Gain in a Semiconductor Waveguide Qubit

J. Harris, D. K. Ferry and R. Akis

Department of Electrical Engineering and Center for Solid State Electronics Research
Arizona State University, Tempe, AZ 85287-5706
Telephone: 480-965-9572 Fax: 480-967-8058 Email: smeger@geekspiff.com

ABSTRACT

Recently, it has been suggested that it may be possible to use combinations of coupled quantum wire waveguides to form quantum computational qubits [1],[2]. However, there are several related problems intrinsic to this approach. First, in order to completely switch the electron probability wave from one waveguide to another, the length of the region in which the two waveguides are coupled must be tuned quite precisely. In addition, even with a well-tuned coupling length, it appears that complete transmission of the probability wave cannot be achieved [2]. Both of these problems may be mitigated by the addition of a bias along the length of the quantum wires – this would effectively alter the coupling length and may also increase the transmission gain. We show that adding this bias does not provide gain to overcome the lack of 100% coupling between the two guides, but can be used to compensate for imprecision in the length of the device's coupling region.

Keywords: qubit, coupled quantum wires, mode-matching, quantum waveguide.

1 INTRODUCTION

The promise of faster and more effective computation has led to the concept of quantum computers, which are hoped to surpass the limits of binary, digital computers [3],[4]. This promise has led in recent years to a rapid development in quantum information theory. The structure of quantum computation has centered around the use of a *qubit* [5], which is a generalization of a simple binary state. An important aspect of quantum computing is that it has been cast with the promise of vastly improved computing speeds, but the qubit alone is not sufficient to produce this speed increase. The coupling of two bits, however, to produce a controlled-not (CNOT) operation has been shown to be sufficient for general computation. One popular example of such a 2-bit system is the Fredkin gate [6]. This simple gate is a controlled-not system: if a "1" is present in the second bit, the first bit is passed through unchanged. On the other hand, if a "0" is present in the second bit, the first bit performs a simple NOT operation (interchanging the strength of the "0" and "1" states).

One recent suggestion for a semiconductor qubit actually utilizes the interference of quantum waves contained within two parallel waveguides [1]. In this system, the active length of a slit through which the wave can interact with the two adjacent guides is used to generate the proposed qubit, as shown in Figure 1. Coupling this device to a second qubit offers the computational basis – the interaction between the two electron probability distributions slows the first down, defeating the transfer. This interaction provides the CNOT function discussed above.

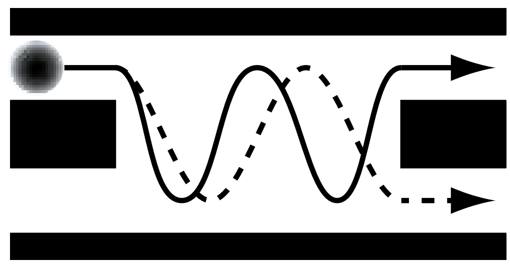

Figure 1: A conceptualization of a waveguide qubit. Bertoni et al [1] have proposed a similar qubit.

The essence of the device's operation is to excite only one of the inputs, and then to control the coupling so as to move the wave between the two outputs. Here, we take the qubit as being in a high-mobility GaAs/AlGaAs heterostructure (at low temperature), in which the waveguides are defined by electrostatic potentials (hardwall potentials are used for this investigation). The two guides are assumed to be 40 and 45 nm wide, and are separated by a potential barrier that is 30 nm thick. An opening is made in this central barrier that allows the two waveguides to couple. This opening length is varied to determine the critical coupling length, the length at which transmission from the input to one of the outputs is at a maximum.

An electron probability distribution in an excited 1-D sub-band oscillates between two quantum wires with a frequency that is dependent upon its energy. Thus, adjusting the coupling length only tunes the device for one sub-band. We assure that only a single sub-band propagates in each isolated waveguide by setting the Fermi energy to 5 meV, roughly halfway between the excitation energies of the first and second sub-bands in the quantum wires. This energy corresponds to a quasi-two-dimensional electron density of 1.4×10^{11} cm^{-2}. While relatively low, this is still well above the metal-insulator transition in such a heterostructure [7]. We denote the top left quantum wire by input one, the top right quantum wire by output one, the bottom left quantum wire by input two, and the bottom right quantum wire by output two.

