Dynamic Analysis of an Electrostatic Micropump

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ABSTRACT

In this paper, a micropump actuated by electrostatic force is dynamically analysed. Mathematical model is established to evaluate the dynamic response of the micropump. Electro-mechanical coupling effect is considered in evaluating the electrostatic force applied on the pump electrode. Boundary Element Method is employed to solve the three dimensional Laplace equation that the potential difference satisfies. A combination of Assumed-mode method and Boundary Element Method is employed to solve the governing equation of the pump diaphragm. Newmark iteration method is utilized to solve the decoupled ordinary differential equations. Deflection of the oscillated diaphragm is demonstrated to evaluate the performance of the electrostatic pump. Amplitude and frequency of the potential difference between the two electrodes are investigated.

Keywords: Micropump, electrostatic force, electro mechanical coupling, three dimensional boundary element method, dynamic analysis, assumed-mode method

1 INTRODUCTION

Recently efforts have been pushed to the study and development of micropumps [4-9]. According to working principles of actuation, micropump can be listed as electromagnetic, electrostatic, piezoelectric and shape memory alloy etc. In the development of micropumps it is essential to predict the performance of the micropump before the prototype is fabricated in order to save production cost and get a better design [3, 9]. During design process of micropumps, modelling of micropumps is essential.

Electrostatic micropump has attracted attention in recent years. In the design and modeling of the electrostatic micropump, electro-mechanical coupling effect is considered. As voltage is applied onto a capacitor, charges are induced on the plates of the capacitor and therefore electrostatic force is generated. As one plate is flexible enough to deform, the distribution of the charges on this plate varies and electrostatic force changes correspondingly, which illustrates a coupling mechanism of electrical and mechanical effect. To study this electro-mechanical coupling effect, Boundary Element Method is employed to evaluate the charge density and corresponding electrostatic force. A combination of Finite Element Method and Boundary element Method is employed to solve this problem[1,2]. Currently, two dimensional and three dimensional static problems have been studied in literatures [1,2] and two dimensional dynamic case is also studied. However, due to the complexity of microstructures, three dimensional dynamic problems have not been studied thoroughly. With this objective in mind, we present a three dimensional micropump driven by electrostatic force to study its performance. Direct Boundary Element Method is used to solve the three dimensional Laplace equation and evaluate the charge density and electrostatic force. Assumed-modes method is employed to solve the governing equation of the diaphragm of the micropump. Numerical simulation is conducted to study the performance of the micropump.

2 DYNAMIC MODEL OF AN ELECTROSTATIC MICROPUMP

The scheme of the electrostatic micropump is shown in Figure 1. Electrode 2 is fixed . As voltage is applied to the electrodes, the thin electrode 1 deforms toward its counterpart electrode 2, which induces bigger volume of the pump container and thus fluid is sucked into the pump chamber. As voltage is released the diaphragm electrode 1 comes back so as to squeeze the fluid out.

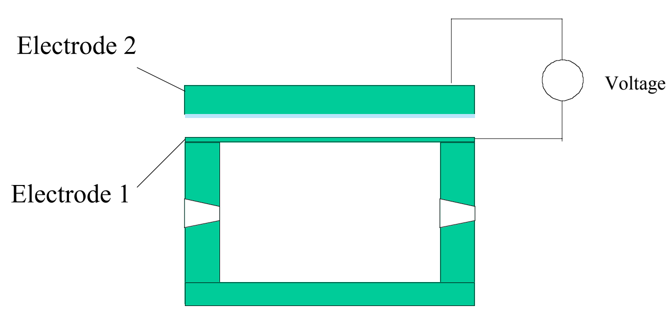


Figure 1 Scheme of the electrostatic micropump

Electrode 1 is a four edge clamped plate and its dynamic equation can be written as the following form [10]:

$$D_0\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)+\rho h\frac{\partial^2 w}{\partial t^2}=p(w(x,y,t)) \quad (1)$$

which can be in the following brief form:

$$D_0\nabla^4 w+\rho h\frac{\partial^2 w}{\partial t^2}=p \quad (2)$$

where $D_0=\frac{Eh^3}{12(1-\mu)}$, E is Young's modulus, ρ is the density of the plate, μ is Poisson's ratio, h is plate thickness, p is the external load applied on the plate, it's the electrostatic force in this application.

3 CALCULATION OF CHARGE DENSITTY AND ELECTROSTATIC FORCE

As a voltage is applied onto the undeformed conductive plates, electrical charges are induced on the surface of the plate, and these charges induce surface normal pressures which are the electrostatic load p and it can be calculated by equation (3)

$$p(x,y)=\frac{1}{2}\frac{q^2(x,y)}{\varepsilon} \quad (3)$$

where p is the normal outward pressure on the plate, q is the surface charge density at point (x,y) on the surface of the conductive plate, and ε is the dielectric constant of the medium in which the plate is placed.

