Electrostatic Spring Effect on the Dynamic Performance of Microresonators

Francis Tay E.H, Kumaran R, B.L.Chua and Logeeswaran VJ

National University of Singapore
Department of Mechanical & Production Engineering
10 Kent Ridge Crescent, Singapore 119260, mpetayeh@nus.edu.sg.

ABSTRACT

This paper reports on the significant discrepancy between the analytical and experimental resonant frequencies of folded beam micro resonators. Experimental results for the resonant frequency showed a consistent 20% discrepancy over theoretical and finite element results.

Possible causes of the discrepancy from tapered cross section of the flexure beams, dimensional variations and electrostatic spring effects are discussed and shown to contribute to the significant difference between analytical and experimental values. IntelliCAD™ electrostatic simulation was done to isolate the electrostatic spring effect and compared with the experimental observations. The compliance due to AC voltage has also been observed in DRIE resonators.

Keywords: resonator, electrostatic spring effect, natural frequency, uncertainty analysis, simulation.

1 INTRODUCTION

Micro resonators are widely used in active microelectromechanical systems (MEMS) devices. Electrostatically driven micro resonators are usually operated at their resonant frequency, which will give the maximum displacement amplitude. This paper reports on the significant discrepancy between the analytical and experimental resonant frequencies of folded beam micro resonators [1,2,3]. Often, the electrostatic spring effects are neglected in folded beam resonators assuming ideal condition. The folded beam resonators have been designed and fabricated using surface micromachined process (MUMPs) and silicon-on-insulator (SOI) technology. Experimental results for the resonant frequency showed a consistent 20% discrepancy over theoretical and finite element results.

This discrepancy is attributed to a few factors, among which is the dynamic electrostatic spring effect. Finite element analysis with fully constrained boundary conditions yields results in good agreement with the analytical calculations with a mismatch of ~1%. The dynamic equation for the resonator and simulation model is described and the analysis of the error contributing factors including electrostatic simulation results are discussed in the following sections.

2 ANALYSIS

Microresonators are interesting mechanical oscillators prominently due to the "field" forced oscillation as compared to typical contact forced oscillation. The lateral comb drive has been widely used in micro-engines, microgyroscopes and micro-switches due to the generation of near constant actuating force for a given vibration amplitude, assuming the linearity of the electrostatic force. On the contrary non-linear electrostatic force exhibits the phenomena of negative electrical spring that provides a softening effect and has been widely used as a tuning parameter for resonant frequency mismatches. Mechanical mass compliance, imperfect boundaries and dimensional parameters variation significantly contribute to the observed frequency discrepancy. An uncertainty analysis against manufacturing variations in the size parameters of the resonators shows this behavior.

2.1 Uncertainty Analysis

The uncertainty in the estimated natural frequency includes the measurement uncertainties of the Young's Modulus, size parameters and the structure mass. The upper bound for the natural frequency expression is given by Rayleigh's energy method as

$$f_{nat} = \frac{1}{2\pi}\sqrt{\frac{2Ehw^3}{l^3\left(M_p + \frac{1}{4}M_t + \frac{12}{35}M_b\right)}} \tag{1}$$

where E=Young's Modulus, h=height, w=width, l=spring length, M_p=plate mass, M_t=truss mass and M_b=beam mass. The mechanical stiffness is derived based on ideal sidewall cross section. The uncertainty neglecting is then

$$\Delta f_{nat} = \sqrt{\left(\frac{\partial f_n}{\partial E}\Delta E\right)^2 + \left(\frac{\partial f_n}{\partial w}\Delta w\right)^2 + \left(\frac{\partial f_n}{\partial M_p}\Delta M_p\right)^2} \tag{2}$$

Data: E=140±22% GPa, w=2e-6±1.8% m & M_p=4.8e-11±5% kg. It is apparent that the contribution of the second term is the largest and accounts for close to 9-10% variation. SEM done on the cross section of the beams show negligible gradient in the sidewall. Figure 1 and 2 shows the SEM on the width of the flexure beams (data was averaged from 20 sampling points) and the boundary of the flexure with the anchor respectively.

