Optimal Design of A Tuning Fork gyroscope and Its Testing Experiment

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ABSTRACT

The eigenfrequency of a MEMS Tuning Fork Gyroscope (TFG) is calculated using a formula derived from Rayleigh method. From this formula, the objective function for the optimal design is established. Then two methods to solve the objective function are introduced. The TFG has also been fabricated using SOI technologies based on the optimum design. Driving testing results show good agreement between the calculated and the experimental eigenfrequencies.

Keywords: optimization, gyroscope, eigenfrequency, testing.

1 INTRODUCTION

Resolution is one of the most important specifications of MEMS gyroscopes. Therefore, it is selected as the objective function for the design optimization. Theoretically the resolution of a MEMS gyroscope is proportional to its eigenfrequency. This means that whenever the minimum natural frequency is calculated, its resolution can be obtained.

Here, a formula for calculating the eigenfrequency of TFG is derived using Rayleigh method. Finite element analysis (FEA) method is also used to prove its validity. Based on the formula, the objective function for the optimal design of TFG is established, and the optimized structure parameters are obtained by solving the objective function using Matlab™ and Excel™.

Further, the optimal design is verified by the fabrication and testing experiment of the TFG.

2 DERIVATION OF THE FIRST TWO EIGENFREQUENCIES FOR TFG

The TFG sketch is shown in Fig.1. We are using Rayleigh's principle to derive its lateral and vertical eigenfrequencies. The Rayleigh principle indicates that the maximum kinetic energy is equal to the maximum potential energy in a mechanical system. To apply the Rayleigh principle to the TFG design, a deflection curve must be proposed first. The proposed curve must comply with the boundary condition of the system.

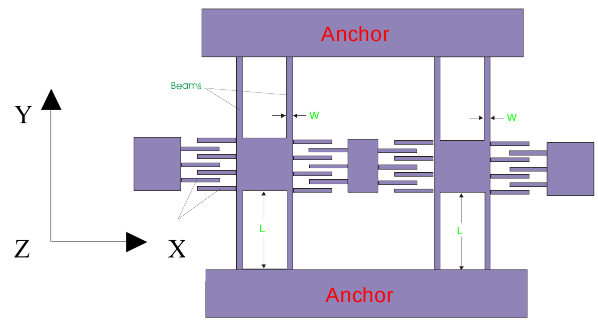

Figure 1: Sketch of the TFG.

Here the deflection curve is suggested as follows:

$$y = y_0\left(1-\cos\frac{\pi}{L}x\right) \tag{1}$$

where x is the width of the beams, L is the length of the beams, and y_0 is the maxim deflection.

The kinetic energy of TFG can be derived as follows:

$$T = 2\omega^2 y_0^2 (M_p + M_c) + \frac{3M_b}{4}\omega^2 y_0^2 \tag{2}$$

where M_p, M_c and M_b are the masses of the prove mass, the combs and the beams.

The potential energy of the system can be derived as follows:

$$V = \int_0^{2L} EI(y'')^2 dx = \frac{8\pi^4 EI y_0^2}{(2L)^3} \tag{3}$$

With T equals to V, then the lateral eigenfrquency can be obtained as follows:

$$\omega_l = \sqrt{\frac{\pi^4 Ehw^3/(2L)^3}{3(M_p + M_c + 0.375M_b)}} \tag{4}$$

With the same process, a similar formula for vertical eigenfrequency can also be obtained.

3 VERIFICATION USING FEA

The precision of the analysis results using Rayleigh method is affected by the suggested deflection curve. Therefore, as a comparison, FEA is used to calculate the eigenfrequencies in lateral and vertical direction of the comb-driven single silicon resonator. The analysis results of three different cases are presented here. The main structure parameters are as follows:

Case1, the depth of the resonator is 10 μm . The length and width of the beam are 280 μm and 2 μm respectively. The length and the width of the proof mass are 200 μm and 194 μm . The gap between the proof mass and the driving or sensing electrodes is 60 μm . The length and width of the combs are 40 μm and 2 μm . There are 25 combs in the proof mass, and there are 24 combs in the driving or sensing electrode.

Case 2, only the width of the beam is changed to 4 μm , the other parameters are just the same as those in Case 1.

Case 3, the depth of resonator is changed to 5 μm , and the length of the beam is changed to 140 μm , and the others parameters are the same as those in Case1.

Method	New method (rad/s)	FEA (rad/s)	Percentage difference
Case 1	2.6088e+5	2.5556e+5	1.69%
Case 2	3.6510e+5	3.4893e+5	4.635%
Case 3	3.7092e+5	3.5725e+5	3.815%

Table 1: The lateral eigenfrequencies of different methods.

Method	New method (rad/s)	FEA (rad/s)	Percentage difference
Case 1	5.2177e+4	5.1937e+4	0.462%
Case 2	1.4604e+5	1.4795e+5	1.291%
Case 3	1.4837e+5	1.5523e+5	4.419%

Table 2: The vertical eigenfrequencies of different methods.

