Efficient Simulation of MEMS Using Element Stamps

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ABSTRACT

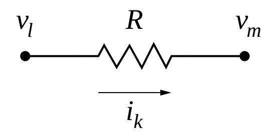
This paper describes an approach to MEMS simulation based on so-called element stamps, which are the building blocks of conventional circuit simulators. Stamps are derived from lumped-constant models of individual electromechanical devices, and built into the simulator. When a system is simulated, the equations describing its behavior can be built and solved in a systematic and efficient way using the stamps of its constitutive elements. This makes the simulation faster and more efficient, compared to simulators that rely on models written in hardware description languages. Numerical results obtained from the simulation of a simple electromechanical system containing a linear comb-drive accelerometer are presented.

1 INTRODUCTION

The combination of MEMS devices with conventional integrated circuit technology can potentially lead to the development of highly efficient, low-cost mixed-technology systems with a wide range of commercial and military applications. Such systems may contain integrated control circuitry, sensors, digital logic, etc., for a total of hundreds to thousands of tightly-coupled electrical and mechanical devices.

One of the crucial needs arising in the design phase of such systems is to verify their behavior by means of simulation. Published results show that lumped-constant models of MEMS devices are sufficiently accurate for many applications [1], [2]. Ordinary circuit simulation also uses lumped-constant models of electronic devices. This makes it possible to model complex mixed-technology systems starting from the models of their constitutive elements. The challenge is to find a way to build automatically coupled systems of equations in the electrical and mechanical domains so that the computational effort required to solve them is comparable to that of ordinary circuit simulation.

This paper proposes an approach to the simulation of MEMS based on an extension of the familiar concept of element stamps for electrical components [3]. Like Kirchhoff's laws, the equations governing the dynamics of systems of constrained rigid bodies are additive with respect to the number of elements contained in the system: adding one body or one constraint results simply in the addition of a certain number of terms to the set of equations describing the system. As a consequence, models of MEMS devices can be represented by so-called stamps, which contain all the terms contributed by the device to the global system of equations.



	v_l	v_m
l	$+G$	$-G$
m	$-G$	$+G$

Figure 1. Resistor stamp derivation

A brief explanation of the concept of element stamp for simulation is given in Section 2, while Section 3 illustrates the derivation of the stamp for a comb-drive accelerometer. This stamp was used to model the accelerometer in a simple electromechanical system, and the simulation results are presented in Section 4.

2 ELEMENT STAMPS

2.1 Electrical Element Stamps

The Modified Nodal Analysis (MNA) method for circuit simulation is based on Kirchhoff's current law (KCL), which states that the sum of all the outgoing currents at each node must be equal to zero:

$$\sum_k i_k = 0. \tag{1}$$

In this equation, each i_k represents the branch current through an element connected to the node. Kirchhoff's current law written at each node in the circuit generates a set of algebraic-differential equations. By applying a suitable numerical integration algorithm, these equations are transformed into a set of algebraic equations, which is then solved by means of Newton-Raphson's method [3]. This requires the repeated solution of a set of linear equations:

$$\mathbf{A}\mathbf{v} = \mathbf{b}. \tag{2}$$

The observation that each element in the circuit contributes to (2) in an easily identifiable way leads to the concept of *element stamp* [3]. For example, consider a resistor connected between nodes *l* and *m*, as shown in Fig. 1, and let *G* be its conductance. The branch current of this particular element, $i_k = G(v_l - v_m)$, appears with a positive sign in the KCL equation at node *l*, and with a negative sign in the KCL equation at node *m*. Therefore the resistor contributes a term equal to $+G(v_l - v_m)$ to the *l*-th equation and a term equal to $-G(v_l - v_m)$ to the *m*-th equation in (2). If v_l and v_m are the *l*-th and *m*-th elements of **v** respectively, the resistor's contribution to the coefficient matrix **A** in (2) adds a quantity equal to $+G$ to entries in positions (l,l) and (m,m) and a quantity equal to $-G$ to entries in positions (l,m) and (m,l).

Therefore the resistor's contribution can be represented by the stamp shown in Fig. 1. The labels in the stamp's leftmost column represent row indices in the coefficient matrix, while entries in the stamp's top row represent unknown variables, and thus correspond to column indices in the matrix. The resistor's contribution to the right-hand side of (2) can also be represented with a stamp, which is omitted here for space reasons.