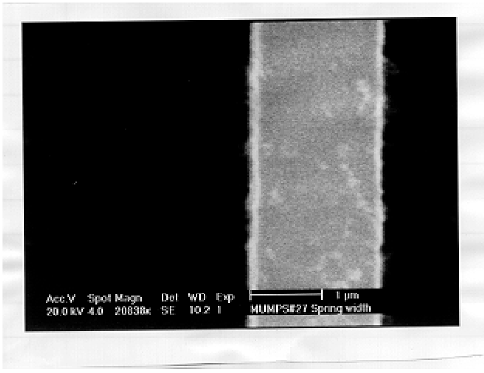

Figure 1. SEM on the flexure width variation

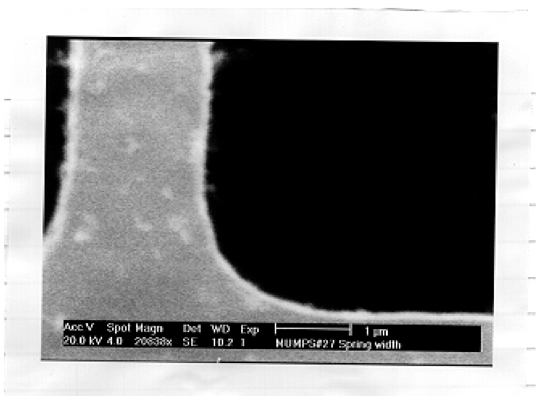

Figure 2. SEM on the flexure boundary with anchor

Figure 2 shows that the boundary condition of the flexure is stiffer than the analytical assumption, which then would increase the upper bound of the resonant frequency rather than decreasing it.

2.2 Dynamic Equation

It has been widely reported that the resonant frequency of MEMS devices vary with applied DC bias voltages and the simulation of this phenomenon has been done but only for DC bias effects [4,5]. For passive devices such as accelerometers, the sensing is typically biased with a DC voltage and the shift in resonant frequency for small displacement can be inferred from the equation of motion:

$$m\ddot{x} + c\dot{x} + \left[k - k_{e,Vdc}\right]x = 0 \qquad (3)$$

where the frequency is given by

$$f = \frac{1}{2\pi}\sqrt{\frac{k - k_{e,Vdc}}{m}} \qquad (4)$$

For active devices such as gyroscopes the detection is DC biased and further actuated with AC using comb drive actuators. The electrostatic forces operating in comb drive resonators have been rigorously described in [6] and it has been shown that the actuation forces (whether with or without ground plate) is dependent on the displacement x, but varies in the order of magnitude. The common equation of motion for a push-pull resonator can be written as [1]

$$m\ddot{x} + c\dot{x} + k_m x = F_0 \sin \omega t \qquad (5)$$

where

$$F_0 = 4V_{DC}V_{AC}\frac{fC}{fx}\sin \omega t \qquad (6)$$

Eqn (5) shows that the effective stiffness is only due to the mechanical compliance and if one considers only the DC bias the modified equation of motion is as in eqn (7).

$$m\ddot{x} + c\dot{x} + \left[k_m - k_{e,Vdc}\right]x = F_0 \sin \omega t \qquad (7)$$

However if the AC signal significantly contributes to the system compliance, the equation of motion could be expressed as in eqn (8).

$$m\ddot{x} + c\dot{x} + \left[k_m - k_{e,Vdc} - k_{e,Vac}(t)\right]x = F_0 \sin \omega t \qquad (8)$$

3 RESULTS & DISCUSSION

An experimental and simulation scheme is proposed here to isolate the effects of non-mechanical compliance component on the resonant frequency using a MUMPs folded beam lateral microresonator. Two fabrication process was used namely, the MUMPs process and SOI technology as shown in Figure 3 and Figure 4. The experimental displacement amplitude for varying AC

