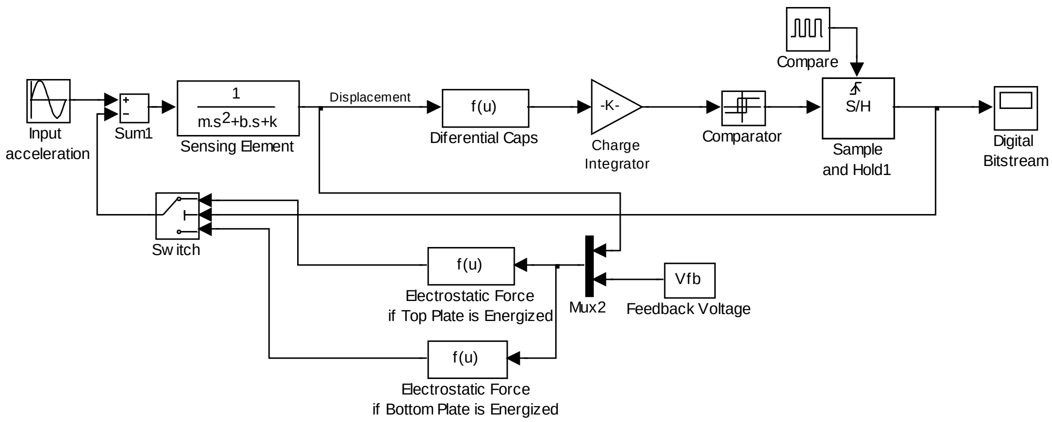
Fig. 1: Block diagram of an accelerometer embedded in a ΣΔM force feedback control loop.

an electrostatic force pulling the mass to the mid position between the electrodes is produced, thus the system stability is improved.

A major design parameter for such an electro-mechanical ΣΔM system is the signal to quantisation noise ratio (SQNR). The quantisation noise should be made small enough so that it does not limit the minimum detectable signal of the sensor, i.e. appreciably smaller than other noise sources such as Brownian noise, electronic thermal noise and noise introduced by interconnects. For the sensors described in the literature so far, the SQNR is mainly determined by the transfer function of the mechanical sensing element which can be approximated by a second order mass-damper-spring system and can be regarded as analogous to the two cascaded electronic integrators commonly used in 2^{nd} order electronic ΣΔM A/D converters [4]. Since the damping of a typical micromachined sensing element can be very high (due to squeeze film effects), the equivalent d.c. gain of the integrator functions is considerably lower than compared with their electronic counterparts. This leads to a much lower SQNR for the electro-mechanical ΣΔM compared with an electronic implementation. The most obvious method to increase the SQNR is to increase the sampling frequency of the system; however, this might not be possible in some cases due to other constraints.

2 SINGLE LOOP ARCHITECTURE

The mechanical sensing element under consideration for this work is a typical bulk-micromachined device fabricated in SOI technology with a capacitive position measurement interface. For simplicity, separate sets of electrodes for sensing and feedback have been assumed here. If operated in ambient air the sensing element is overdamped and consequently exhibits a dominant pole behaviour. A



Simulink model of the sensing element incorporated in a single loop ΣΔM feedback arrangement is shown in fig.2. The sensing element is described by a second order transfer function relating force to deflection of the proof mass. Here, the simplifying assumption has been made that the damping coefficient is constant (a valid assumption for small proof mass deflections [5]). The deflection of the proof mass is converted to a differential change in capacitance which, in turn, can be measured by a capacitive position measurement interface. In the model this also has been assumed to be a simple gain block. This signal is subsequently subjected to a clocked comparator which provides the output in form of a pulse density modulated bitstream. It also controls a switch determining to which electrode a feedback voltage is applied to. The electrostatic feedback force on the proof mass is given by:

$$F_{el} = \operatorname{sgn}(D)\frac{1}{2}\frac{\varepsilon_o A_{fb} V_{fb}^{\ 2}}{(d_0 + \operatorname{sgn}(D)x)^2} \tag{1}$$

where ε_0 is the dielectric constant, A_{fb} the area of the feedback electrode, V_{fb} the feedback voltage, d_0 the nominal gap between proof mass and the electrodes to either side, x the proof mass deflection from its rest position and D the comparator output assumed to be +1 or –1.