We have previously described our motivation for using the full wave function solution, and described a means by which the desired transfer from one output to the other can be accomplished by the use of an applied magnetic field [2]. In this paper, we illustrate the use of an applied electrostatic potential to tune the length of the interaction region and thereby increase the transmission gain. It is hoped that the use of an applied bias can thus mitigate imprecision in the coupling length, making the device easier to realize physically. We also investigate the possibility of using an applied electrostatic bias to switch electron current from one output to the other.

2 TRANSMISSION CALCULATIONS

We perform our calculations by solving Schrödinger's equation on a discretized lattice (with a lattice constant of 3.75 nm) using a variation of the Usuki method of mode-matching via the scattering matrix [8],[9]. The device is partitioned into 'slices' and the one-dimensional Schrödinger equation is solved for each slice. The kinetic energy terms of the Hamiltonian are approximated using a finite-difference approach and are thus mapped onto a two-dimensional tight-binding model, coupling the slices together and giving the (discretized) Schrödinger equation

$$\left(E_F - H_j\right)\psi_j + H_{j,j-1}\psi_{j-1} + H_{j,j+1}\psi_{j+1} = 0\,. \qquad (1)$$

This tight-binding model is then used to calculate a transmission for each slice, which enables one to determine the total transmission across the device.

To find the current, we solve the Landauer-Büttiker formula. Since we assume that our device is at low temperature, this equation is

$$I = \frac{2e}{h}\int_{\mu_d}^{\mu_s} dE\, T\left(E\right) \qquad (2)$$

where μ_s is the quasi-Fermi level at the source (the input side of the device) and μ_d is the quasi-Fermi level at the drain (the output side of the device). We apply the bias across the device such that it raises the source quasi-Fermi level and lowers the drain quasi-Fermi level. Thus, to perform the numerical integration over transmission, we find the transmission for a discrete range of energies, each of which has a position-dependent quasi-Fermi level.

3 RESULTS

In order to investigate the effects of an applied source-drain bias upon our device, we began by finding a coupling length that would result in a large transmission from input one to output two for a given Fermi energy. Because we are only concerned with single-mode quantum wires, we picked a Fermi energy that was in the middle of the range for which only a single mode would be excited, namely 5 meV. We performed simulations for a range of different window lengths and looked for a coupling length that resulted in nearly all of the electron probability transiting through output two (see Figure 2). The length we chose was 375 nm.

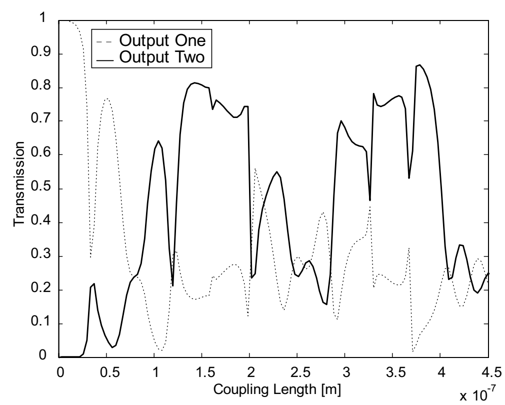

Figure 2: The transmission through output two is at a maximum at 375 nm and varies strongly with coupling length.

The electron probability density for this coupling length is shown in Figure 3. It is clear that electrons are most likely to exit the device through output two. In fact, the transmission through output two is 86.24% of the input transmission, while the transmission through output one is 3.817%. The remainder of the input density is reflected (3.448% through input one and 6.056% through input two).

Armed with this knowledge, we can begin to investigate the effects of an applied source-drain bias V_{sd} upon this device. We apply the bias in such a way that it raises the quasi-Fermi level in the input wires while lowering the quasi-Fermi level in the output wires. For the coupling length and Fermi energy we have chosen, a slightly negative source-drain bias increases the transmission through output two (see Figure 4). The maximum transmission is 86.84%, only 0.6% higher than the original transmission, which is not much of an improvement. This result indicates that although our coupling length could have been a bit more precise, applying a bias will not dramatically increase the gain in the system. It may, however, be used to 'patch' a small imprecision in the coupling length. Interestingly, Figure 4 also shows that the dominant output transmission appears likely to switch when a larger positive bias is applied to the device, indicating that it may be feasible to use an applied bias to switch the device.