The set of equations (2) can be built simply by adding the contribution of each element in the circuit through the corresponding stamp. This is possible because (1), from which (2) is derived, is a linear equation in the branch currents i_k. If element stamps are built into the simulator, the equations can be assembled and solved in a very efficient manner.

2.2 MEM Element Stamps

Most of the research work on MEMS simulation has focused on distributed-constant methods of analysis [4]. Recently, however, efforts have been made to develop lumped-constant models for MEMS, by exploiting the fact that most, if not all MEMS structures are built from a common set of basic (or atomic) elements, such as beams, anchors, plates, and so on [2], [5]. Lumped-constant models are less accurate than distributed-constant models, but greatly reduce the computational effort required for simulation. Published results indicate that the accuracy of lumped-constant models of MEMS devices is sufficient for many applications [1], [6].

Conventional circuit simulation also relies on lumped-constant models of electronic devices. This fact suggests that efficient MEMS simulation can be achieved by extending the concept of element stamp to MEMS devices and their basic constitutive elements. As shown next, this can be done simply by replacing Kirchhoff's current law with the Newtonian equations of motion.

For simplicity, it will be assumed that MEM elements can be modeled as rigid bodies, and that their motion is restricted to three-dimensional translations and rotations around a fixed axis. The methodology for the derivation of stamps, however, remains valid even if these restrictions are removed.

Under these assumptions, the equations of classical mechanics governing the dynamics of rigid bodies can be expressed as follows:

$$\begin{aligned} \sum_k \mathbf{F}_k &= 0 \\ \sum_k T_k &= 0, \end{aligned} \tag{3}$$

where $\sum \mathbf{F}_k$ is the sum of all the forces acting on the body, and $\sum_k T_k$ is the sum of the force torques with respect to the axis of rotation. The forces in the first equation include the inertial force: $\mathbf{F}_p = -\dot{\mathbf{p}}$, where $\mathbf{p}$ is the body's linear momentum. Similarly, the torques in the second equation include the inertial torque: $T_m = -\dot{m}$, where *m* is the body's angular momentum around the axis of rotation.

The similarity between (3) and (1) is apparent. This means that element stamps can also be used to build the equations in (3). Some MEMS devices, however, can also function as electrical components: for example, the two sets of fingers of a comb drive form a capacitor. Thus a complete description of the device's behavior must involve both the mechanical and the electrical domains. As a consequence, the stamp of a MEMS device will combine both mechanical and electrical variables and equations, as illustrated in the next section.

3 ACCELEROMETER STAMP

The stamp of a specific MEMS device (or of its constitutive elements) is obtained by modeling its electromechanical behavior. As an example, consider the simple comb-drive accelerometer shown in Fig. 2. In this device, a displacement of the central comb structure creates a differential capacitance between the comb and the static fingers, as shown in the enlargement in Fig. 2. This differential capacitance can be detected by circuits connected to the accelerometer.

From a mechanical standpoint, the central comb can be modeled as a mass connected to a spring and subject to acceleration and electrical forces. According to this model, the

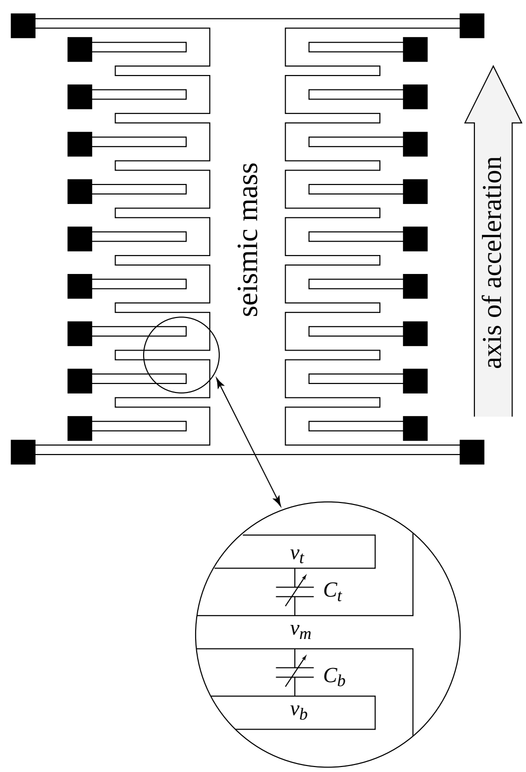


Figure 2. A linear comb-drive accelerometer [7]

