# System Level Simulation of an Electrostatically Levitated Disk

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## ABSTRACT

This paper describes the derivation of a system-level model for a micromachined disk which is levitated by electrostatic forces. Such a system has many potential applications for inertial sensors, micro-motors, microfluidic and micro-optical devices. As a simulation tool Matlab/Simulink was used since it can easily handle different domains such as electrical and mechanical without special arrangements. Multi-axis electro-mechanical sigma-delta modulators were chosen to control the various degrees of freedom of the levitated disk. Of special interest is the power-up phase at the end of which the control system has to ensure that the disk is centred between the top and bottom electrodes.

***Keywords***: Electrostatic levitation, inertial sensors, system-level simulation.

## 1 INTRODUCTION

Electrostatic forces are commonly used for actuation of micromachined devices e.g. force-balanced inertial sensors, microfluidic valves and micro-optical mirrors. However, in all these devices a mechanical connection exists between the actuated part and the substrate, the properties of which are subject to considerable process tolerances and cannot be changed easily once the device has been manufactured. In this paper a micromachined disk is suggested which is levitated by electrostatic forces. This has many potential applications such as accelerometers with online dynamic characteristics tuning, gyroscopes if the disk is spun and the precession of the disk induced by the Coriolis force is measured, microfluidic mixers and pumps, frictionless bearings for micro-motors and micro-optical light choppers. Surprisingly little work has been undertaken in this direction [1,2].

The system, as shown in fig.1, consists of a nickel disk manufactured by electroplating which is encaged by sets of electrodes on top and bottom and electroplated pillars at the sides. The manufacturing process will be described elsewhere in detail. Each set of electrodes forms five capacitors which are used for both sensing the position of the disk as well as actuating it in such a way that it is maintained at or close to the centre position.

In this work the system level simulation of such a levitated disk is presented. Simulink was chosen as a simulation tool since it is ideal for the simulation of a micromachined system because it easily can handle different domains such as electrical and mechanical without special arrangements. The system is unstable in the open-loop configuration, consequently, the control strategy has to be considered with great care. Here, multi-axis electro-mechanical sigma-delta modulators are used [3] to control the various degrees of freedom of the disk. This prevents the possibility of electrostatic latch-up [4] and also results in an inherently digital system.

## 2 DERIVATION OF THE MODEL

In order to derive a model capturing the governing features of the system the following building blocks have to be considered:

- The dynamic behaviour of the disk,
- conversion from the mechanical to the electrical domain, i.e. the position measurement interface,
- the multi-axis electromechanical sigma-delta modulator,
- the compensator,
- the reset mechanism, i.e. conversion from the feedback voltage to the electrostatic forces and moments including the effects of the current position of the disk.

### 2.1 Disk Dynamics

The dynamic behaviour of the levitated disk is characterized by three second order differential equations:

$$
\begin{aligned}
&m\ddot{z} + b_z \dot{z} = F_{ext,z} \\
&I_x \ddot{\varphi} + b_\varphi \dot{\varphi} = M_{ext,x} \qquad (1) \\
&I_y \ddot{\theta} + b_\theta \dot{\theta} = M_{ext,y}
\end{aligned}
$$

where m is the mass of the disk, z the displacement from the mid-position between the electrodes, φ and θ the angular deflections about the x and y axis of the disk, $b_z$, $b_\varphi$ and $b_\theta$ the damping coefficient in z, φ and θ direction respectively, $I_x$ and $I_y$ the moments of inertia of the disk about the corresponding axis and $F_{ext,z}$, $M_{ext,x}$ and $M_{ext,y}$ the external force and moments in the corresponding directions.