Figure 5 shows the effect of an applied bias on the current in the device. As expected, increasing the bias increases the current. There are no conductance plateaus because the device never has more than a single excited mode.

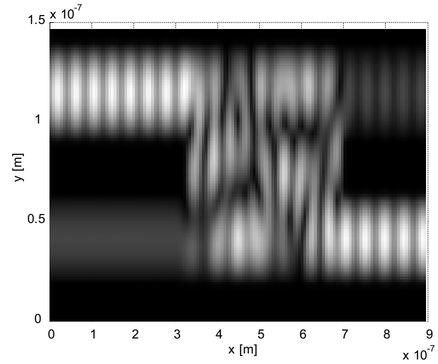

Figure 3: Electron probability density for the coupling length we have chosen. The majority of the probability density exits the device through output two.

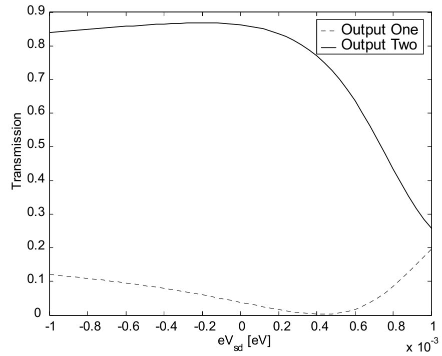

Figure 4: The application of a small bias can be used to correct imprecision in the coupling length.

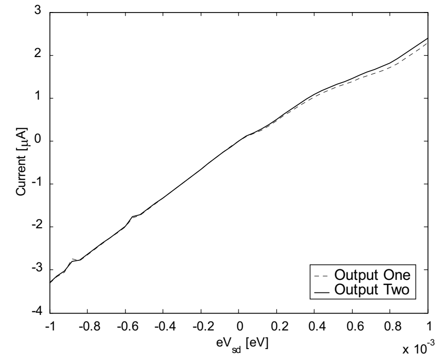

Figure 5: The current varies linearly with applied bias. There are no plateaus because the device is single-mode.

In order to investigate whether it is possible to switch the electron probability from one output to the other by the application of an applied bias, we simulated the device's response to larger biases. Figure 6 shows the device's response. Although output one dominates for a portion of the bias range, the transmission never becomes large enough to seriously consider using the bias as a means of switching the device. We cannot increase the bias more than is shown in Figure 6 because we don't want our system to have more than a single excited mode. As in Figure 5, the current computed in this simulation varies linearly with the applied bias.

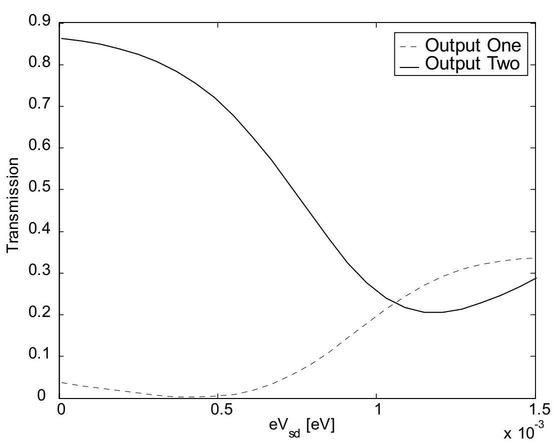

Figure 6: The application of a large bias cannot be used to switch the device.

We were also interested in seeing how the coupled quantum wire system responds to applied biases that were large enough to cause transitions between 1-D sub-bands. Figure 7 shows a schematic illustration of this phenomenon. As the bias becomes larger, more of the energies that are integrated to solve the Landauer-Büttiker formula have output quasi-Fermi levels that are lower than the energy of the lowest sub-band, resulting in zero transmission for that energy and thus lowering the overall current. This result differs from that of a quantum-point contact, for example, in which there is a two-dimensional electron gas outside of the QPC [10], and hence, there are an infinite number of 1-D sub-bands. Accordingly, Figure 8 shows that as the bias continues to increase the current begins to drop.

