Full Wave Electromagnetic Analysis And Model Order Reduction For Complex Three Dimensional Structures

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ABSTRACT

Progress in MEMS and packaging design has made significant changes in the requirements for modeling tools. In order to design a modern microdevice or interconnect system it is no longer sufficient to limit the analysis to quasielectrostatic modeling due to smaller size and higher operation frequencies of the microdevices. On the other hand, a 'full-wave' analysis produces huge systems of equations which require tremendous amounts of computational resources. The method described in this paper is a combination of three-dimensional full-wave coupled RLC electromagnetic analysis and passive Arnoldy based model order reduction algorithm and allows to solve wider range of design problems including miniaturization and high frequency modeling problems.

Keywords: MEMS, model order reduction, full-wave electromagnetic model, CAD, interconnect, packaging

1 INTRODUCTION

During the last few years much work has been done in the field of coupled three dimensional electromagnetic analysis, however a major problem arises from difficulties with integration of a 'full-wave' Greens Function for electromagnetic field. The described method uses power series expansion to handle these integrals and uses an analytical formula described in [3] to handle singularities arising from the use of this method. Another difficulty with full-wave coupled modeling is the size of the resulting system, which is sparse but of infinite dimension. The adoption of the passive model order reduction algorithm [1] allows us to reduce the size of the resulting system from infinitely large to fairly small. It is estimated that the number of unknowns in the reduced system can be about two orders of magnitude less than in original system.

2 INTEGRAL EQUATION FORMULATION

Several integral equation-based approaches have been used in modern simulation tools to determine impedance, Z, associated with a given interconnect structure or package. The formulation given here is the standard integral equation approach based on a 'superposition of the sources' point of view. From the Maxwell's equation using Laplace transformation we can write:

$$\nabla \times \overline{E} = -s\mu \overline{H} \tag{2.1}$$

$$\nabla \times \overline{H} = s\varepsilon \overline{E} + \overline{J} \tag{2.2}$$

$$\nabla \cdot (\varepsilon \overline{E}) = \rho \tag{2.3}$$

$$\nabla \cdot (\mu \overline{H}) = 0 \tag{2.4}$$

where s is the Laplace transform complex frequency.

Divergence of (2.2) combined with (2.3) gives the charge conservation,

$$\nabla \cdot \overline{J} = -s\rho \tag{2.5}$$

Now, we will eliminate the field quantities, $\overline{E}$ and $\overline{H}$, in favor of the current density, $\overline{J}$. From Gauss's Law of magnetic flux (2.4), the magnetic flux can be written as

$$\mu \overline{H} = \nabla \times \overline{A} \tag{2.6}$$

where $\overline{A}$ is the vector potential. Applying this to (2.1),

$$\nabla \times (\overline{E} + s\overline{A}) = 0 \tag{2.7}$$

This implies that there exists a scalar function Φ, such that

$$-\nabla \Phi = \overline{E} + s\overline{A} \tag{2.8}$$

where Φ will be called the scalar potential. Now we require one final relation to relate the current density, $\overline{J}$ to the vector potential, $\overline{A}$. To uniquely determine $\overline{A}$, we must also specify the divergence. We will use the Lorentz gauge,

$$\nabla \cdot \overline{A} = -\varepsilon \mu s \Phi \tag{2.9}$$

and using (2.6) in (2.2), and the identity, $\nabla\times(\nabla\times\bar{A}) = \nabla(\nabla\cdot\bar{A}) - \nabla^2\bar{A}$, we can write that

$$\nabla^2\bar{A} - \varepsilon\mu s^2\bar{A} = -\mu\bar{J} \tag{2.10}$$

and thus

$$\bar{A}(\bar{r}) = \frac{\mu}{4\pi}\int_{V'}\frac{\bar{J}(\bar{r})e^{s/c|\bar{r}-\bar{r}'|}}{|\bar{r}-\bar{r}'|}dv' \tag{2.11}$$

where V' is the volume of all conductors and $c = \frac{1}{\sqrt{\mu\varepsilon}}$.

