HDL-A-model of a Micromachined Tuning Fork Gyroscope

J. Eichholz, J. Quenzer, O. Schwarzelbach, M. Weiß and B. Wenk

FhG-ISIT, Fraunhoferstr. 1, D – 25524 Itzehoe, Germany, eichholz@isit.fhg.de

ABSTRACT

The HDLA-model presented describes the electrical and mechanical behavior of a micromachined silicon tuning fork gyroscope. This gyro will be excited by electrostatic forces and the Coriolis induced torsion motion will be read out capacitively. To control the behavior of the gyro several control nodes are available. In addition the model includes capacitors for a force feedback operation and for tuning of the torsional resonance frequency. The design of the element is described by 25 mechanical parameters. Verification of the model is done by developing concurrently a mathematical model that solves the differential equations. In addition the simulations are compared with measurement results. The model can be used for operation point analysis, transient and frequency simulations.

Keywords: HDLA-model, gyroscope, electrical simulations

1 INTRODUCTION

The model was mainly developed to support the electronic design design for the excitation and readout circuits. This work was done within the European project STARS (ESPRIT No. 25629) using VerilogA language. This language allows to transform mechanical quantities like angle and torque, or position and force into the electrical quantities voltage and current and therefore enables the modeling and simulation of electromechanical systems. Especially for complex systems like gyroscopes this tools is helpful to support the design of electronics. Beside the well known Coriolis force term

$$\overrightarrow{F_C} = 2m\vec{v} \times \vec{\Omega} \qquad (1)$$

we also take into account the angular acceleration $\dot{\Omega}$. Especially for the electronic design, equation (1) is not sufficient to predict the real behavior. Using a model description for an electrical Spice-like simulator is the only possibility to simulate such a complex two domain system in a suitable time, especially when a force feedback loop is considered.

2 ORGANISATION OF THE MODEL

Fig. 1 illustrates the mechanical design of a tuning fork gyro [1] fabricated in surface micromachined silicon. The two separated masses A and B, connected using wide and stiff crossbars, will be excited with electrostatic forces by applying a voltage on the inner electrode C and will move antiparallel in x-direction. An angular rate in y-direction will force the plates in a torsional vibration that can be detected. The tuning fork is fixed to the substrate at the anchor points D and E. The outer electrodes F and G are used to control amplitude and phase of the mechanical reference vibration. To get highest amplitudes the gyro will be excited at resonance frequency. Underneath each mass of the structure three capacitor plates are located: for measuring the Coriolis induced movement, i.e. the gyro signal, for force feedback usage, and to apply a static voltage to trim the torsional resonance frequency.