The charges distributed on the plate surface satisfy

$$q(x,y)=\varepsilon\frac{\partial\phi(x,y)}{\partial n} \quad (4)$$

ϕ is the electrostatic potential of the conductor, n is the inward normal to a conductor at (x,y). The electrostatic potential ϕ, in the region exterior to the conductor, satisfies Laplace's equation

$$\nabla^2\phi=0 \quad (5)$$

The three dimensional Laplace equation (5) is solved with three dimensional Boundary Element Method. The 3-D domain is configured as Figure 2.

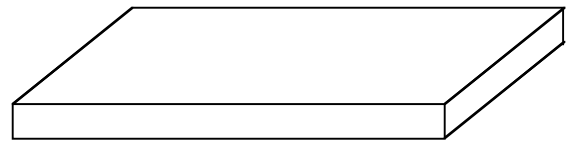
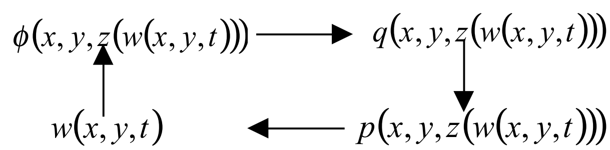


Figure 2 Configuration of the 3-D domain

There are mixed boundary conditions. In Figure 2, the potential of the top surface (electrode 2) is assumed zero, and the potential of the bottom plate (electrode 1) is the applied voltage. The potential gradient of all the other four side surfaces are zero. Dimension of the top and bottom plate is the same as $100\mu m\times 100\mu m\times 1\mu m$ and the distance between the two electrodes is $2\mu m$.

4 SOLUTION OF THE GOVERNING EQUATION

Illustration of electromechanical coupling effect:



Assume deflection $w(x,y,t)$ of electrode 1 as

$$w(x,y,t)=\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}W_{i,j}(x,y)\eta_{i,j}(t) \quad (6)$$

where

$$W_{i,j}(x,y)=\psi_i(x)\varphi_j(y) \quad (7)$$

is the normal shape function

in which

$$\begin{aligned}\psi_i(x)&=a_i(\cosh k_i x-\cos k_i x-\\&\quad\alpha_i(\sinh k_i x-\sin k_i x)) \quad (i=1,2,\ldots)\\ \varphi_j(y)&=b_j(\cosh k_j y-\cos k_j y-\\&\quad\alpha_j(\sinh k_j y-\sin k_j y)) \quad (j=1,2,\ldots)\end{aligned} \quad (8)$$

where $\alpha_p=\frac{\cosh k_p l-\cos k_p l}{\sinh k_p l-\sin k_p l}$ for $p=i$ or j

Substitute (3) into system equation (2), we get

$$D_0 \sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\left(\nabla^4 W_{i,j}(x,y)\right)\eta_{i,j}(t)+\rho h \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} W_{i,j}(x,y)\ddot{\eta}_{i,j}(t)= p(w(x,y,t)) \quad (9)$$

Multiply $W_{r,s}(x,y)$ on each side of equation (9) and integrate over the plate area we get

$$\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\eta_{i,j}(t)\int_{\Omega} D_0\left(\nabla^4 W_{i,j}(x,y)\right)W_{r,s}(x,y)dxdy + \sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\ddot{\eta}_{i,j}(t)\int_{\Omega}\rho h W_{i,j}(x,y)W_{r,s}(x,y)dxd = \int_{\Omega} p(w(x,y,t))W_{r,s}(x,y)dxdy \quad (10)$$

According to orthogonal condition, equation (9) can be transformed to a series of decoupled equations

$$\ddot{\eta}_{r,s}(t)+\omega_{r,s}^2\eta_{r,s}(t)=\int_{\Omega} p(w(x,y,t))W_{r,s}(x,y)dxdy$$

(11)

Here we take an interception of the vibration mode to r=1, 2, 3 and s=1, 2, 3.

The natural frequencies of the plate are:

$$\omega_{i,j}^2=\frac{D_0}{\rho h}\left(k_i^4+k_j^4+2\rho h k_i^2 k_j^2\right)\int_{\Omega} F_{i,j}(x,y)W_{i,j}(x,y)dxdy$$

(12)

in which

$$F_{i,j}(x,y)=\left[\cosh k_i x+\cos k_i x-\alpha_i(\sinh k_i x+\sin k_i x)\right]\left[\cosh k_j y+\cos k_j y-\alpha_j(\sinh k_j y+\sin k_j y)\right] \quad (13)$$

Then equation (11) could be solved with Newmark numerical method with electrostatic force interactively solved with Boundary Element Method at each time step.

