On Design of a Backplate used in a Hearing Aid

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ABSTRACT

In this work topology optimization is used to optimize the compliance or the eigenfrequencies of prestressed plates. The prestress is accounted for by including the force equivalent to the prestressing and adding the initial stress stiffness matrix to the original stiffness matrix. The calculation of the sensitivities is complicated because of the initial stress stiffness matrix, but the computational cost can be kept low by using the adjoint method. The topology optimization problem is solved using the SIMP method (Simplified Isotropic Material with Penalization) in combination with MMA (Method of Moving Asymptotes). Numerical examples of minimizing compliance or maximizing the first eigenfrequency of plates are given. The example is a MEMS (Micro Electro Mechanical Systems) application involving a backplate used in a microphone for a hearing aid.

***Keywords*:** Topology optimization, Prestress, SIMP, Eigenfrequency, MEMS design.

1 INTRODUCTION

The subject of this paper is topology optimization using a material distribution concept and mathematical programming. For an overview of topology optimization see [3] and references herein. The main contribution of the present paper is the introduction of prestress in topology optimization. The object of the optimization is not the prestress distribution in the structure as the prestress is considered to be given as a result of e.g. residual stresses from the manufacturing process. The structures addressed are plates and the motion of interest is out of plane.

Two different objectives will be used in the optimization problems. The first objective is compliance (elastic energy) as is the case in many papers dealing with topology optimization, cf. [3]. The second objective is the optimization of eigenfrequencies.

The main motivation for this paper is to introduce prestress of plates in connection with topology optimization. This is then used to optimize the performance of a backplate of a silicon microphone used in a hearing aid. The microphone is a rather big MicroElectroMechanical Systems (MEMS) with size of $2\,mm \times 2\,mm$ as indicated in figure 1. Different possible designs for the microphone exist, see e.g. [1] or [2].

The principle of the microphone is that a sound pressure wave will deflect the diaphragm, see figure 1, leading to a capacitive change between the backplate and the diaphragm. To maximize this change in capacitance it is vital that there are holes in the backplate to let the pressure through. On the other hand the backplate should be as stiff as possible. In order to increase the stiffness of the backplate it is often chosen to employ prestressing. One example of a presently used backplate can be seen in figure 2. There are a number of papers on the analysis of the backplate see e.g. [4] or [5], but no optimization of the design has been performed. The center part of the backplate is dense with small holes and it is chosen to keep the center part fixed during optimization. It is therefore the design task to connect the center part to the boundaries. Although the center part is not solid we may calculate a set of material data using e.g. the Hashin–Shtrikman bounds and then treat the center part as being solid with the new material data. By doing this we will treat the center part as being isotropic. It is thus assumed that the structure of the center is isotropic. The idea is now that we may use the same material to connect the center to the boundaries. This leads to a standard topology optimization formulation.

Although the design might indicate that there could be a problem in relation to squeeze film and damping, the practical build backplates show no problems.

Section two of the paper briefly describes the sensitivity analysis necessary for doing the optimization of compliance/elastic energy or eigenfrequency, the method of optimization is also discussed. Finally an example of the optimization of the backplate used in a hearing aid is given. Here the results shows a great improvement compared to backplate designs presently used in microphones.

2 SENSITIVITY ANALYSIS AND METHOD OF OPTIMIZATION

This section is kept very short, for further information see [7]. The first objective of optimization is compliance which we will define as

$$W = \frac{1}{2}\{F_{ex}\}^T\{D_{ex}\} \tag{1}$$

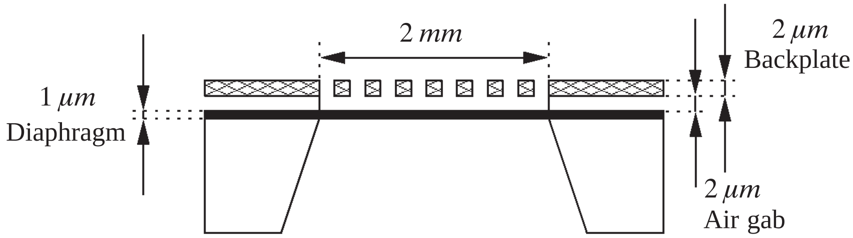


Figure 1: Schematic side view of hearing aid. The black part is the diaphragm while the crosshatch part is the backplate. There is air between the two parts.

