Principles of Electron Wave Computing Using Quantum Resistor Networks

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Abstract

Quantum resistor networks are interconnected thin wires or electron conducting wave guides. A multiple input-output quantum network is capable of performing massive parallel computing,similar to optical computing. Basic logic functions and arithmetic computation have been demonstrated to be possible on very simple structures. The basic principles of utilizing such quantum resistor networks are very important and are presented here.

Keywords: Quantum computing, Aharonov-Bohm effect, Quantum effect devices, Quantum networks.

Introduction

In recent years ,the capability of fabricating ever smaller man-made thin wire or conducting path continues to advance. A thin wire with a thickness of near atomic dimension could be within our reach in the near future. Improved itching and lithography techniques as well as the use of atomic force microscopy can help us to come to this realization. Inside such a thin wire, electron propagates like a wave at very low temperature, very similar to microwave in a wave guide, since all inelastic scatterings are not significant. In that situation, an entire new scheme of computing will emerge by using those interconnected thin wires. This kind of quantum computing is totally different from those spin-based quantum computing, or spintronics, by others. The use of quantum resistor networks offers a much smaller space to perform computing than the current microelectronics based processor. In addition, a large multiple input-output network structure allows massive parallel and fault tolerant computing., very similar to optical computing, another branch of wave computing. In this paper, four basic principles of utilizing such quantum resistor network (QRN)[1-3] are presented. Those principles are summarized from the mesoscopic physics principles that have been developed in the past two decades or so by others.

Principle 1: Landauer-Buttiker Theory for QRN.

Landauer-Buttiker formulation [4] of quantum conductance provides the evaluation method of electron transport within a QRN. A QRN can be considered as a net that consists of many nodes and bonds that connect two adjacent nodes. Inside a bond, the propagation of electron wave is quasi one-dimensional at Fermi energy at zero temperature. It is then clear that the transport of electron wave from one location to the other in a QRN is described by Landauer-Buttiker theory. It is in that sense that an interconnected wire network is a resistor network even though a QRN is neither a passive nor an active network . A QRN is simply a massive network that route and re-route many channels of electron wave from input locations to output locations with desirable results. This is analog to a convergence lens that can route a huge amount of light channels into a focused spot and hence can perform a Fourier transform. Thus any logic function or computation is directly determined by the transmission probability between input and output of that network, which is a subsystem of a quantum parallel computing machine.

A QRN is mounted between two reservoirs for computing purpose. One of them is maintained at a higher voltage or chemical potential than the other. Coherent electrons are injected from the high voltage reservoir to the net. Interference of waves occurs at every node point before the arriving of electron at the low voltage reservoir with intended results. Thus the computing principle is based on the interference principle rather than on the switching principle of transistor used by traditional microelectronics. In a transistor, current is a scalar quantity. Low current (or voltage) and high current (or voltage) of a circuit determine “zero” and “one” in a digital system. On the contrary, an electron wave in a QRN is a two-dimensional vector with well-defined amplitude and phase. The results at outputs are then the vector sum due to individual input. Therefore “zeros” and “ones” in a QRN can be obtained by aligning those two-dimensional vectors properly.

Transport of electron wave in a QRN can be conveniently evaluated using node-equation approach developed by the author earlier [1]. At each node point, a Kirchhoff current conservation law must be satisfied. This results in connecting wave function of a node point with all the wave functions of its neighboring or connected nodes. Thus for each node point, there is an associated linear node equation, very much like the Kirchhoff current law in elementary circuit theory. A summary of those rules is given in the author's recent paper [2].

Principle 2: Anderson Localization Always Prevails.

Anderson localization always prevails in any QRN. Electron wave for computing differs greatly from microwave or optical wave in terms of how elastic and inelastic scatterings affect its propagation in a network. Microwave can stay coherent over a large distance without suffering significant scatterings. Optical wave is very susceptible to Rayleigh and Brillouin scatterings. However, electron wave is quite different in nature . When an electron wave is used in the elastic scattering domain, it is a very highly reflective wave. Any node point in a network is an elastic scattering center, where an electron wave always prefers to turn backward or turn around through the next available exit nodes than to the forward direction towards the low voltage reservoir. This means that even if we have a "periodic network", whether it is constructed as a square lattice network ,a hexagonal lattice network or any other, mounted between two reservoirs, electrons injected from the high voltage reservoir will not arrive at the low voltage end if the distance is large. Most of the electron will either turn back or circulate in loops inside the network. In another word, there is absolutely no Bloch wave for a periodic network at all . At first glance, this fact seems to run against our intuition from elementary solid state physics. However, this fact is nothing but another statement of Anderson localization. In the original Anderson localization theory, electrons are scattered among those elastic scattering centers that are randomly distributed in a disordered materials. The backward scatterings resulted in localization of electrons. In a "gedanken" experiment , if we were able to move the positions of those randomly distributed scattering centers around a little so that they are all now periodically placed, then of course the same Anderson localization would still prevail. It is very important to note that the existence of Bloch wave in a periodic lattice is true only in singly-connected space. A QRN is a multiply-connected space. Therefore localization of electrons in a periodic QRN is to be expected rather than be surprised. A QRN is then indeed a man-made structure to demonstrate Anderson localization effect.