Several simulations were carried out in which the amplitude of the input signal was increased and the SQNR was calculated from the output bitstream. The resultant diagram is shown in fig. 3. The simulation result is compared with a simulated, optimised second order modulator and with the SQNR predicted by theory; the maximum SQNR for a full scale input signal is equivalent to the dynamic range and can be calculated by [4]:

$$DR = \frac{3}{2}\frac{2L+1}{\pi^{2L}} M^{2L+1} \tag{2}$$

where L is the order of the modulator and M the oversampling ratio.

For L=2 and an oversampling ratio M=64 the maximum SQNR is nearly 80dB and the simulated, optimised 2^{nd} order modulator almost reaches this value. The ΣΔM with the sensing element, however, exhibits a nearly 30dB loss in performance. This is due to the low gain of the integrators at low frequencies and the overdamped characteristics which lead to a SQNR typical for a 1^{st} order modulator, thus making this control approach very problematic for overdamped accelerometers.

3 MULTI LOOP ARCHITECTURE

To alleviate this problem, in this paper it is suggested for the first time, to apply a higher order ΣΔM architecture to a micromachined accelerometer. The fundamental approach is to cascade the electro-mechanical ΣΔM comprising the capacitive sensing element with a purely electronic ΣΔM. This results in a modified 'MASH' ΣΔM structure in which the quantisation error of the first ΣΔM is fed to a second modulator; this provides further noise shaping of the quantisation noise. In this paper a simple 1^{st} order modulator was chosen which provided the overall SQNR sufficient to meet the requirements for this work. If even better noise shaping was required, a higher order modulator could be used as well. The approach is illustrated

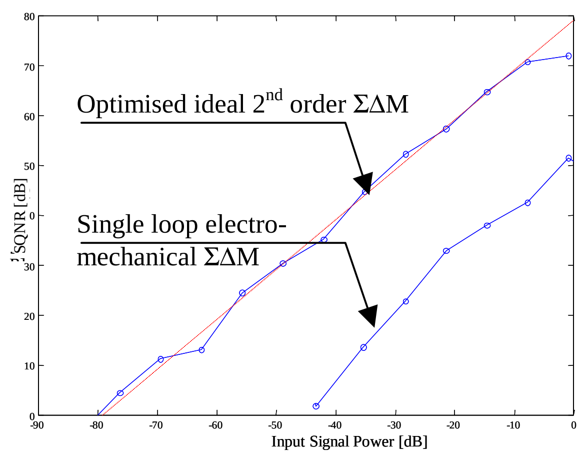


Fig. 3: SQNR with the sensing element incorporated in a single loop electro-mechanical ΣΔM compared with an optimised, ideal 2^{nd} order ΣΔM. The dashed line shows the theoretically predicted SQNR.

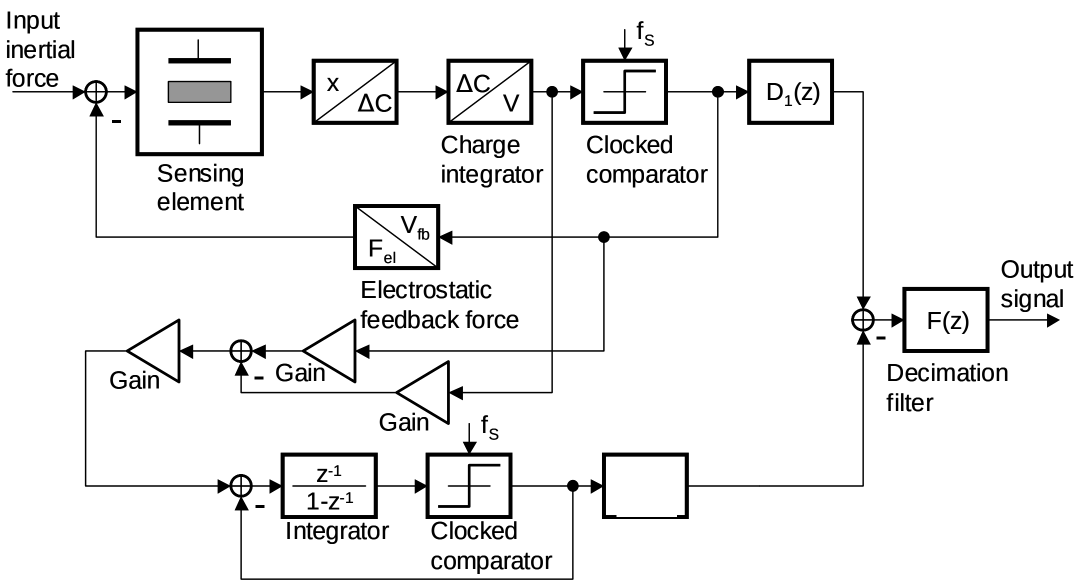


Fig. 4: Block diagram of the micromachined accelerometer incorporated in a MASH ΣΔM force feedback loop.

in block diagram form in fig. 4.

