

Analysis of an Electrostatic Microactuator with the help of Matlab/simulink: transient and frequency characteristics

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ABSTRACT

The article is focused on the development of a numeric simulator dedicated to the electrostatic microactuator. It's based on the use of Simulink (Matlab ©). The electrostatic microactuator is regarded as a model « rigid plate » associated to a spring with an electrode above. The simulation is based on the mechanical law governing the working. By this way, transient simulation of the electrostatic microactuator can be obtained. The pull in effect is taken into account in the simulation and instability of electrostatic actuation is shown with the influence of the damping factor. The results have been extended to a frequency characterisation in order to highlight the resonant frequency shifting with the polarisation applied on the electrode. This result is supported by a modelisation in small signals mode.

Keywords: Electrostatic, Modelisation, Frequency shifting, Numeric method.

1 INTRODUCTION

With the miniaturisation of the system, the possibility to have an actuator is classically done via the electrostatic forces [1]. But the modelisation of such a system is a real problem. In static mode, analytic results can be developed in order to underline the non linear characteristics of electrostatic actuation and estimate displacement [2]. In dynamic mode, it's more complex because of the link between the electrostatic force and the displacement. With the use of Simulink [3], it becomes possible to obtain numeric simulation of such a system. By this way, the transient and frequency comportment of an electrostatic actuator can be simulated and analysed. We concluded the article with the study of the frequency resonant shifting with a dc voltage.

2 MODEL OF ELECTROSTATIC MICROACTUATOR

The classical model used in order to modelise the working of electrostatic microactuator is to consider a rigid plate attached by a spring and submitted to an electrostatic field (Fig. 1):

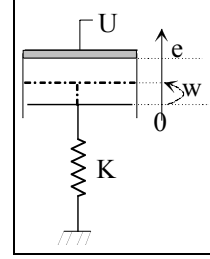


Figure 1 : Simple model of an electrostatic actuator

To achieve the simulation of the system represented in figure 1, the calculation is based on the mechanical law governing the electrostatic actuator which can be expressed as follow:

$$m \frac{d^2 W}{dt^2} + \lambda \frac{dW}{dt} + KW = \frac{\epsilon U^2}{2(e - W)^2} S \quad (1)$$

Where W is the deflection, m the mass, λ the damping factor, K the spring value. K depends on the geometry of the microstructure. The excitation is represented with the electrostatic pressure through a gap e applied on the plate surface S , with U the voltage and ϵ the permittivity. The mass can be expressed with the geometrical characteristics of the plate : $m = \rho h S$, with ρ the volumic density and h the plate thickness.

Equation (1) shows clearly the non-linearity of the electrostatic microactuator. The excitation depends on the plate deflection. There is no analytic solution of this equation. We can express the solution of the deflexion in a static mode [4]. In this case, all derivation are null. We obtain the classical voltage limitation (U_s) [5] due to the instability of the electrostatic excitation, this instability provokes the sticking of the plate on the electrode.

$$U_s < \sqrt{\frac{8Ke^3}{27\epsilon S}} : \text{The equilibrium is stable (} W < e/3 \text{)}$$

$$U_s > \sqrt{\frac{8Ke^3}{27\epsilon S}} : \text{The equilibrium is unstable. Plate is stucked on the electrode}$$

In order to evaluate the transient characteristics, we have to use a numeric simulator. Simulink offers the possibility to make this simulation by adding different function blocks.

3 MATLAB IMPLEMENTATION

Equation (1) has been implemented by the use of integrator. The main simulation structure is in figure (2) where we can control all the excitation parameters :

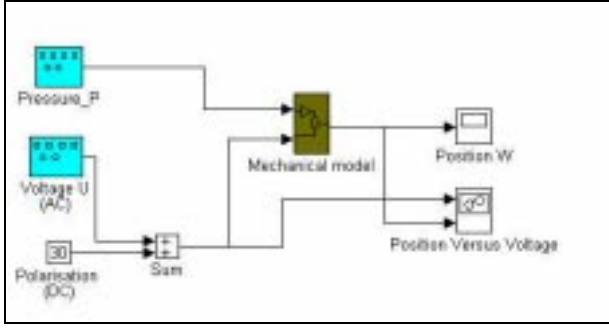


Figure 2: Main structure of the simulator

The mechanical model is taken from equation (1) and achieved the acceleration estimation:

$$\frac{d^2 W}{dt^2} = \frac{\epsilon U^2}{2(e - W)^2} + P_{\text{pneu}} - \frac{\lambda}{S} \frac{dW}{dt} - \frac{K}{S} W \quad (2)$$

By integrating the acceleration, the speed and the position can be estimated. The simulink schematics of equation (2) is depicted in figure (3). The electrostatic pressure is calculated at each calcul step and then becomes a variable parameter with the deflection W . In this block, all the physical parameters are controllable.