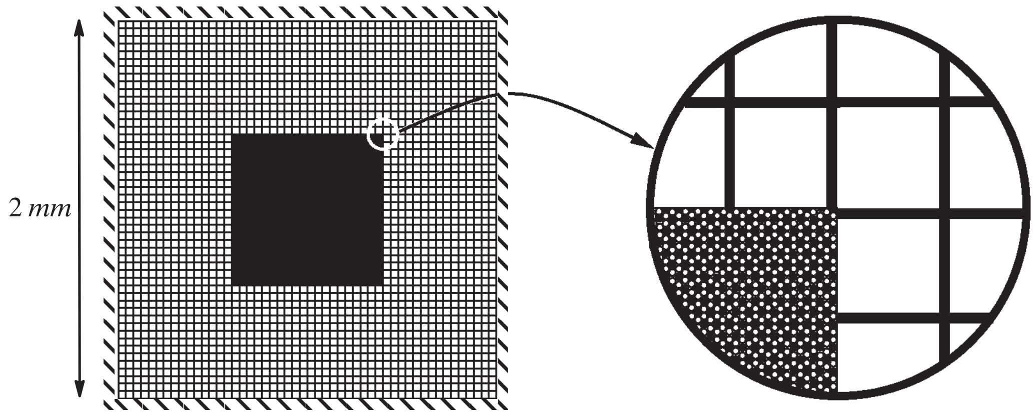


Figure 2**:** Conventional design of backplate, the black center part is dense with very small holes, while the rest of the structure have bigger holes.

where $\{F_{ex}\}$ are the external forces and $\{D_{ex}\}$ is the displacement due to the external forces. The second objective is the first eigenfrequency which may be found by solving

$$U - \omega^2 T = 0 \tag{2}$$

where ω is the eigenfrequency, U is the double of the potential energy and T is the specific kinetic energy. In a topology optimization using the SIMP model we express the stiffness and mass matrices as

$$[S] = \sum_{e=1}^{NE} \rho_e{}^{\alpha}[s_e] \quad \text{and} \quad [M] = \sum_{e=1}^{NE} \rho_e[m_e] \tag{3}$$

where ρ_e is the relative element density factor of element e in the finite element discretization. The relative density factor is restricted by $0 < \rho_{\min} \leqq \rho_e \leqq 1$. The minimum density factor $\rho_{\min}$ is introduced to avoid numerical problems when calculating the inverse of the stiffness matrix (a typical value could be $\rho_{\min} = 0.001$). The element stiffness matrix is given by $[s_e]$ and the element mass matrix by $[m_e]$. Typical values of the exponent is $\alpha = 3$. The relative density factors are the design parameters of the optimization problem. The sensitivities needed is therefore

$$\frac{dW}{d\rho_e} \quad \text{and} \quad \frac{d\omega^2}{d\rho_e} \tag{4}$$

The calculation of the sensitivities (4) is complicated due to the prestressing, but by using the adjoint method, see [10], it is still possible to keep the computational costs low. The adjoint method is preferable when the number of constraints are low compared to the number of design variables, as is the case here.

Applying the penalization scheme (3) are likely to produce checkerboard design, so to prevent this a filtering is used on the sensitivities of the optimization. For more details on this filter see [8].