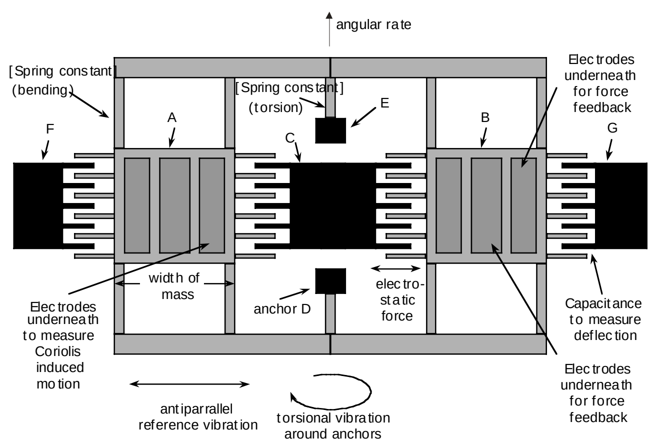

Fig. 1: Schematic of a micromachined tuning fork gyro

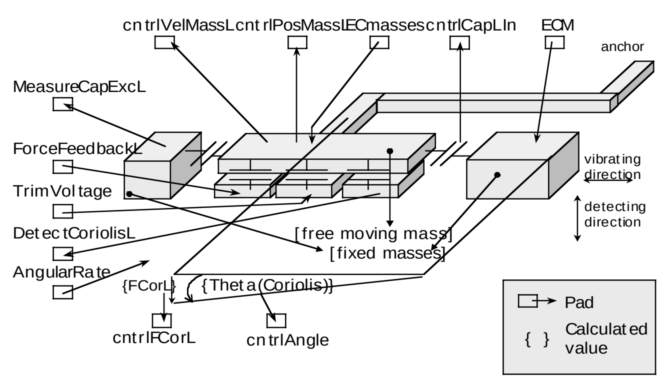

Fig. 2: Electrical pins of the gyro model

Consequently the model has 10 pins that will be connected to an ASIC as can be seen in Fig. 2. In addition to these pins 9 control-pins, all marked with „cntrl“ as prefix, are introduced to control the mechanical behavior of the gyro. With these pins the capacitance of the comb structures used for excitation, the position and velocity of the mass and the Coriolis force can be monitored for each mass. This is very useful to analyze the influence of mismatch and asymmetries.

2.1 Parameters and Buttons

The model uses 25 different parameter shown in Fig. 3, that define the mechanical design. Here only a quarter of the schematic of Fig. 1 is shown. The index “L” indicates that a corresponding parameter for the right part exists, so that asymmetries can be simulated.

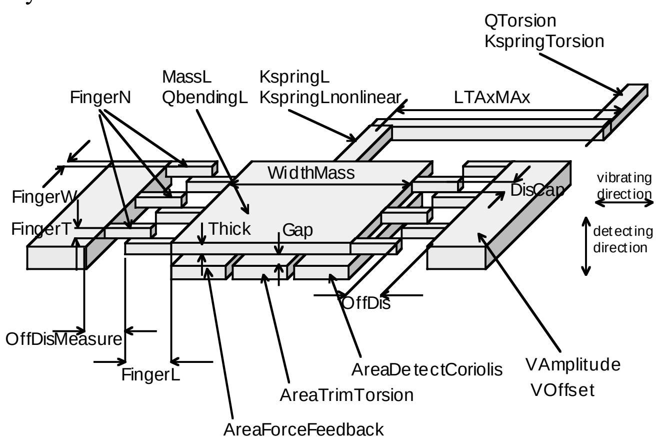

Fig. 3: Explanation of used parameters

Radiobuttons as shown in Fig. 4 are introduced to decide to use more precise and time consuming calculations or not. „verbose“ on button „Output“ will print important parameters on the screen, like:

- bending and torsional damping coefficients and resonance frequencies including the effect of the trimming voltage
- excitated deflection at resonance and available gap (including a warning when deflection exceeds gap)
- moment of inertia and change of moment of inertia at resonance
- the quality factors in bending and torsion direction and the quality factors used for the calculation.

„Include fingertips“ will add the area built by the fingertips of the comb structures to the capacitor, see Fig. 4, i.e. from the electrical point of view the capacitors become larger and from the mechanical point of view an extra portion of force depending on the position of each mass is added.

Obviously applying a readout voltage on plates underneath the free moving masses for measuring introduces additional forces that may disturb the gyro. With the button „Force_of_measure“ it can be decided to simulate with or without these forces.

The differential equation (4) includes time dependent, periodic coefficients on the left hand side. For large values of modulation of the moment of inertia ΔJ and for large quality factors of torsion (Q_t>20,000) parametric resonance may occur. Simulations as well as numerical calculations showed that at certain conditions, parametric resonance can occur in the transient simulation before steady state is reached. Due to the imperfect vacuum inside a gyro the gas damping lowers the torsional Q so that these periodic coefficients can normally be neglected. To reduce simulation time for the transient response „parametric resonance“ can be suppressed by pushing the no-button.

To reach the steady state the time of a transient solution depends on the Q value and is approximately $2\pi Q/\omega_{res}$. To reduce the transient response the Q-correction can be used, where the calculations will be done with Q=100. results This can be used only at matched frequencies $\omega_{exc}=\omega_{resb}=\omega_{rest}$, and a constant angular rate.