3.1 Theory

The MASH ΣΔM architecture approach relies on the cancellation of the quantisation noise from all but the last loop. For the mathematical analysis the clocked comparator is replaced by a gain block (termed k_{Q1} for the first loop and unity gain for the second loop) and some added white noise, which is the usual approximation made in ΣΔM theory. Furthermore, for small proof mass deflections the conversion from a proof mass deflection ($x \ll d_0$) to a voltage can be represented as a simple gain factor, k_{po}. The electrostatic force also can be assumed of constant magnitude, k_{fb}. With these simplifications a model suitable for mathematical analysis can be derived and is shown in fig.5.

After a somewhat cumbersome derivation, which is omitted here for brevity, it can be shown that noise of the first loop is shaped by:

$$NTF_{Q_1} = \left(\frac{k_{Q1} D_1(z)}{1 + M(z) k_{po} k_{Q1} k_{fb}} - k_{Q1} z^{-1} D_2(z) \right) Q_1$$

$$F_{el} = \mathrm{sgn}(D) \frac{1}{2} \frac{\varepsilon_o A_{fb} V_{fb}^{\,2}}{(d_0 + \mathrm{sgn}(D) x)^2}$$ where M(z) is the z-domain equivalent of the force-displacement transfer function of the sensing element, for the other symbols refer to fig. 5.

If now $D_1(z)=z^{-1}$ and $D_2(z)=1/(1+M(z)k_{fb}k_{po}k_{Q1})$ are chosen, the noise transfer function NTF_{Q1} equals to zero and hence the quantisation noise of the first loop is totally cancelled. This approach requires a precise knowledge of the M(z) and k_{Q1}, but is in practice not possible. M(z) depends on the absolute values of the spring constant, mass and damping coefficient of the sensing element, the former two are subject to considerable manufacturing tolerances, and the latter is inherently nonlinear due to squeeze film damping effects. k_{Q1} is the effective gain of the quantiser in the first loop and depends on the magnitude of its input signal, thus only an average value can be estimated. However, as described in the next section, the overall system performance is fairly tolerant of gain factor mismatches.

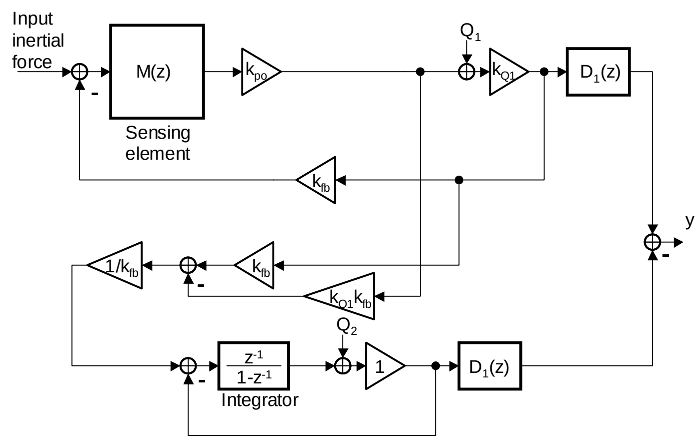


Fig. 5: Block diagram of the micromachined accelerometer incorporated in a MASH ΣΔM suitable for mathematical analysis.