In the case of maximizing the first eigenfrequency the formulation of the optimization problem to be solved is

$$\begin{aligned} &\textit{maximize:} && \omega_1{}^2 \\ &\textit{subject to:} && \sum_{e=1}^{NE} \rho_e V_e - V \leqq 0 \\ & && 0 < \rho_{\min} \leqq \rho_e \leqq 1 \quad , \quad e = 1,\ldots, NE \end{aligned} \tag{5}$$

where V_e is the volume of element e and V is the volume of the totally allowed material. In the case of minimizing compliance we just change the objective in (5). The optimization method used is MMA (Method of Moving Asymptotes) see [9].

When optimizing eigenfrequencies of plates using topology optimization we have to pay special attention to low density areas, when the problem is not formulated as a reinforcement problem, see Pedersen [6].

The plates are modelled using rectangular plate elements. The element used in the modelling is an isoparametric 4 node Mindlin Plate element with 5 degrees of freedom per node (no “drilling” d.o.f.). The membrane and the bending part of the stiffness matrix are calculated with 4 Gauss points while the shear part is calculated using 1 Gauss point to prevent shear locking.

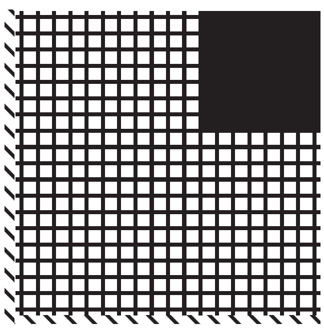
3 OPTIMIZATION OF BACKPLATE

The dimensions of the backplate is given in figures 1 and 2. The passive area occupies an area of $0.8mm \times 0.8mm$ at the center. The material used is silicon with the material data

$$E = 180 \cdot 10^9 \; N/m^2 \qquad \nu = 0.06 \qquad \hat{\rho} = 2300 \; kg/m^3$$

The center part of the the backplate is filled with 75% material. The prestress in the plate is very high $\sigma_{11} = 3.0\;10^8 \; N/m^2 = \sigma_{22}$. To compare the results, one quarter of the conventional backplate design is modelled with 75×75 element as shown schematically in figure 3.

The volume percentage of the center part is fixed at 75% in order to model the porous material while the stiffeners in the grid structure that support the center are solid. The values of the compliance and first eigenfrequency of this design are also given.



Volume percent $= 49.4\%$

$W = 7.0\;10^{-14}\,J \qquad \omega_1 = 0.19 \; MHz$

Figure 3**:** One quarter of a conventional backplate design modelled with 75×75 elements

The holes in the center part of the backplate are so small that to model them using a fin grid would make the number of elements too big for practical computations. Instead it is assumed that the holes are placed in the center part in such a way that the center part can be regarded as being isotropic. We may then use the Hashin–Shtrikman bounds to find a set of corresponding material parameters.

These bounds are given by

$$E(\rho) = \frac{\rho E^0}{3 - 2\rho} \qquad \nu(\rho) = \frac{1 - \rho(1 - \nu^0)}{3 - 2\rho} \qquad (6)$$

where ρ is the relative density. Using these bounds the material data used in the whole design domain are

$$E = 90 \cdot 10^9 \; N/m^2 \qquad \nu = 0.1966 \qquad \hat{\rho} = 1725 \; kg/m^3$$

The first example is compliance optimization where the external force is a distributed surface force acting on the passive area. Because of symmetry, only one quarter of the structure is used in the optimization and it is discretizised into 40×40 elements. The result of the optimization is shown in figure 4. Two different designs are shown based on different volume percentages. It is seen that in the case of 50% volume, the optimized design has a compliance value of $3.4\;10^{-14}\,J$ compared to the conventional design with a compliance of $7.0\;10^{-14}\,J$.

The second example is the maximization of the first eigenfrequency of the backplate and the result is given in figure 5. Again it is seen that the optimized result is better than the conventional design, in this case the improvement is 23%. It is seen from the results that minimization of compliance an the maximization of first eigenfrequency, in the case of evenly distributed surface force, gives almost identical results. This is expected because of the similarity between the mode of the first eigenfrequency and the static deflection due to the surface load.