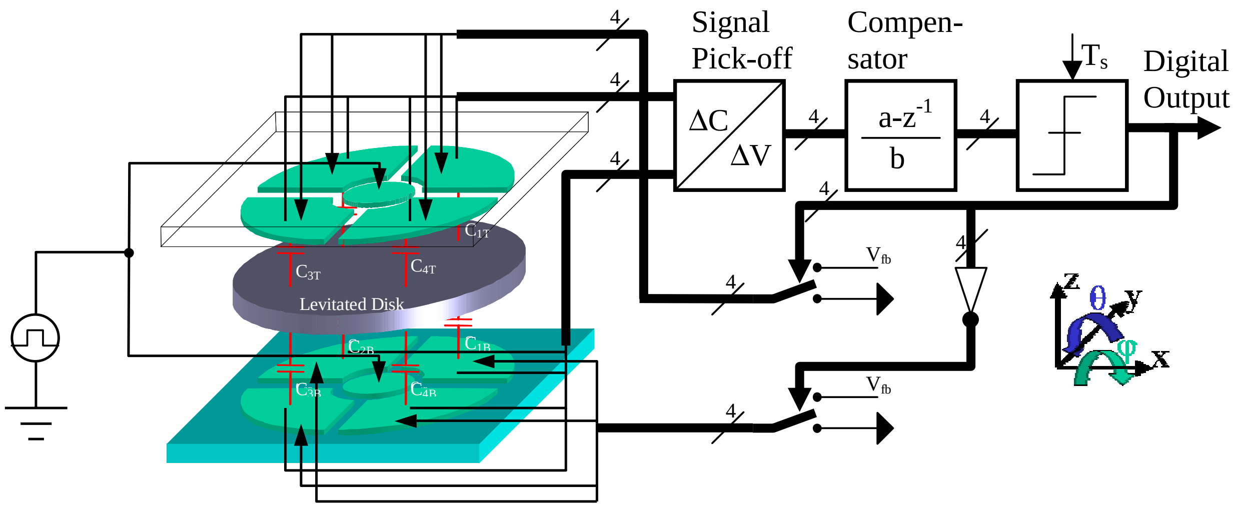


Fig. 1: Levitated disk and control system.

The main difference to the usual dynamic equations describing a micromachined proof mass is that a mechanical spring force term is missing. The effective spring constant of the disk is entirely controlled by the external electrostatic forces and moments. As a drawback it is obvious that the system cannot operate in an open loop mode, as usually possible for inertial sensors.

A further building block is required to simulate the physical constraint of the disk, this is achieved by saturation blocks.

## 2.2 Position Measuring Interface

The position of the disk, characterized by its z, φ and θ coordinates, has to be measured. This is achieved by measuring the differential capacitance of the four outer, pie-shaped electrodes. The system level model does not simulate an electronic implementation, however, it is envisaged to apply an excitation voltage to the round, middle electrodes and sense the currents from the outer electrodes from which it possible to measure the four differential capacitances between the corresponding top and bottom electrodes.

The information about z, φ and θ deflections is encoded in the differential capacitance values. Two possible control approaches are therefore possible. Firstly, one could extract the position information at this point of the control loop which would result in a three-axis sigma-delta modulator control scheme. Secondly, a sigma-delta loop with four paths is possible, one for each differential capacitance. The information about disk position and external inertial forces and moments is then encoded in the digital bitstream providing the output signals of the system. This approach was chosen here since the decoding of the position information can then be undertaken in the digital domain with obvious advantages.

For the system level model an analytical expression for the capacitance between the pie-shaped electrodes and the disk has to be derived. The expression for this geometry is very lengthy and it was decided to use an approximation for a square electrode. As an example the expression for the top-right segment is given:

$$C_{1T} = \varepsilon_0 \int_0^R \int_0^R \frac{1}{-x\theta + y\varphi + z_0 - z} dxdy =$$

$$= \varepsilon_0 \frac{\ln(R\varphi - R\theta + z_0 - z)(-R\varphi + R\theta - z_0 + z) + \ln(R\varphi + z_0 - z)(R\varphi + z_0 - z)}{\varphi\theta} \dots$$

$$\dots \frac{\ln(-R\theta + z_0 - z)(-R\theta + z_0 - z) - \ln(z_0 - z)(z_0 - z)}{\varphi\theta} \qquad (2)$$

where $\varepsilon_0$ is the dielectric constant of vacuum (assumed to be the same as for air), R is the disk radius, $z_0$ the nominal distance between disk and electrodes when the disk is at mid-position.

Furthermore, small angular displacements are assumed so that tan(φ) ≈ φ and tan(θ) ≈ θ and the usual approximation of parallel-plate capacitors is made. The expressions for the other electrodes can be derived by symmetry considerations. The use of the equation is illustrated in fig. 2a.

This square plate approximation is justified by the fact that in the forward path of the sigma-delta modulator control system an ideal comparator is located whose output is not determined by a numerical exact solution but rather governed by the sign of its input signal.

It should be noted that the expression for the capacitance has a singularity at φ = 0 or θ = 0, which causes a simulation error at zero deflection and incorrect numerical results for very small angles due to numerical noise. This problem can be overcome by using a linearized version of eq. (2) around φ = 0 and θ = 0, both expressions are then implemented in the model, depending on the magnitude of the angular displacements either the linerized or full expression is used.

## 2.3 Sigma-delta Modulator and Compensator

The building blocks for the sigma-delta modulator can be easily simulated by a relay and a sample and hold. The preceding compensator is required to stabilize the loop and ensure a high-frequency limit cycle in the unforced condition by adding a zero at a frequency at approximately 1/10 of the sampling frequency [5].