	v_t	v_m	v_b	x	u_x
t	$\frac{\partial q_t}{\partial v_t}$	$\frac{\partial q_t}{\partial v_m}$		$\frac{\partial q_t}{\partial x}$	
m	$-\frac{\partial q_t}{\partial v_t}$	$-\frac{\partial q_t}{\partial v_t}-\frac{\partial q_b}{\partial v_b}$	$-\frac{\partial q_b}{\partial v_b}$	$-\frac{\partial q_t}{\partial x}-\frac{\partial q_b}{\partial x}$	
b		$\frac{\partial q_b}{\partial v_m}$	$\frac{\partial q_b}{\partial v_b}$	$\frac{\partial q_b}{\partial x}$	
x				1	$-\frac{h}{2}$
u_x	$\frac{h}{2}\frac{\partial F_e}{\partial v_t}$	$\frac{h}{2}\frac{\partial F_e}{\partial v_m}$	$\frac{h}{2}\frac{\partial F_e}{\partial v_b}$	$\frac{h}{2}(k+\frac{\partial F_e}{\partial x})$	$m+\frac{h}{2}b$

Figure 3. Accelerometer stamp

accelerometer dynamics are described by the following pair of equations:

$$\begin{aligned}\dot{x} &= u_x \\ m\dot{u}_x &= -bu_x - kx - F_e(v_t, v_m, v_b, x) + ma,\end{aligned} \tag{4}$$

where *x* is the comb's displacement along the axis of acceleration, F_e is the electrical force generated by the charges on the accelerometer's fingers, and *a* is the acceleration of the accelerometer's frame of reference.

A simple lumped-constant model of the accelerometer's electrical behavior consists simply of two capacitances C_t and C_b between the comb structure and the fixed fingers:

$$\begin{aligned}i_t &= \dot{q}_t(v_t, v_m, x) = \frac{d}{dt}[C_t(x)(v_t - v_m)] \\ i_b &= \dot{q}_b(v_b, v_m, x) = \frac{d}{dt}[C_b(x)(v_b - v_m)] \\ i_m &= -(i_t + i_b) = \\ & -\frac{d}{dt}[C_t(x)(v_t - v_m) + C_b(x)(v_b - v_m)].\end{aligned} \tag{5}$$

Using a parallel-plate model for these capacitances gives the following expressions for C_t and C_b:

$$\begin{aligned}C_t &= \frac{\varepsilon_0 A}{d - x} \\ C_b &= \frac{\varepsilon_0 A}{d + x},\end{aligned} \tag{6}$$

where *A* is the capacitors' equivalent area, and *d* is the at-rest value of the gap between fingers. This capacitance model yields the following expression for the electrical force on the comb:

$$\begin{aligned}F_e &= -\frac{1}{2}\frac{\partial}{\partial x}(C_t v_{tm}^2 + C_b v_{bm}^2) = \\ &= -\frac{\varepsilon_0 A}{2}\left[\frac{v_{tm}^2}{(d-x)^2} - \frac{v_{bm}^2}{(d+x)^2}\right],\end{aligned} \tag{7}$$

where $v_{tm} = v_t - v_m$ and $v_{bm} = v_b - v_m$.

Equations (4) through (7) provide a complete description of the accelerometer's electromechanical behavior. The corresponding stamp contains five rows: three for KCL equations (5) at each of the accelerometer's electrical terminals, and two for the mechanical dynamics equations in (4). Similarly, there are five columns in the stamp, one each for the five variables v_t, v_m, v_b, x, u_x. Expressions for the stamp entries are obtained by first applying a numerical integration method, such as the trapezoidal method, to the device equations, and then linearizing the resulting expression with respect to the device variables. The resulting stamp is shown in Fig. 3. The contributions of these equations to the right-hand side of (2) can also be collected in a stamp following a similar procedure.

4 NUMERICAL RESULTS

To verify the effectiveness of this approach to MEMS simulation, the accelerometer stamp described in the previous section was incorporated into a circuit simulator. A simple system containing the accelerometer, shown in Fig. 4, was simulated to verify its response to a $3g$ acceleration impulse starting at $t = 0.5$ ms and lasting 3 ms. The input voltages were two 4 KHz sinusoids in phase opposition. The corresponding displacement of the accelerometer's comb structure and the accelerometer's output voltage as a function of time are shown in Fig. 5. It can be seen that around $t = 3$ ms, when the oscillation of the comb mass has almost died down, the amplitude of the output voltage settles at a constant value, which is determined by the C_t/C_b ratio. It can also be seen that the frequency of the oscillation in the comb displacement is twice the voltage frequency, consistent with the fact that the electrostatic force on the comb mass depends on the square of the input voltages. The simulation of this system required less than 0.4 s on a Sun Ultra 1 workstation running Solaris 2.6.