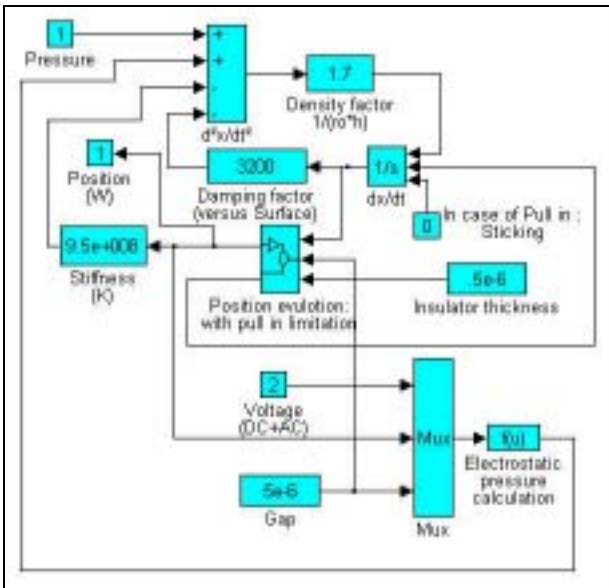


Figure 3: Implementation of the mechanical law under simulink

By including specific conditions on the position value, the pull-in effect, due to instability of electrostatic actuation [6], is taken into account (fig. 3).

- the position can't be greater than the gap (minus an insulator thickness located on the electrode).
- if the plate is stuck, the speed is null.

By this way, we can control the excitation voltage (shape, frequency) and the mechanic characteristic (m , K , λ) in order to evaluate their influence on the deflexion.

4 TRANSIENT RESULTS

4.1 Static results

First, we achieve simulation in order to validate the principle. In a quasistatic mode, the voltage is increased slowly until the sticking effect then decreased to zero. Results are depicted in figure (4).

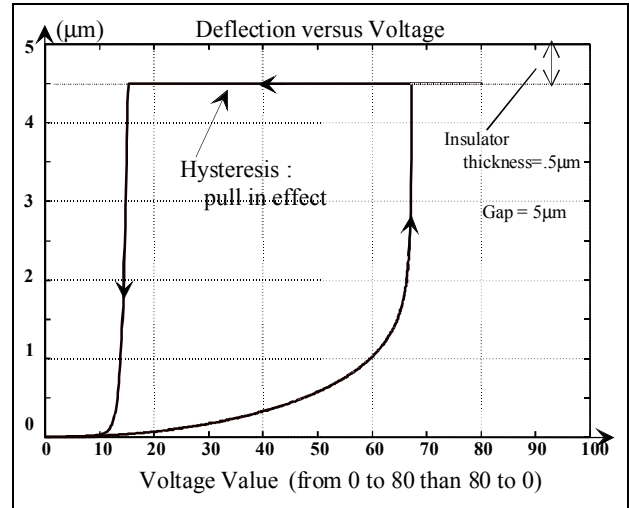


Figure 4: Static results (hysteresis and pull in effect)

For the simulation we used for the values : $1/\text{ph} = 1.7 \text{ kg}^{-1}\text{m}$, $\lambda/S = 3200 \text{ m}^{-2}$, $K/S = 9.5\text{e}8 \text{ Nm}^{-3}$, a gap e of $5\mu\text{m}$ with an insulator thickness of $.5\mu\text{m}$ and $\epsilon = 8.85\text{e}^{-12} \text{ F.m}^{-1}$.

The estimation of the spring value K can be obtained with the mechanical comportment of the structure under pneumatic pressure. In the case of a plate, the evaluation of K [7] is then :

$$\frac{K}{S} = \frac{16D_0 h^3}{0.02S^3}$$

where D_0 is the elastic fluxural rigidity.