To derive the similar relation for Φ, use (2.8) in (2.3) which gives

$$\nabla\cdot\varepsilon(-\nabla\Phi + s\bar{A}) = \rho$$

Applying (2.9) we will have

$$\nabla^2\Phi - \varepsilon\mu s^2\Phi = -\rho$$

And integral relation between the charge density and scalar potential is then

$$\Phi(\bar{r}) = \frac{1}{4\pi}\int_{V'}\frac{\rho(\bar{r})e^{s/c|\bar{r}-\bar{r}'|}}{|\bar{r}-\bar{r}'|}dv' \tag{2.12}$$

To derive an integral equation for the potentials and currents alone, we note that inside the conductors, by Ohm's law

$$\bar{J} = \sigma\bar{E} \tag{2.13}$$

where σ is the conductivity. Equation (2.13) can be used so that the charge density is essentially zero on the interior of the conductor. Substituting (2.13) and (2.3) in (2.5) we can see that

$$\frac{d\rho}{dt} = -\frac{\sigma}{\varepsilon}\rho \tag{2.14}$$

and it gives us that

$$\rho = \rho_0 e^{-\frac{\sigma}{\varepsilon}t} \tag{2.15}$$

this states that the charge inside a conductor is zero for all time apart from any initial charge which dissipates very quickly. So we can see that charge is restricted to the surface and (2.5) becomes

$$\nabla\cdot\bar{J}(\bar{r}) = 0, \quad \bar{r} \text{ - inside of conductor} \tag{2.16}$$

$$\bar{n}\cdot\bar{J}(\bar{r}) = -s\rho(\bar{r}), \quad \bar{r} \text{ - on a surface of a conductor} \tag{2.17}$$

For the region of surface of a conductor corresponding to terminals we allow for external current, $\bar{J}_t$ and then (2.17) will become

$$\bar{n}\cdot(\bar{J}(\bar{r}) - \bar{J}_t) = -s\rho(\bar{r}) \tag{2.18}$$

Substituting (2.11) and (2.13) into (2.8) we will obtain following integral equation

$$\frac{\bar{J}(\bar{r})}{\sigma} + \frac{s\mu}{4\pi}\int_{V'}\frac{\bar{J}(\bar{r})e^{s/c|\bar{r}-\bar{r}'|}}{|\bar{r}-\bar{r}'|}dv' + \nabla\Phi(\bar{r}) = 0 \tag{2.19}$$

Significance of the $e^{s/c|\bar{r}-\bar{r}'|}$ can be seen from the ratio of the wavelength to the characteristic length of the structure:

$$s/c|\bar{r}-\bar{r}'| \ll 1 \Rightarrow e^{s/c|\bar{r}-\bar{r}'|} \approx 1 \tag{2.20}$$

and we can see that for high frequencies we should use $\frac{e^{s/c|\bar{r}-\bar{r}'|}}{|\bar{r}-\bar{r}'|}$ kernel which we will call 'full-wave' and for low frequencies we can use $\frac{1}{|\bar{r}-\bar{r}'|}$ kernel which is electromagnetoquasistatic approximation.

3 DISCRETIZATION

We will divide our structure into filaments of rectangular cross-section inside, with the current components assumed to flow along the filament edges. Each filament is surrounded by panels which are not connected to each other with different charge densities on each of them. In order to properly capture skin and proximity effects in a conductor, the cross section of the conductor can be divided into bundle of parallel filaments.