Fig. 4: Radiobuttons to control the model performance

3 EQUATIONS

For the reference movement in x-direction the model follows the differential equation

$$\ddot{x} + \frac{\omega}{Q}\dot{x} + \frac{k}{m}x + \frac{k_{nl}}{m}x^3 = \frac{F(t)}{m} \qquad (2)$$

with ω: excitation frequency, Q: quality factor of bending motion, k: linear spring constant and k_{nl}: nonlinear spring constant.

The electrical driving force is

$$F(t) = \varepsilon_0 n \frac{h}{d} V^2(t) \quad (3)$$

with n: amount of fingers of comb, h: height of fingers, d: air gap between fixed fingers and fingers of the moving mass, V(t): applied electrical voltage.

The equation of motion of the tuning fork due to an angular rate that is solved by the model is [2]

$$\ddot{\varphi}(J_0 + \Delta J \cos(\omega t)) + \dot{\varphi}(c_t - \Delta J \omega \sin(\omega t)) + \varphi k_t = -J_0 \dot{\Omega}(t) - \Delta J \dot{\Omega}(t) \cos(\omega t) + \Delta J \omega \Omega(t) \sin(\omega t) \quad (4)$$

with J_0: moment of inertia of a non-vibrating tuning fork, ΔJ: modulation of moment of inertia due to the vibration, c_t: torsion damping coefficient, k_t: torsion spring constant, Ω: applied angular rate.

4 SIMULATION RESULTS

The next two figures show different transient simulations of the steady state response of the Coriolis induced torsion angle in comparison with calculations of the mathematical model. In both cases the upper picture is the MATHEMATICA-result followed by the SpectreS simulation result. It can be seen that both cases are matching.

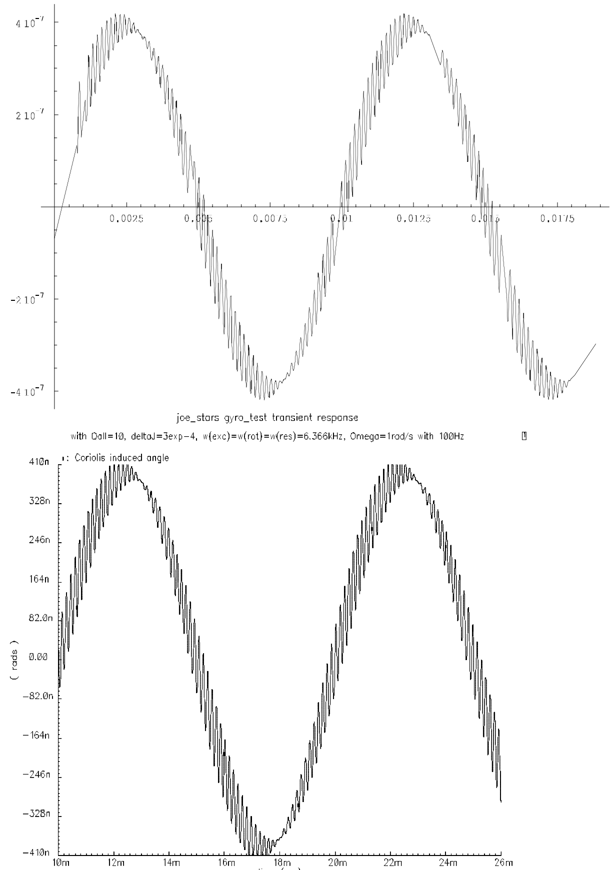

Figure 5: Results with $Q_t=Q_b=10$, $\Delta J/J_0=0{,}3e\text{-}3$, $\omega_{exc}=\omega_{resb}=\omega_{rest}$, and $\Omega=1rad/s \sin(2\pi 100Hz\ t)$

To perform a simulation with the pure VerilogA-model as shown in Fig. 2 the symbol must be connected to three voltage sources. The voltage source for reference vibration has to be connected to the node "ECM". The voltage source that defines the angular rate must be connected to "AngularRate"-pin. Finally, needed for any electrical simulation, "ground" has to be fixed to "ECmasses" representing the mechanical position "0".