## 2.4 Feedback Arrangement

In the feedback path, each of the eight top and bottom electrodes is either energized by the feedback voltage or held at zero potential, depending on the output state of the four comparators. The control system has to ensure that the electrode further away from the disk is energized, consequently forces and moments are generated to move the disk back to the mid-position parallel to the top and bottom electrodes.

Provided the disk is held at zero potential (which is assumed here) the magnitude of the electrostatic force in the z-direction can be calculated and is given by:

$$F_{z,1T} = \frac{1}{2}\varepsilon_0 V_{fb}^2 \frac{\partial C_{1T}}{\partial z} \tag{3}$$

for the top right electrode, where $V_{fb}$ is the feedback voltage which is either a fixed voltage or zero depending on the state of the comparator output. The result of using the equation is illustrated in fig. 2b.

As a minimum value the electrostatic force must at least be able to counterbalance the gravitational force on the disk, however, any larger inertial forces would make the disk touch the electrodes, hence the system momentarily inoperative. Simple force calculation reveal that a feedback voltage of 40 V can counterbalance approximately 100 g inertial force for a disk with a thickness of 200 μm and a radius of 0.5 mm, which is believed to be sufficient.

The magnitude of the moment generated by the feedback voltage can be calculated and is given by:

$$M_{y,1T} = \frac{1}{2}\varepsilon_0 V_{fb}^2 \int_0^R \int_0^R \frac{\partial C}{\partial z}\, y \; dxdy \tag{4}$$

for the moment about the y-axis for the top right electrode. The result of using the equation is illustrated in fig. 2c.

Both the expressions for the force and moments suffer from the same singularity as the capacitance equation causing numerical problems. The same method as described for the capacitance was chosen to circumvent the problem. Since the expression for the moments contain a squared term of one angular displacement in the denominator the error tolerance boundaries have to be chosen tighter.

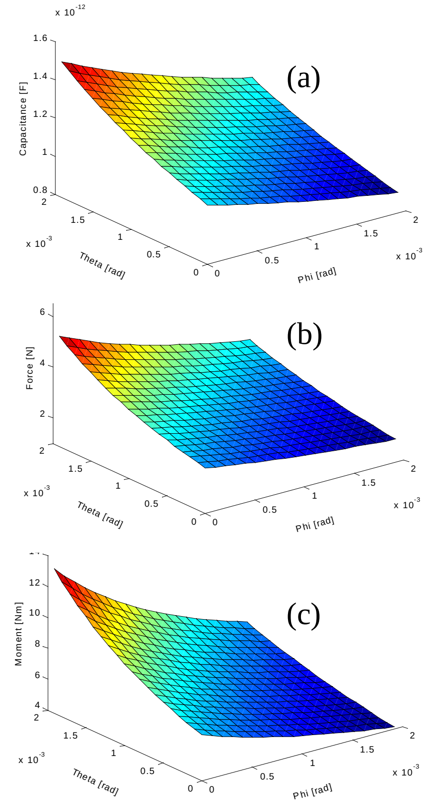


Fig. 2: Capacitance between the top-right electrode and the disk ($C_{1T}$) (a), electro-static force along the z-axis (b) and moment about the y-axis (c) as a function of the two tilt angles φ, θ for a feedback voltage of $V_{fb}$ = 10 V applied to the same electrode and a nominal gap of 2 μm.

Above expressions also assume square electrodes, for pie-shaped geometry the expressions are too unwieldy.

## 2.5 Overall Model

Fig. 3 shows the overall model implemented in Simulink. The nonlinear expressions have been directly implemented as function blocks. One sampling period is divided in a position measuring phase and a actuation phase, the delay between and the length are important parameters determining the loop stability since they add further lag.

# 3 RESULTS

Simulations with initial conditions assuming the disk already at the mid-position show the expected behaviour; the four output signals from the comparators are repetitive sequences of two high periods and two low periods, i.e. a

stable (2,2) mode which is the lowest possible mode for a second order sigma-delta modulator [4].

