Hydrodynamic Force on a Plate Near the Plane Wall

M.-U. Kim*, K.-W. Kim**, J. H. Kim*** and B. M. Kwak****

Korea Advanced Institute of Science and Technology,
373-1, Kusong-Dong, Yusong-Ku, Taejon, 305-701, Korea
*mukim@cais.kaist.ac.kr, **leo@cais.kaist.ac.kr,
***jhkim@khp.kaist.ac.kr, ****bmkwak@khp.kaist.ac.kr

ABSTRACT

Hydrodynamic force on a plate moving in a viscous fluid near the plane wall as a model of a microelectromechanical system(MEMS) is considered. The flow field is described by the linearized Navier-Stokes equation. The method of matched asymptotic expansions(MAE) is used, in which the physical domain is subdivided into three regions, interior, edge and outer region respectively. The solution of the Reynolds equation is compensated by so called edge effect. The damping coefficients compared with the experimental results show good consistency.

Keywords: Microelectromechanical system(MEMS), Linearized Navier-Stokes equation, Matched asymptotic expansion(MAE), Edge effect, Viscous damping.

1 INTRODUCTION

A hydrodynamic analysis of microstructures moving in a viscous fluid is one of the main topics of microelectromechanical systems(MEMS). The smaller the mechanical structures are, the more important the viscous air damping becomes and in MEMS(typically 1~100μm) it is one of the main factors determining the dynamic responses and the performance characteristics of devices. For a micromachined accelerometer[1,2], proper specification of the viscous air damping is needed to achieve a flat response over the wide frequency range and to avoid the resonance. In particular, the viscous air damping in resonant microdevices governs the amplitude of microdimensional motions[3]. Thus, a sound understanding and close estimation of the viscous air damping is one of the most important issues in the design of microstructures.

For a microstructure moving in a viscous fluid, the flow field is governed by the linearized Navier-Stokes equation

$$\frac{f\vec{V}}{ft} = -\frac{1}{\rho} \quad p + \nu \quad {}^{2}\vec{V} \tag{1}$$

where, $\vec{V}$ denotes the velocity vector, p the pressure, ρ the density and ν the kinematic viscosity of fluid. The solution of (1) can be used to approximate the viscous air damping in MEMS. Kim[4] measured the viscous air damping for three types of oscillating motions(squeezing, tilting and sliding) of microdevices and compared with the solution of the Reynolds equation. More than 40% of discrepancies between the numerical and the experimental results were reported and he speculated the major source of the discrepancies would be the edge effect. In this paper, we consider the flow due to a plate moving near a plane wall taking into account the edge effect.

2 MATHEMATICAL FORMULATION

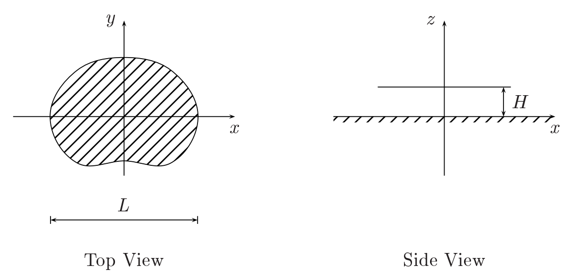

Fig.1. geometry

The geometrical configuration of the problem is shown in Fig.1. The fluid occupied the upper half of the (x, y, z) - space, $z > 0$, and a plate of the characteristic length L is located at $z = H$ parallel to the plane wall, $z = 0$. The motion of the plate is a steady, arbitrary translational motion near the plane wall. The governing equations and the boundary conditions for the flow field can be written in a normalized form

$$\begin{aligned} ?\vec{V} &= 0 \\ {}^{2}\vec{V} &= \quad p \end{aligned} \tag{2}$$

$$\begin{aligned} \vec{V} &= 0 \quad \textit{on the wall} \\ \vec{V} &= \vec{V}_0 \quad \textit{on the plate} \end{aligned} \tag{3}$$

where, $\vec{V}_0$ denotes the velocity of the plate. Because the governing equations and the boundary conditions are linear, we need only to consider two kinds of motions of the plate, namely, the broadside and the edgewise motion.