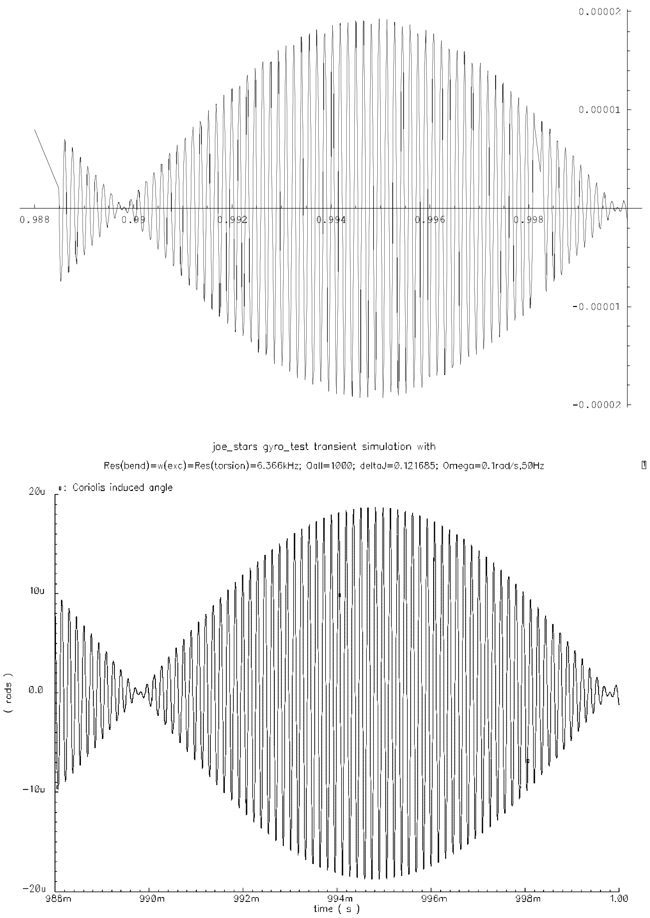

Figure 6: Results with, $Q_b=Q_t=1000$, $\Delta J/J_0=121{,}7e\text{-}3$, $\omega_{exc}=\omega_{resb}=\omega_{rest}$ and $\Omega=1rad/s \sin(2\pi 50Hz\ t)$

The results in both figures are the Coriolis induced angle measured at the pin "cntrlAngle" of the model. The x-axis are not identical, because the MATHEMATICA-solution directly shows the steady state response whereas the electrical simulation needs a certain time. Both simulations are done without Q-correction, excluding the fingertips and with the possibility of a parametric resonance.

4.1 Effect of nonlinear Spring Constant

As mentioned before the gyro will normally excitated at resonance to achieve the largest possible amplitude. Mechanical springs show the known physical effect, that a spring becomes stiffer, when the deflection exceeds a certain limit. This is described with a cubic term as written in equation (2). Fig. 7 shows the mathematical solution of the physical effect, that the amplitude has two stable solutions, i.e. is depending on the sweep direction. This behavior can not be observed in the electrical ac-analyses, because the simulator linearizes at the DC operating point and thus does not show the form of a breaking wave. Only a shift of the resonance peak can be found similar to the mathematical solution due to the dependency of the static deflection on the voltage.

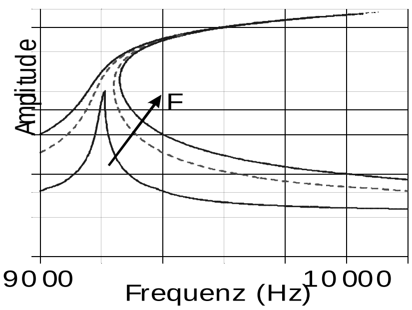

Figure 7: Analytical evaluation of the nonlinear spring constant

In the time domain the dependency on the sweep direction can be observed. In both Figures 8 and 9 the transient simulation is used, where the model has a low Q=10 and the excitation frequency is swept slowly.