3.2 Simulation Results

A Simulink model was implemented of the MASH ΣΔM force feedback loop as shown in fig. 6. The model for the first loop is identical to fig. 2, the second loop is a

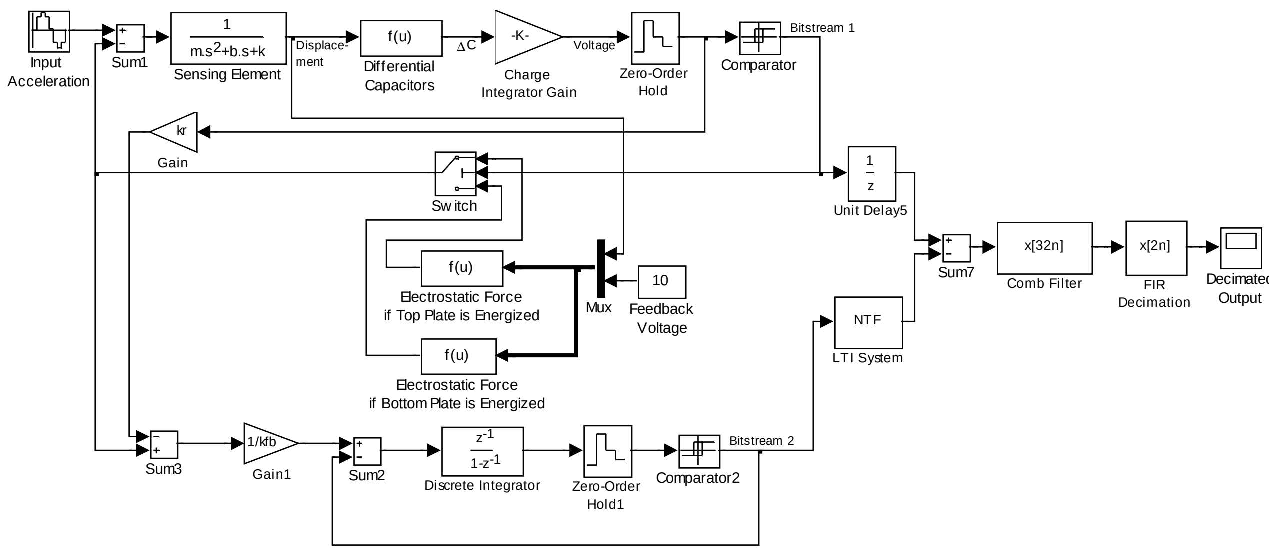


Fig. 6: Simulink simulation model of the MASH ΣΔM force feedback loop.

standard first order ΣΔM. The block labelled NTF realises the transfer function $D_2(z)$ as described in the previous section. The same simulations as for the single loop system were carried out. The simulation result is presented in fig.7. For comparison, the simulation result for the single loop system is shown as well. It is obvious that the SQNR has increased by nearly 30dB and hence is similar to that of an idealized 2nd order ΣΔM. Further simulations were carried out in which the critical system parameters were varied from their ideal calculated values, the SQNR only decreased insignificantly for up to 10% mismatch.

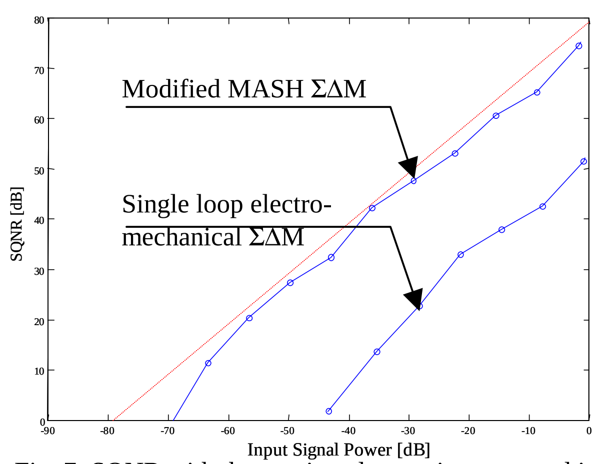


Fig. 7: SQNR with the sensing element incorporated in the MASH ΣΔM compared with a single loop approach.

4 CONCLUSIONS

In this paper a new approach for the control system design of closed loop inertial sensors is suggested, which is suitable for micromachined accelerometers and gyroscopes. It is based upon the incorporation of a micro-mechanical sensing element in a higher order ΣΔM control loop. A modified MASH ΣΔM was identified as a suitable architecture. This provides considerably better noise shaping compared to a single loop control system in which the dynamics of the sensing element mainly determine the noise shaping. If squeeze film damping effects make the dynamics of the sensing element overdamped, a single loop approach only provides first order noise shaping which may be insufficient for many applications. The modified MASH ΣΔM control system improves the SQNR ratio by nearly 30dB and makes the noise shaping similar to a 2nd order modulator.

The simulation work presented here was aimed at illustrating the feasibility of the approach. A hardware implementation using a typical bulk-micromachined accelerometer sensing element is currently pursued.

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