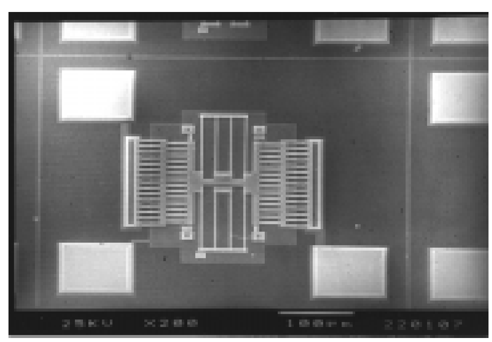

Figure 3: MUMPS folded beam resonator

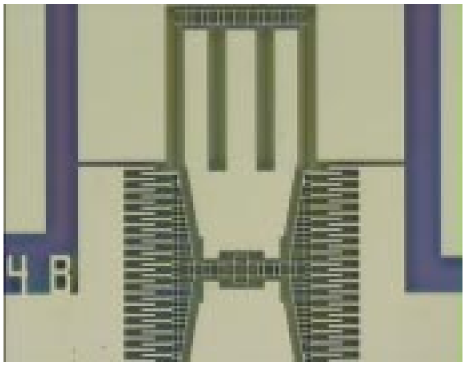

Figure 4: DRIE 40µm SOI folded beam resonator.

voltage is shown in Figure 5.

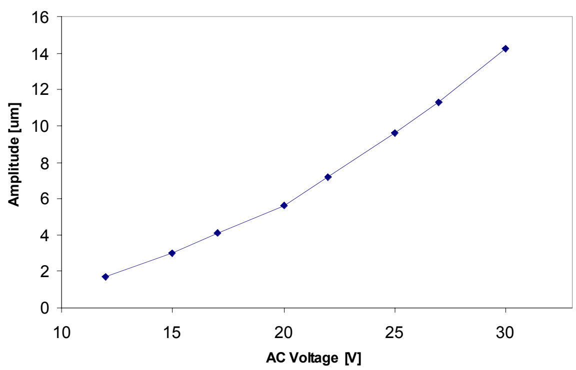

Figure 5. Graph of amplitude against AC voltage

From this data, simulation, using IntelliCAD was done to extract the maximum static electrostatic force corresponding to each maximum displacement. Figure 6 shows the simulation model used in IntelliCAD while Figure 7 shows the finite element analysis model used in ANSYS™. With the simulation results, the graph of force amplitude against displacement amplitude is obtained. From vibration theory, the displacement amplification factor is given by

$$\frac{k_{eff}X}{F} = Q \tag{9}$$

which can be expressed as $F = k_{eff}X/Q$; the gradient k_{eff}/Q can be obtained from Figure 8. The value of Q has been found experimentally to be ~34.