If the current density in each direction inside of each filament is assumed to be constant, then the approximation to the unknown current distribution can then be written as

$$\bar{J}(\bar{r}) = \sum_{i=1}^{b} I_i w_i(\bar{r})\bar{l}_i \tag{3.1}$$

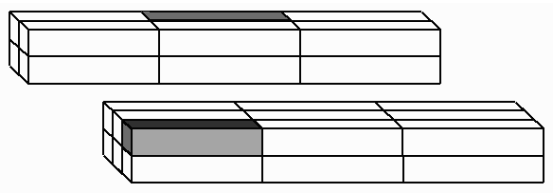

Figure 1: Two conductors discretized into filaments and panels. Each side of each filament is a panel. Different colors on sides of marked filaments correspond to different surface charges.

where I_i is the current inside the filament i, $\bar{l}_i$ is a unit vector along the direction of the current and $w_i(\bar{r})$ is the weighting function which has a value of zero outside filament i and $1/a_i$ inside, where a_i, if the cross section area of filament. By defining the inner product of two vector function, $\bar{a}$ and $\bar{b}$, by

$$(\bar{a},\bar{b}) = \int_V \bar{a}\cdot\bar{b}dv \tag{3.2}$$

and following the method of moments, a system of b equation can be generated by taking the inner product of each of the weighting functions with the vector integral equation (2.19). Using current continuity equation (2.18) we can write in a matrix form

$$\begin{bmatrix} R+sL & P \\ D & sI \end{bmatrix}\begin{bmatrix} i \\ q \end{bmatrix} = \begin{bmatrix} 0 \\ i_t \end{bmatrix} \tag{3.3}$$

where i is a vector of filament currents, q is a vector of panel charges, i_t is a vector of terminal currents,

$$R_{ij} = \frac{l_i}{a_i\sigma} \tag{3.4}$$

are elements of the diagonal matrix of filament DC resistances,

$$L_{ij} = \frac{\mu}{4\pi a_i a_j}\int_{V_i}\int_{V_j'}\frac{\bar{l}_i\cdot\bar{l}_j e^{s/c|\bar{r}-\bar{r}'|}}{|\bar{r}-\bar{r}'|}dv' \tag{3.5}$$

are elements of the partial inductances matrix,

$$P_{ij} = \frac{1}{4\pi\varepsilon a_j}\int_{S_j^-}\frac{e^{s/c|\bar{r}_i-\bar{r}'|}}{|\bar{r}_i-\bar{r}'|}ds - \frac{1}{4\pi\varepsilon a_j}\int_{S_j^+}\frac{e^{s/c|\bar{r}_i-\bar{r}'|}}{|\bar{r}_i-\bar{r}'|}ds \tag{3.6}$$

are elements of the potential coefficient matrix, D is the coefficient matrix corresponding difference of currents and charges, and I is the identity matrix. This discretization does not explicitly enforce zero charge inside of the conductor. The advantage of this formulation is that the zero internal charge condition can be used as a way to check the accuracy of the model, and this formulation does not produce unnecessary boundary conditions for each filament.

Equation (3.3) can be written in terms of single frequency dependent matrix M(s)

$$M(s)x(s) = b \tag{3.7}$$

4 POWER SERIES REPRESENTATION

The numerical calculation of integrals contained in (3.5) and (3.6) requires much computational time due to the singularity of the kernel if it is performed directly. To solve this problem we expanded the Greens function into a power series using polynomial with an infinite radii of convergence. Now we can represent each integral as infinite sum of integrals of powers of $s/c|\bar{r}-\bar{r}'|$. The first integral which contains $\frac{1}{|\bar{r}-\bar{r}'|}$ can be computed analytically [3]. The rest do not contain a singular kernel and could be computed numerically in reasonable period of time. Examples of good polynomial expansion include Hermintian polynomials expansion and even Taylor series expansion which have infinite radii of convergence for the expansion of exponential functions.