4 CONCLUSION

This paper focus on the influence of prestress in relation to topology optimization of plates. This is exemplified by the optimization of a practical MEMS application in which we optimize the performance of a backplate used in a microphone for a hearing aid. The results are topologically different than the design presently used, showing that an improvement of the objective is possible. This is the case for both the compliance minimization and the maximization of first eigenfrequency.

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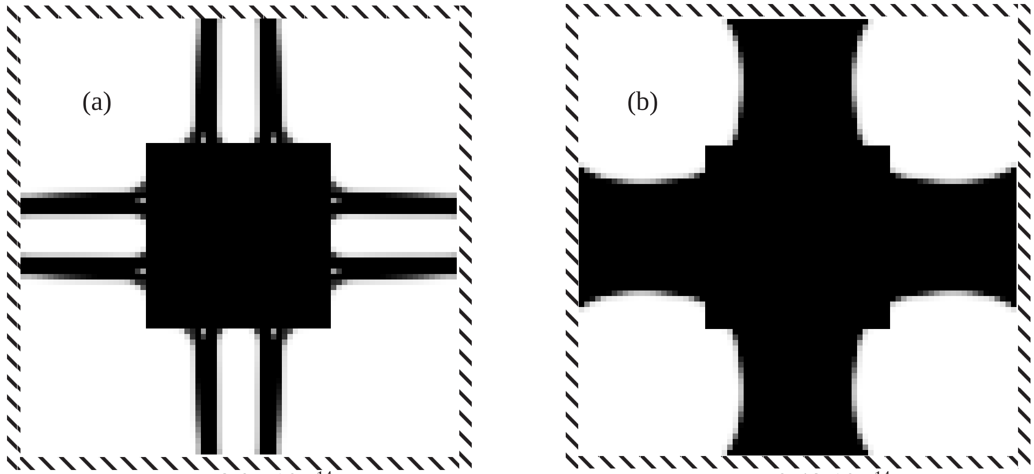
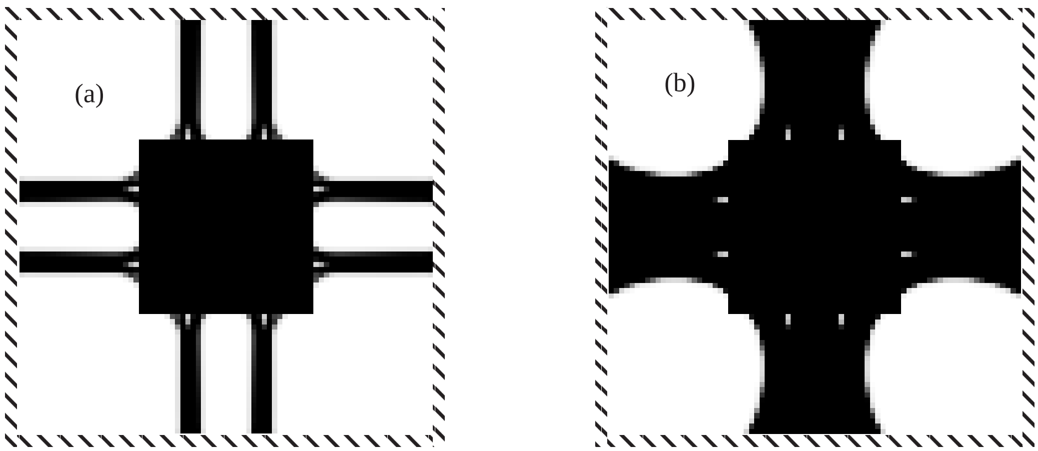


$W = 8.97\ 10^{-14}\ J$ $\qquad$ $W = 3.42\ 10^{-14}\ J$

Figure 4: Optimized designs when optimized for compliance, (a) 30 % material, (b) 50% material.



$\omega_1 = 0.161\ MHz$ $\qquad$ $\omega_1 = 0.233\ MHz$

Figure 5: Optimized designs when optimized for maximum first eigenfrequency, (a) 30% material, (b) 50% material.

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