4 OPTIMAL DESIGN OF TFG

The resolution of TFG can be described as follows:

$$\Omega_{\min} = \frac{\omega A_s}{2QA_{d\max}} \tag{5}$$

where ω is the eigenfrequency of the MEMS gyroscope; A_s is the smallest deflection of the proof mass, which can be assumed from the specification of the sensing circuit; Q is the quality factor of the MEMS gyroscope, which can be assumed according to the environment condition; $A_{d\max}$ is the maximum vibrating amplitude for the primary mode of the MEMS gyroscope, which may be assumed according to the length of the comb.

From equations (4) and (5), the objective function can be established as:

$$F_{\min} = \frac{\Delta\delta}{2\vartheta_{\max} Q}\sqrt{\frac{\pi^4 Ewh^3/(2L)^3}{3(M_p + M_c + 0.375M_b)}} \tag{6}$$

We use both Matlab™ and Excel™ to solve the objective function (6). Suppose that the beam length changes from 300 μ m to 400 μ m, the beam width changes from 2 μ m to 10 μ m. The width and the length of the proof mass change from 300 μ m to 400 μ m respectively. Then, the optimised structure parameters are obtained (in Table 3).

Parameters	Results
The length of beams	400 μ m
The width of the beams	2 μ m
The number of the combs	40
The width the proof mass	400 μ m

Table 3: The optimised results for the TFG.

5 FABRICATION RESULTS

According to the optimised structure parameters, one round of fabrication using SOI technologies has already completed. The SEM pictures of the TFG structure are shown in Figs. 2 and 3. The measured average structure parameters are as follows: The length of beam is about 398 μ m, and the width of beam is about 1.9 μ m. The width of the comb is about 2.0 μ m, and its length is about 59 μ m. The width and the length are also about 400 μ m and 396 μ m respectively. From formula (4) and using the measured parameters, the actual lateral eigenfrequency of TFG can be calculated. Then the optimized parameters can be obtained by solving equation (5).

Figure 2: Combs and proof mass of TFG.

Figure 3: Top view of TFG.

6 TESTING EXPERIMENT

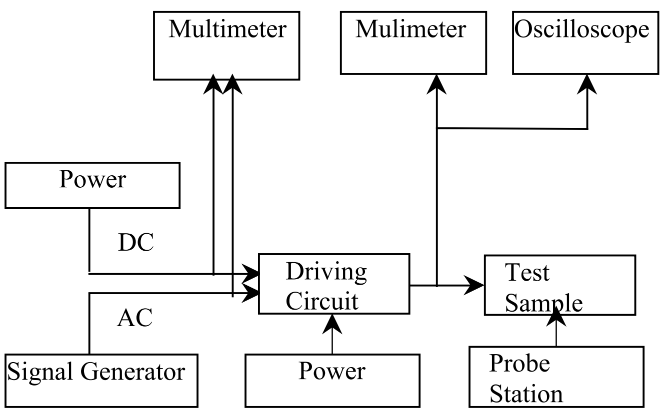

Figure 4: Block diagram of the testing.

The experiment setup for the driving testing is shown in Figure 4.

Figure 5: Vibrating with different amplitudes.

Figure 5 shows us the vibrating TFG with different amplitudes. The picture in the right hand side of Figure 5 shows us the vibrating with an amplitude of 10.0 μ m, and the left one shows us the vibrating with an amplitude of 5.0 μ m.

The driving testing results are shown in Table 4.

DC (V) **AC (V)**	**10.0**	**20.0**	**30.0**
0.5	2.6k	3.02k	3.35k
1.0	3.06k	3.83k	4.3k
1.5	3.4k	4.35k	5.0k
2.0	3.87k	4.8k	
2.5	4.13k	4.28k	

Table 4: The driving testing results.

The testing results are processed using linear curve-fitting method. The percentage of the maximum variance is under 5%, and the fitted value of the eigenfrequency is 2.2 kHz.

Using equation (4) to calculate the lateral eigenfrequency, we must know the exact value of Young's Modulus. Usually this value is provided by the wafer supplier. However, it is relatively difficult to test the Young's Modulus of SOI wafer because of its complicated components. The vender failed to give us a reference.

Actually we are now developing a very practical method to measure the Young's Modulus of SOI wafer. The initially tested value of Young's Modulus is 140 Gpa, which is quite reasonable and encouraging. As we know, the Young's Modulus of the wafer will become much lower than the single crystal silicon wafer after it has been highly doped.

Using the tested value of Young's Modulus, 140 Gpa, the calculated lateral eigenfrequency according to equation (4) is 2.7 kHz.

The difference of the calculated and the experimental lateral eigenfrequencies is under 23%. Even using the extreme value of Young’s modulus, 170Gps, it is still under 36%.

7 CONCLUSIONS

The formula derived by us to calculate the eigenfrequency of TFG is shown to be close to that of both FEA and experimental results. The optimal TFG design also been fabricated using SOI technologies.

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