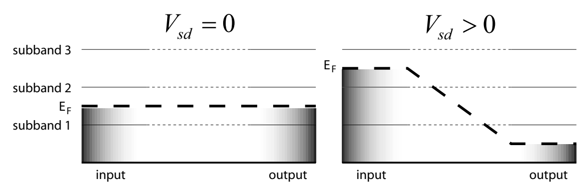

Figure 7: As the applied bias is increased, the Fermi level in the output drops below the energy of the lowest mode, cutting off current.

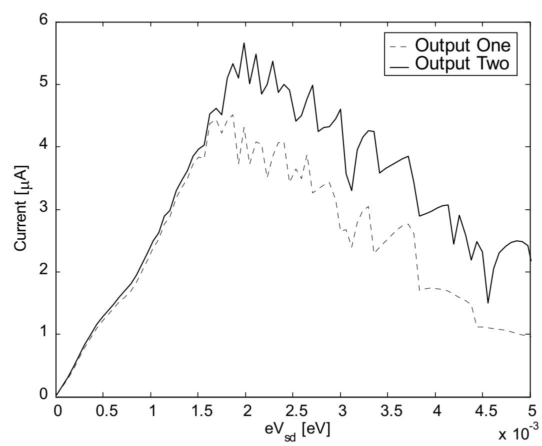

Figure 8: With increasing bias, the current decreases because the output quantum wire has no available sub-bands.

Finally, we investigated whether an applied bias can be used to correct imprecision in the coupling length. To test this, we used a device in which the coupling length was set about 5% away from the optimal coupling length we calculated earlier, that is, a new coupling length of 356.25 nm. For this device, the unbiased transmission through output two drops to 77.63%, an appreciable change. Figure 9 shows the current through this device as the applied bias is increased. In order to keep the quantum wires single-mode, we cannot use an applied bias that results in an energy higher than about 1.5 meV, corresponding to the potential at which the current begins to drop due to the lack of output sub-bands discussed above. The transmission (not shown) increases linearly with the simulated bias, so the highest transmission, corresponding to a potential drop of 1.5 meV, is about 82%. Thus, even though we did not regain our maximum transmission, the application of an applied bias brought us about half of the way. With a smaller imprecision in coupling length, the application of an applied bias would bring the transmission closer to its maximum value. For example, when the coupling length is 2% away from it's optimized value, an applied bias of -1.08 meV brings the transmission through output two up to 85.02% from an unbiased value of.53.07%, illustrating both the sensitivity of the device to the coupling length and the dramatic changes possible by applying a bias.

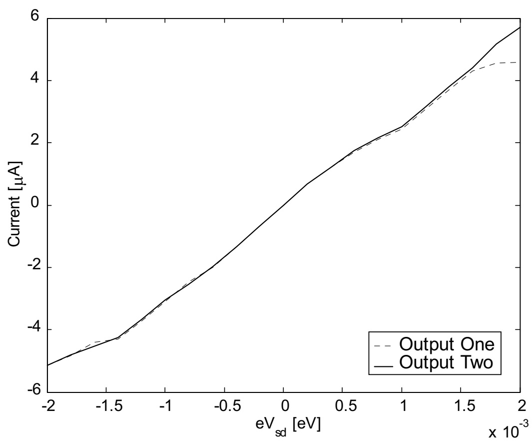

Figure 9: This plot shows the highest bias that we can use to correct an imprecise coupling length.

4 SUMMARY

We have described the motivation behind using coupled quantum wires as a basis for a quantum-computational qubit. The qubit would operate by exciting one of the inputs to the coupled quantum wire system and using a second qubit (possibly with a different geometry) to create a potential field that would control the output chosen by the original excitation. In order to achieve this goal, it will be necessary to maximize the reliability of the coupled quantum wire device. To this end, we have shown that the application of an applied electrostatic source-drain bias can be used to compensate for imprecision in the length of the coupling region that determines which output is chosen by an unbiased excitation.

We have also found that it is difficult to switch the output electron probability distribution from one output to the other by the application of an electrostatic bias. Although a large enough bias may switch the device that we simulated, the bias will excite additional 1-D sub-bands first. We have not ruled out the possibility of using an applied source-drain bias to switch the device, but such an application would need to take care to assure that it is not exciting additional 1-D sub-bands.

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