5 CONCLUSIONS

This paper has suggested an approach to MEMS simulation based on element stamps, a concept initially developed in the

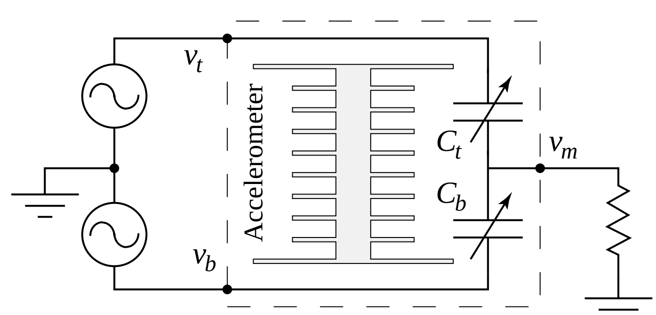


Figure 4. A simple electromechanical system

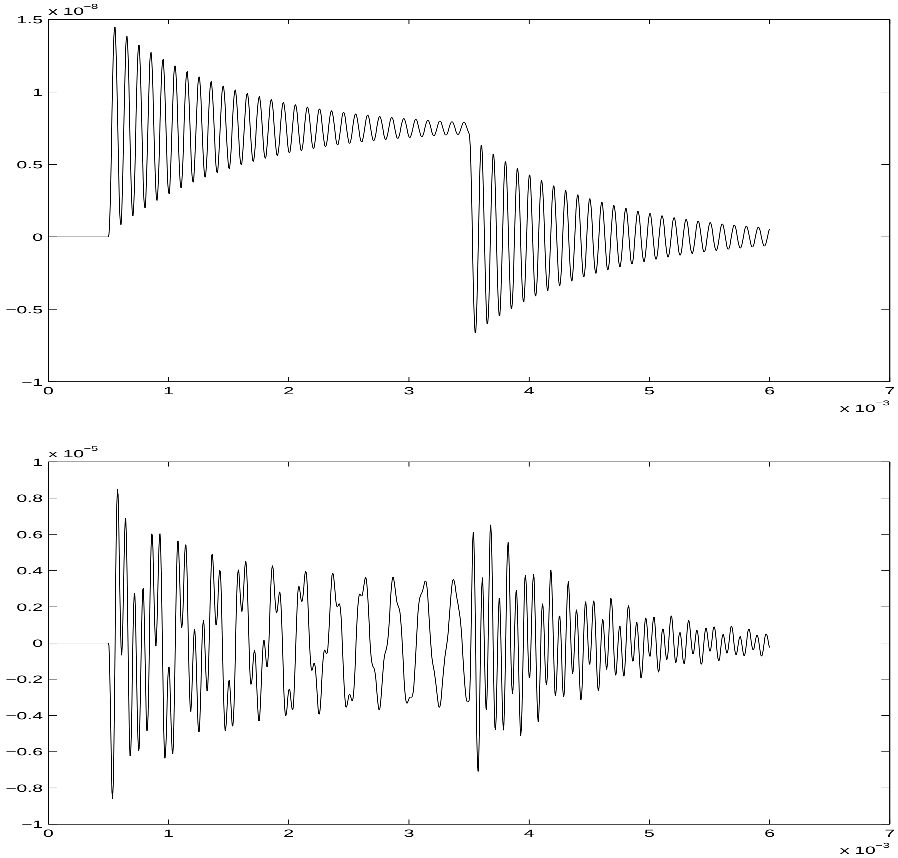


Figure 5. Accelerometer's response: displacement (top) and output voltage (bottom) vs. time

context of circuit simulation. Element stamps are derived from device models, and provide a compact and efficient way to add the contribution of a particular element to the set of equations describing the overall behavior of a system.

Representing MEMS devices with stamps makes it possible to include built-in models for them in simulators, and to handle them no differently than other circuit elements, such as capacitors or transistors. On the one hand, this means that a simulator that relies on stamps can handle only devices described by its built-in models. This is unlikely to be a major drawback, however, because most MEMS devices are built from a limited set of elementary elements [5]. On the other hand, the use of element stamps greatly reduces the computational effort required for simulation, compared to general-purpose simulators like SABER [1] or MATLAB [6] that rely on user-provided HDL models. This becomes a significant advantage in the simulation of mixed-technology systems containing hundreds or even thousands of electronic and MEMS devices.

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