5 MODEL ORDER REDUCTION

In order to reduce computational time, an Arnoldy based passive model order reduction algorithm [1] was used. Let us consider equation (3.7):

$$M(s)x(s) = b$$

and let us present M(s) in a from:

$$M(s) = \frac{1}{s^p+1}M_t + \frac{s^p}{s^p+1}M_t \tag{5.1}$$

where M_t is power series expansion of M(s) with infinite radii of convergence. This form is useful to compute the integral contained in (3.5) and (3.6) as it was described in Section 4. We will then generate first order infinite dimension matrix in a way similar to that described in [2].

$$\left[I + \frac{s}{s^p + 1} \begin{vmatrix} \tilde{M}_0 & \tilde{M}_1 & \cdots & \tilde{M}_N \\ -I & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & -I & 0 \end{vmatrix} \right] \begin{vmatrix} x_0 \\ x_1 \\ \vdots \\ x_N \end{vmatrix} = \begin{vmatrix} b \\ 0 \\ \vdots \\ 0 \end{vmatrix} \qquad (5.2)$$

$$y = [c^t \quad 0 \quad \cdots \quad 0] \begin{vmatrix} x_0 \\ x_1 \\ \vdots \\ x_N \end{vmatrix}$$

where $x_k = s^k x$, $\tilde{M}_k = M_0^{-1} M_{k+1}$ for k+1<p and $\tilde{M}_k = M_0^{-1}(M_{k+1} + M_{k+1-p})$ for k+1 ≥ p.

Now we can apply the Arnoldy process to the new systems with $\tilde{s} = \frac{s}{s^p + 1}$. The full algorithm can be described as follows

1. Calculate M_0
2. set n=0 and set some p>1
3. Calculate zero order transfer function
4. n++
5. Calculate M_n
6. Update p such as $\left| \tilde{s} \max_{i,j} \tilde{M}_{i,j} \right| < 1$ for any M_n and $\tilde{s}$
7. Update M_l for all l from 0 to n
8. Calculate n-th order transfer function
9. Check convergence of transfer function
10. If converged, calculate the reduced model, else back to step 4

Using the properties of power series expansion with infinite radii of convergence and the definition of passivity, it is possible to show that the presented algorithm will produce a passive reduced model. The proof is based on the fact that we do not have to truncate the expansion of M(s) in order to calculate the n-th order of reduced model and that the expansion of transfer function will converge for any value of $\tilde{s}$.

6 TEST COMPUTATIONAL EXAMPLE

The Figure 2 demonstrates the advantage of the described method. A simple example of terminated transmission line was taken to illustrate our method. As can be seen, the size of the system can be tremendously reduced without much loss of accuracy of the solution. In this example, the size of the system was reduced by two orders of magnitude with relative error less then 0.1%. Thus we have reduction of four orders of magnitude in memory required for the system and two orders of magnitude in computational time.

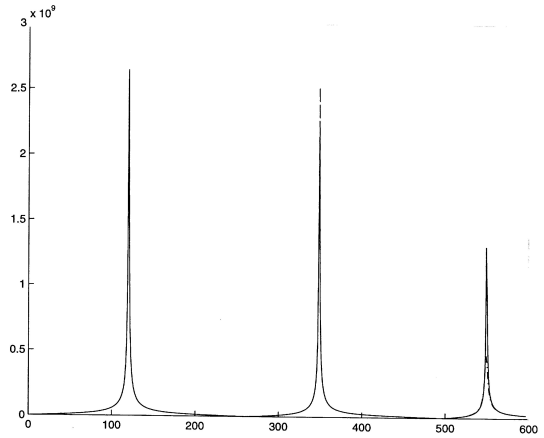

Figure 2: Solution of system of 7500 unknowns generated with 50th order series expansion (solid line) and solution of reduced system of 72 unknowns (dashed line). On a horizontal-axis shown frequency f(10^5 Hz), on a vertical-axis shown admittance

7 SUMMARY AND FUTURE WORK

The presented model gives a robust tools for computation of three dimensional full-wave models. The major difficulty is the complexity of the system. The effitient way to solve this problem is to reduce the number of non-zero elements of the system matrix. This can be achieved by using such methods as a fast multi-pole algorithm and precorrected-FFT. Another problem is how to optimize the reduction algorithm in terms of getting optimal performance; this requires optimization of the way in which p is calculated and it can be achieved by analysis of behavior of M as function of p. Thus the described method provides researchers and engineers with new robust tools for the design of MEMS and packaging systems which combines accuracy of full-wave modeling with acceleration by reduction algorithm.

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