5 SIMULATION RESULTS

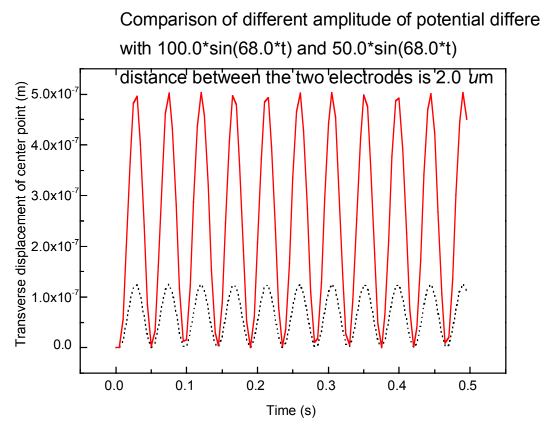


Figure 3 Response under different amplitude of potential difference between two electrodes

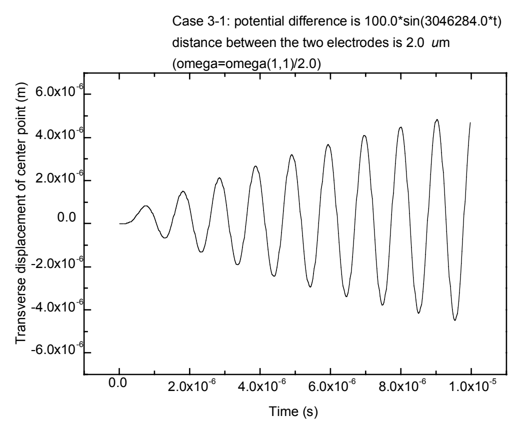


Figure 4 Response as driving frequency is equal to half of the natural frequency of the diaphragm

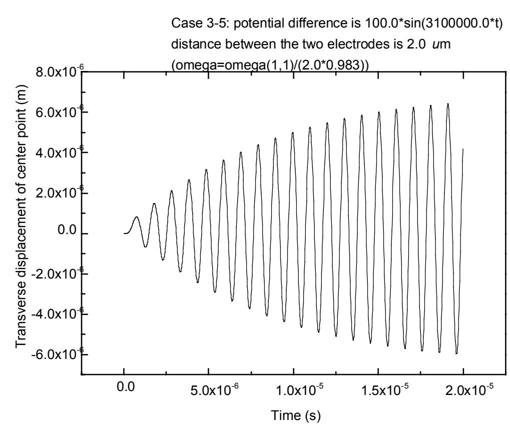


Figure 5-1 Response as driving frequency is slightly bigger than the first natural frequency

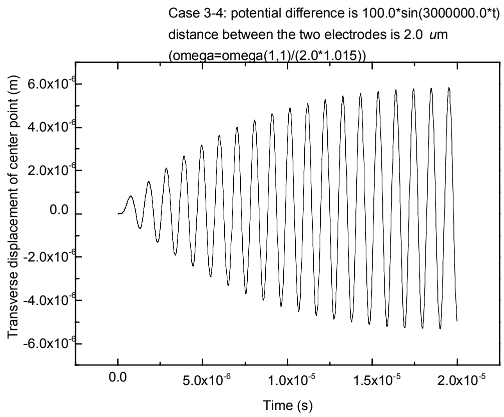


Figure 5-2 Response as driving frequency is slightly smaller than the first natural frequency

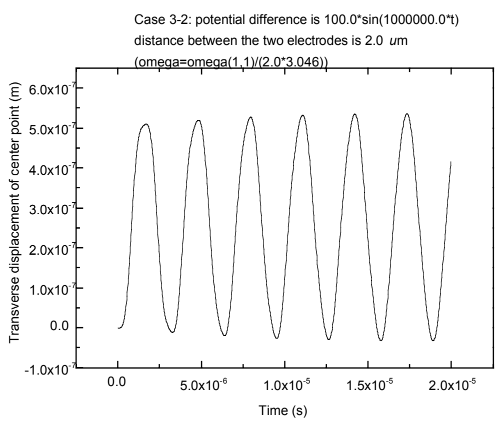


Figure 6 Response as driving frequency is far away from the first natural frequency

6 CONCLUSIONS

1. The vibration element of the micropump has been simplified as a clamped thin plate. Interaction of mechanical deflection of the plate and the electrostatic force has been investigated and implemented.

2. A new method combining the assumed-mode method and Boundary Element Method has been developed for modelling of the electrostatic micropump.

3. Newmark method has been employed for solution of plate deformation since Newmark method is an implicit method which always gives a convergent solution. Only if the time step is properly selected, the Newmark iteration provides reasonable result.

4. Deflection of the diaphragm of the micropump is of nonlinear relationship with the amplitude of the potential difference between the two electrodes.

5. As frequency of the driving voltage is close to half of the first natural frequency of the vibration diaphragm, the diaphragm goes to a resonant vibration state.

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