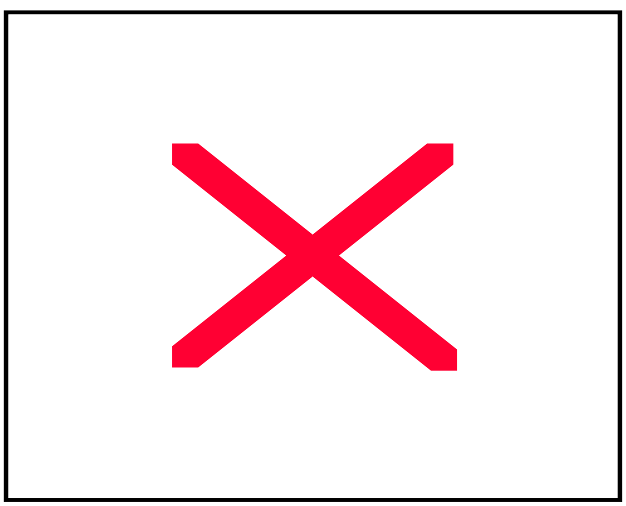

Figure 8: AC-sweep in transient simulation from low to high frequencies

In Figure 8 the frequency started at 4 kHz and ended at 9 kHz, as the lower curve indicates. Figure 9 shows the result starting at 9kHz and ending at 4kHz. Both middle curves show for comparison the behavior of a pure linear spring. The upper results show the predicted dependency of the starting frequency.

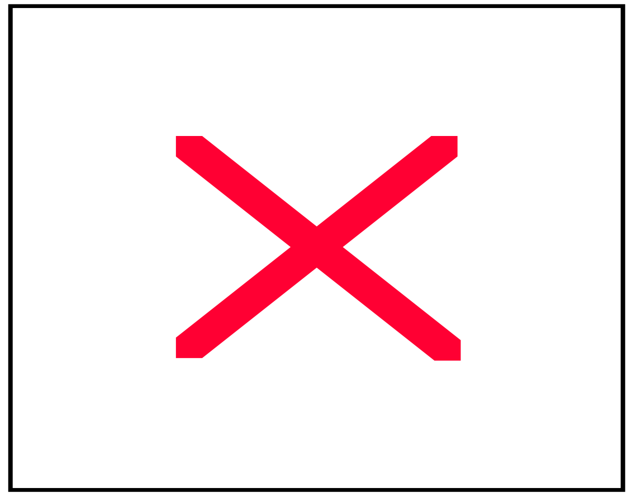

Figure 9: AC-sweep in transient simulation from high to low frequencies

First measurements with test devices that are excitated in x-direction at resonance frequency show the expected behavior, as the measured curve in Fig. 10 presents.

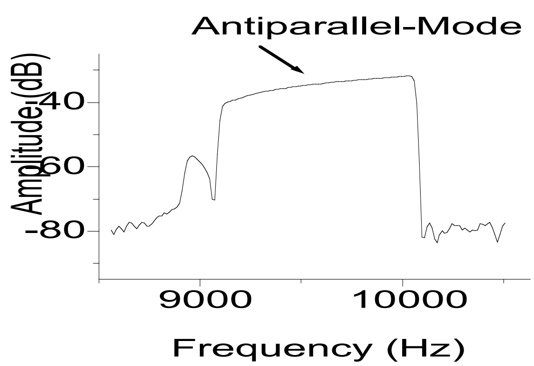

Figure 10: deformation of resonance peak due to nonlinear spring constant

5 CONCLUSION

With the presented VerilogA-model the behavior of a tuning fork gyro can be modeled in detail in the electrical domain. This allows an optimized design of the related electronics and reduces the number of re-designs.

REFERENCES

[1] J. Bernstein et. al., “A Micromachined Comb-Drive Tuning Fork Rate Gyroscope”, Proc. MEMS ‘93, p. 143-148.

[2] G. Hunt et. al., “Development of an accurate tuning-fork gyroscope”, Proc. Instn. Mech. Engrs. Vol. 179, p. 130-139, 1964.