The predicted sticking voltage is 63 V as we found in simulation. The quasistatic simulation makes appear the square evolution of the position versus the voltage, the

sticking effect and the hysteresis phenomenon. The membrane unsticked for a voltage lower than the sticking voltage.

4.2 Dynamic results

The second step was to examine the transient comportment of the deflexion. The sticking voltage has been calculated in the static mode. But we have to take into account the influence of the damping factor because with low damping factor, the position can attain twice its final value, due to the transient.

Then, transient simulations have been achieved with an echelon voltage of 60 V for an estimated sticking voltage of 63V. By varying the damping factor, we drawn the position evolution versus time. Results are depicted in figure (5).

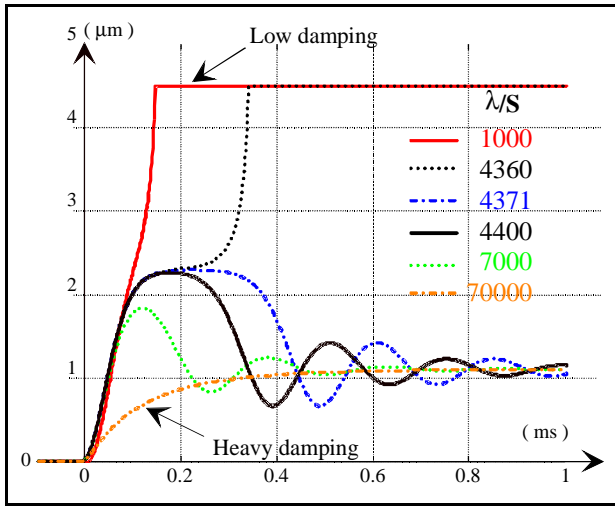


Figure 5 : Position evolution with several damping factor with a same voltage excitation (echelon of 60 V)

With heavy damping ($\lambda/S = 70000$), the position is stable and the plate attain its predicted final value. But, with the decreasing of the damping factor, the non-linear excitation attract the plate on the electrode and provokes an underisable sticking ($\lambda/S = 4360; 1000$). The system becomes unstable in spite of unsticking voltage used. Then, a system with low damping factor may pull in for a voltage lower than the voltage obtain in a static mode.

5 FREQUENCY RESULTS

The principle validated, the transient simulation of the deflexion can be extended to a frequency characterisation [8]. The equation used is the dynamic equation (1).

It's a second order differential equation. In case of a sinusoidal excitation (thermal or pneumatic case), the

resonant frequency of the system is : $\omega_0 = \sqrt{\frac{K}{m}}$

Here, the electrostatic excitation is non linear :

$$\frac{\epsilon U^2}{2(e - W)^2}$$

- the excitation depends on the square voltage (frequency is doubled).
- the position W modifies the excitation value.

We will study the case of a small alternative excitation (AC) with a polarisation (DC) :

$U = U_{dc} + U_{ac} \cos(\omega t)$ with U_{dc} a constant voltage polarisation and U_{ac} the sinusoidal voltage amplitude, with ω the frequency. We will supposed $U_{ac} \ll U_{dc}$ (small dynamic displacement).

So, we can express the deflexion as the results of the deflexion (W) due to the voltage polarisation (W_{dc}) plus the alternative voltage (W_{ac}) :

$$W = W_{dc} + W_{ac}$$

W_{dc} is a constant and is the position obtained in a static mode with U_{dc} as excitation. W_{ac} is the position due to the alternative excitation. So the two equations governing the deflexion are:

$$m \frac{d^2(W_{dc} + W_{ac})}{dt^2} + \lambda \frac{d(W_{dc} + W_{ac})}{dt} + K(W_{dc} + W_{ac}) = \frac{\epsilon(U_{dc} + U_{ac})^2}{2(e - (W_{dc} + W_{ac}))^2} S \quad (3)$$

$$KW_{dc} = \frac{\epsilon U_{dc}^2}{2(e - W_{dc})^2} S$$

In very small displacement ($W_{ac} \ll W_{dc}$), with a one order development, we can write:

$$\frac{\epsilon(U_{dc} + U_{ac})^2}{2(e - (W_{dc} + W_{ac}))^2} S = KW_{dc} (1 + 2U_{ac}/U_{dc} + 2W_{ac}/(e - W_{dc}))$$

Then, equations (3) become :

$$m \frac{d^2(W_{ac})}{dt^2} + \lambda \frac{d(W_{ac})}{dt} + K_{dc} W_{ac} = (2KW_{dc}/U_{dc}) U_{ac} \quad (4)$$

$$KW_{dc} = \frac{\epsilon U_{dc}^2}{2(e - W_{dc})^2}$$

With $K_{dc} = K(1 - 2W_{dc}/(e - W_{dc}))$.