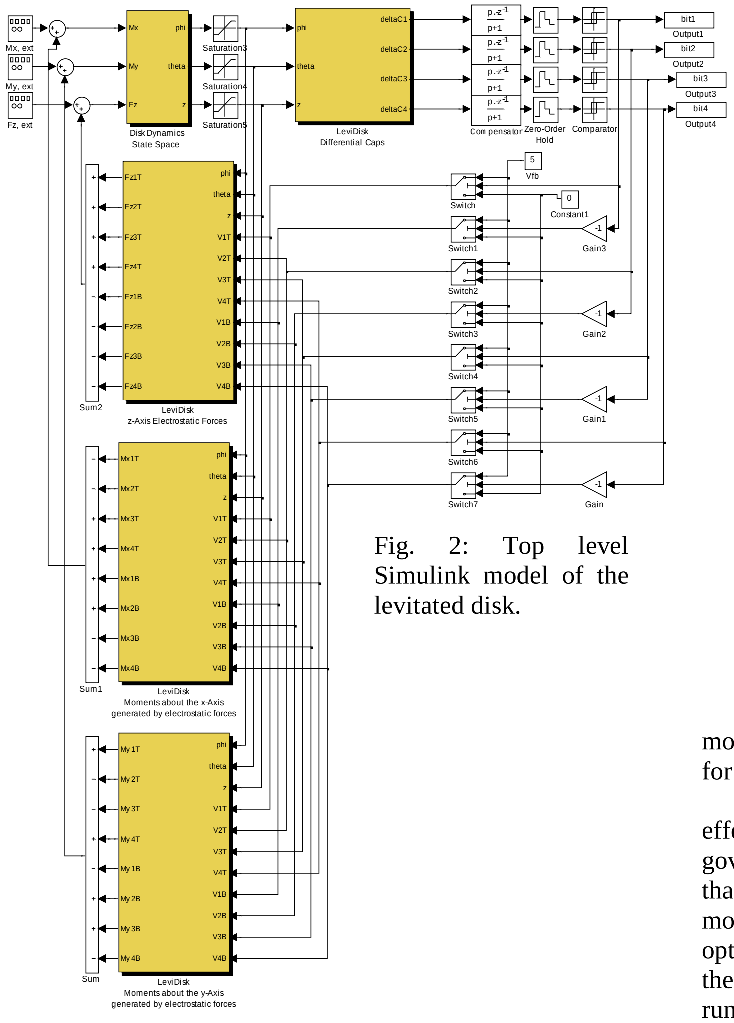

Fig. 2: Top level Simulink model of the levitated disk.

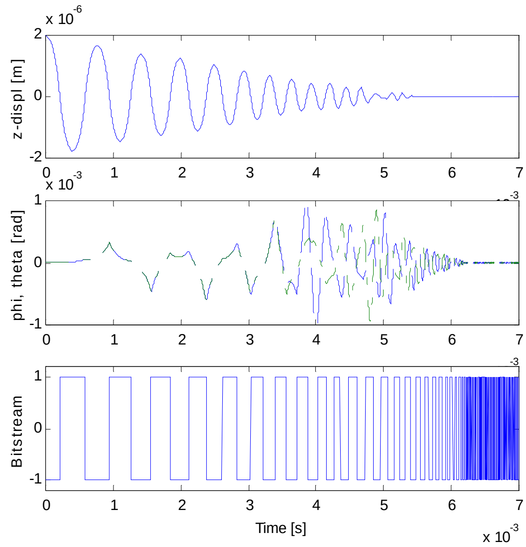


Fig. 4: Simulation result showing the start-up phase. The disk reaches the middle position after approximately 7 ms. Upper plot: displacement of the disk in z-direction; middle plot: tilt of the disk about x-axis (dashed) and y-axis (dash-dot); bottom plot: one of the four output bitstreams.

Of special interest is the power-up phase, at the beginning of which the disk lies flat on one set of electrodes, i.e. initial conditions $z = z_0$, $\varphi = 0$ and $\theta = 0$. For a system with $R = 500$ µm, $z_0 = 2$ µm and a disk thickness of 200 µm the simulation result is shown in Fig. 4. It is obvious that the disk is forced to the centre position between the electrodes and the tilt angles are zeroed as well. This simulation does not take into account any surface forces which might cause stiction of the disk to the electrodes. Such a behaviour is extremely difficult to describe analytically and will be investigated in the hardware implementation. Since the disk will be fabricated from electroplated nickel the surface roughness will be considerable hence will reduce the contact area between disk and electrode which it is believed to alleviate the problem.

## 4 CONCLUSIONS

For any control system comprising a sigma-delta modulator system level simulation is often the only way to predict its behaviour since a complete analytical description is extremely difficult if not impossible. Consequently, the model for the levitated disk has proved to be a valuable aid for the design of the proposed system.

Although the model presented here does not consider all effects which could be included, it still describes the governing features of the system. Especially the parameters that determine the geometry, capacitance, sigma-delta modulator dynamics and stability etc. can be assessed and optimised. A trade-off between the degree of realism and the simulation time is also important. A typical simulation run of the presented model takes about 10-20 minutes on a modern PC.

Further work will concentrate on the inclusion of the behaviour of lateral deflections and simulation of spinning the disk about its main axis.

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