3 METHOD OF SOLUTION

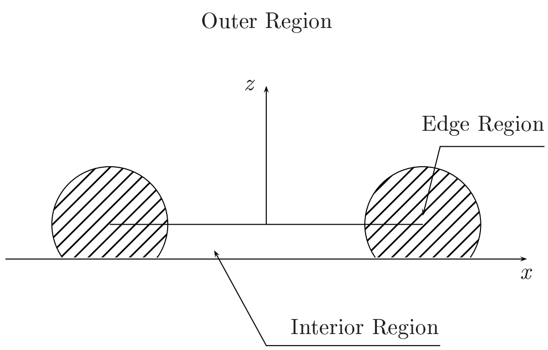

Fig.2. subdivision of the physical domain

We use the method of matched asymptotic expansions(MAE). In this method the physical domain is subdivided into three regions, which we call the interior, the edge and the outer region respectively(Fig.2). For each region we renormalize the coordinate system , the velocity components and the pressure. Expanding the velocity components and the pressure as asymptotic series of every order of $\varepsilon(=H/L)$ and collecting the terms of the same order of ε, we obtain the governing equations and the boundary conditions of every order of ε for each region. Their solutions and proper interregional matching procedures determine the flow field.

4 RESULTS AND DISCUSSIONS

We compared the damping coefficients obtained from our analysis with the experimental results of Kim. He measured the damping coefficients of thin squared plate in a squeezing motion. Plate either closed or has 1, 4, 9 holes of the same area ratio(ratio of the area of holes with respect to the area of closed part). Closed plate has a side of 57μm or of 80μm. He also obtained the solutions of the Reynolds equation by the finite element analysis(FEA) and reported more than 40% of discrepancies between the numerical and the experimental results. He speculated the major source of the discrepancies would be the edge effect. We got to know the solutions of the Reynolds equation correspond to the solution of order ε^{-3} and the edge effect to the solution of order ε^{-2} of our analysis.

Fig.3 shows the damping coefficients for the closed plates and Fig.4~Fig.6 shows the damping coefficients for the plates with 1, 4, 9 holes of the area ratio 3.4%, 6.8% and 13.6% respectively. Our analysis shows the damping coefficients larger than those from the solutions of the Reynolds equation and the differences are due to the edge effect. Our analysis somewhat underestimated the damping coefficient but taking into account some possible errors in experiments the discrepancies may not be considerable.

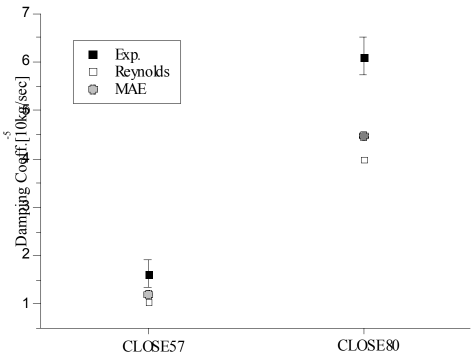

Fig.3. closed plates

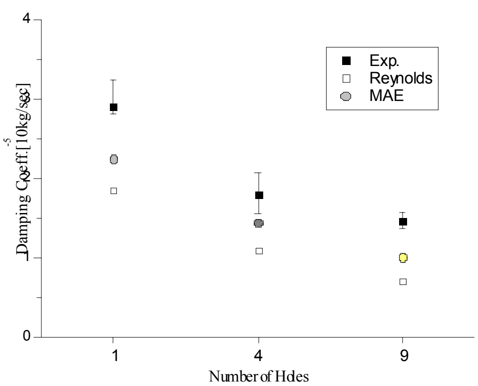

Fig.4. area ratio 3.4%

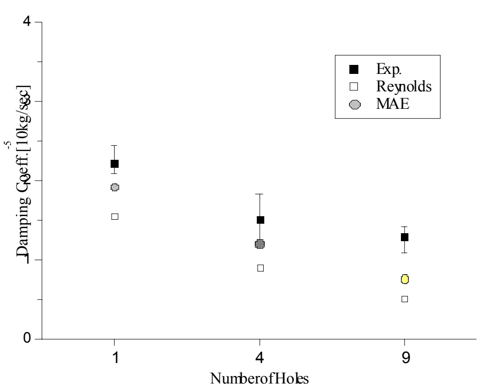

Fig.5. area ratio 6.8%

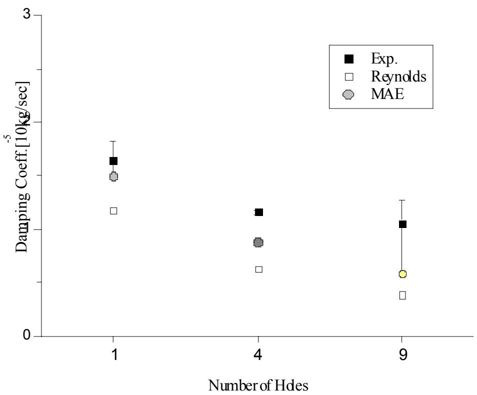

Fig.6. area ratio 13.6%

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