K_{dc} is an equivalent spring dependant of the polarisation.

Thus, the polarisation acts as a mechanical modification of the spring value. Furthermore, in equation (4), we can notice that the non-linear excitation becomes, in small signals, a linear excitation. We can then express the frequency resonant of the system ω_{dco} :

$$\omega_{dco} = \sqrt{\frac{K_{dc}}{m}} = \omega_0 \sqrt{1 - \frac{2(Wdc/e)}{1 - (Wdc/e)}} \quad (5)$$

This frequency is also dependant of the polarisation. We can then obtain a resonant frequency control with the polarisation:

$$\text{For } U_{dc}=0 \Rightarrow \omega_{dco}/\omega_0 = 1$$

$$\text{For } U_{dc}=U_{sticking} (W_{dc}/e = 1/3) \Rightarrow \omega_{dco}/\omega_0 = 0$$

In figure (6), the evolution of the frequency shifting is drawn versus the initial position. Simulated points are compared versus the model with good results.

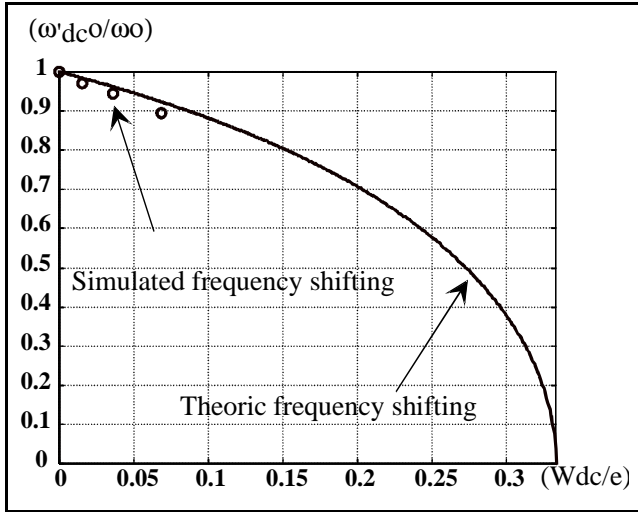


figure 6: Normalised resonant frequency evolution versus normalised dc position (Wdc/e)

The resonant frequency can be controlled from ω_0 down to zero with the dc polarisation voltage from zero up to the sticking limit voltage.

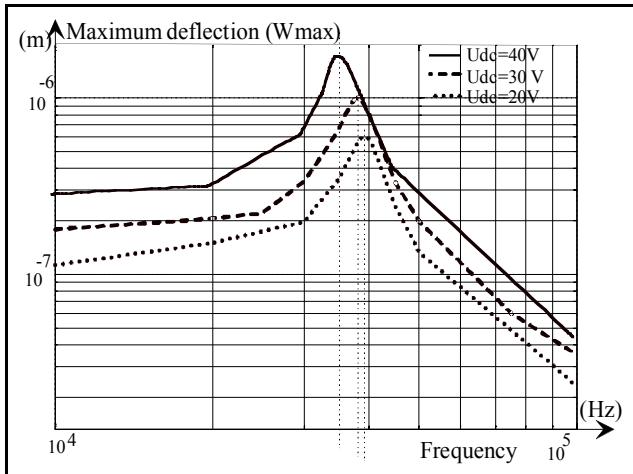


Figure 7: Frequency results (Case of Three DC polarisation {20 V; 30 V; 40 V}): resonant frequency shifting

In figure (7), the frequency results are depicted for three different polarisations:

6 CONCLUSION

By the use of a very simple numeric simulator (Simulink), the study of the electrostatic microactuator can be widely depicted. Static and dynamic simulation results have been presented. First, the influence of damping factor has been developped and studied on the transient characteristics. Then, application to a frequency characterization enables to highlight the frequency shifting obtained with the used of a voltage polarisation (DC).

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