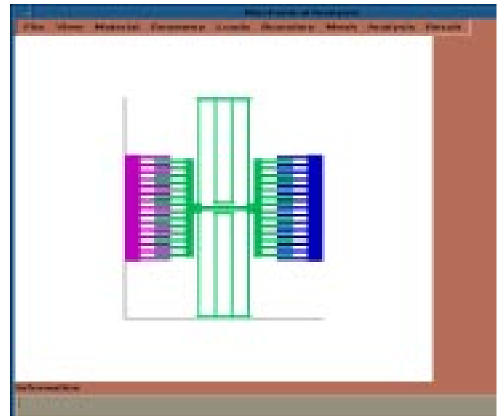

Figure 6. IntelliCAD model

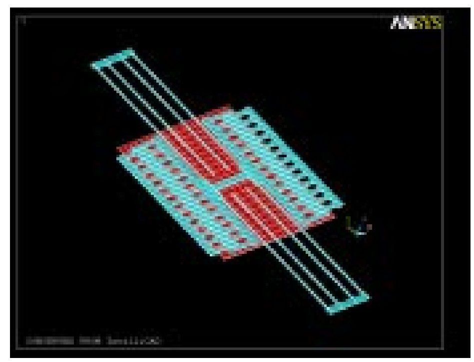

Figure 7. FEA model in ANSYS

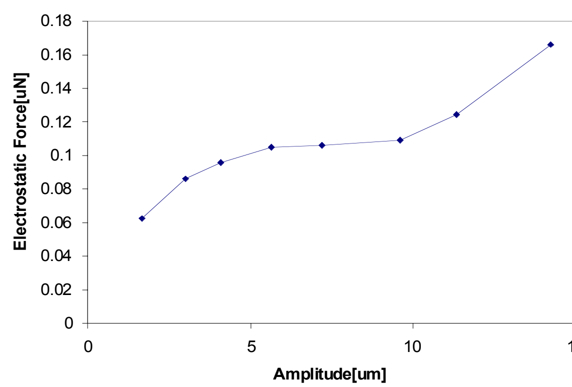

Figure 8. Force amplitude against displacement amplitude

The k_{eff} is found to be approximately 0.42N/m but the k_m is 0.5N/m. This verifies that there does exists a non-mechanical stiffness-reducing component in the system. This component has been widely attributed in literature to be a function of V_{DC}. From our experimental results with MUMPS resonators we are unable to establish a significant decrease in the resonant frequency by varying the AC voltage amplitude while maintaining the DC component. However from similar experiments conducted on DRIE comb-drive resonators it is found that for a fixed V_{DC} value of 7.6V and by varying the AC voltage amplitude from 2.5V to 5V, the resonant frequency does decrease by as much as 40%. Figure 9 shows the lateral actuating electrostatic force distribution for varying displacement. The increasing slope manifests the dependency of the force with displacement for increasing DC voltage.

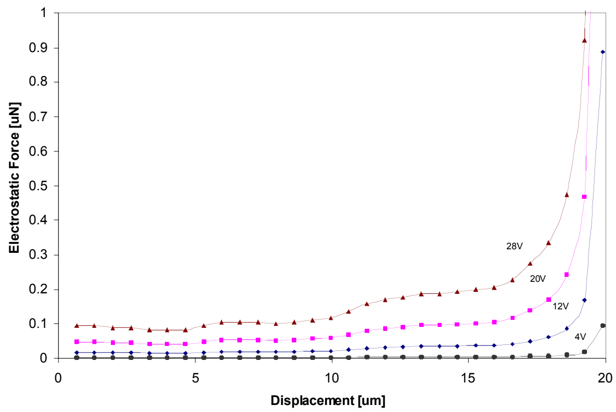

Figure 9. Static electrostatic force for varying displacement

It is evident that the AC voltage does contribute to the change in compliance from its design value, which is the mechanical stiffness. Therefore in active devices such as comb-drive actuated vibrating microgyroscope, the effect of both DC and high AC voltages on the resonant frequency have to be considered.

4 CONCLUSION

The discrepancy between the experimental and design values in the resonant frequency of MUMPS fabricated resonators have been discussed. The significant contribution of the manufacturing variations in the size parameters in the resonant frequency variation has been highlighted. A simulation and experiment scheme has also been proposed to isolate the effect of the dynamic electrostatic spring effect. This attempt does show a discrepancy but we are unable to verify them experimentally with MUMPs resonators while experiments done on the DRIE resonator does show that the resonant frequency decreases as V_{AC} is increased while maintaining a fixed V_{DC} value. Though in many devices the magnitude of AC signal used is small the existence of the AC compliance effect has been shown from our experiment.

ACKNOWLEDGEMENT

The authors would like to acknowledge the help of Peck Kong Eng and Tan Yee Yuan for the actuation and resonant frequency characterization data and C.Gallegos and D. Keating of IntelliSense Corp. for their feedback on simulation.

REFERENCES

[1] W.C. Tang, T.C.H Nguyen & Roger T. Howe, "Laterally driven polysilicon resonant microstructures," Sensors & Actuators A, 25-32, 1989.

[2] W.C. Tang, "Electrostatic comb drive for resonant sensor and actuator applications", Ph.D. Thesis, University of California at Berkeley, Nov 1990.

[3] T. Hirano, T. Furuhata, K.J. Gabriel, & H. Fujita, "Operation of sub-micron gap electrostatic comb-drive actuators," IEEE Int. Conf Solid State Sensors & Actuators, 873-876, Transducers '91.

[4] V.L. Rabinovich, M.Deshpande & J.R.Gilbert, "Prediction of mode frequency shifts due to electrostatic bias," IEEE Int. Conf Solid State Sensors & Actuators, 17-23, Transducers '99.

[5] Y.He, J.Marchetti, C.Gallegos & F.Maseeh, "Accurate fully-coupled natural frequency shift of MEMS actuators due to voltage bias and other external forces", Proc. MEMS'99, 321-325, 1999.

[6] W.A.Johnson & L.K.Warne, "Electrophysics of micromechanical comb actuators," Journal of Microelectromechanical Systems, 4(1), 49-59, 1995.