If Anderson localization prevails for any QRN, periodic or aperiodic network, then it is impossible for electron wave to propagate from an input terminal to an output terminal with large transmission probability. Consequently, any QRN can be declared as a useless network under the tyranny of Anderson localization effect. However this is true only if there is no external control mechanism. This external mechanism can be provided by the next principle

Principle 3: Use Aharonov-Bohm effect.

Use Aharonov-Bohm effect as the tuning mechanism for QRN. The original Aharonov-Bohm (A-B) effect can be understood by the electron wave transport through a clean metallic ring with two terminals. One of the terminals is connected to the high voltage reservoir, where electrons are injected into the ring. Electron wave will traverse the ring through both the upper and lower arms of the ring before joining together at the output terminal. This is similar to Mach-Zehnder effect in optics. Because of the interference effect of electron waves from the upper and lower arms, the transmission probability at the output terminal will depend on the phase difference of the two waves. In A-B effect, this phase difference can be tuned by an external electric or magnetic fields.

In the situation when an external magnetic flux is used, the transmission probability is a periodic function of the applied flux. If the total number of atoms in the ring is an even number, this period is single in the unit of elementary flux, hc/e. However, if the total number is an odd number, then there is universal double periodicity [1]. This implies that there exists two basic classes of two terminal A-B rings. Furthermore, in each class, there is a scaling relation so that the transmission behavior of two different A-B rings can be identical from small atomic-size ring to a mesoscopic one.

If a QRN is constructed, there will be many closed loops, which are identical to having rings with more than two connected terminals. This suggests that a generalization of A-B rings to more than just two terminals. Thus a large tunable QRN consists of many multi-terminal A-B rings with an applied magnetic flux in each of the ring. This will allow QRN to be useful for quantum computing.

A generalized A-B rings with three terminals [2] and four terminals have been investigated [5] to demonstrate the powerful capability of electron wave computing.

As mentioned in principle 2 that an electron wave is highly reflective. In order to perform logic function or computation, those reflected waves (or back propagated waves) may not be part of the computation. If this is the case, then those unwanted wave must be removed from further computation. This important dilemma faced in any quantum wave computing , as was first raised by Landauer [7], can be solved as I address the next principle.

Principle 4: Use Quantum Circulator in QRN.

Use quantum circulator to dump unwanted computation is very important. We have investigated electron wave transmission through three-terminal generalized A-B rings. There are four classes of such rings, each has its own scaling relation. In one of the classes, there exists a quantum circulator that is worth special mentioning here. If the three terminals are labeled as A,B and C. A quantum circulator is such that an input from terminal A will be totally routed to terminal B at particular applied flux and hence there is no localization effect. Similarly, input from B will be totally routed to C and C to A at the same flux. Therefore if terminals A and B are connected in between any computational path , then the forward going wave will be transmitted from terminal A to terminal B while the reflected wave from terminal B will be dumped into terminal C and hence will not interfere with the incoming wave.

As we have shown [2,3], three terminal A-B rings, including quantum circulator, can provide a wide range of logic functions in a single ring. Those include the logic functions of IF-THEN, AND,OR,XOR and INVERT. Higher order functions, such as half adder or full adder can be constructed from those simple logic gates.

In a large NxN input-output network, a quantum circulator will be needed at each appropriate location to ensure that proper parallel processing computation can be achieved.

The trimming of unwanted computations by the use of quantum circulators also implies that refreshing of inputs at certain stage of computation is required. This is the same requirement needed in the logic function of INVERT. A supply line concept can be provided. For example, in XOR-gate, one of the input can be considered as the supply line, while the other input now becomes the inpt that is to be inverted. This makes QRN neither a passive nor an active network. A QRN is simply a large parallel processing network of routing large inputs with desirable results.

A four terminal generalized A-B ring has also been investigated recently [5]. There are three basic classes of such rings, each with its own scaling relation. There is one distinctive advantage of using four terminal A-B rings over the three terminal ones. With two-inputs-two-outputs arrangement, a half adder can be constructed in a single ring. What is even more impressive is that when three-inputs-one-output arrangement is used, a three-bit computation for a full adder is now possible. The “carry” part and “sum” part of a full adder can be constructed from just two four terminal A-B rings. This replaces about two dozen transistors needed in traditional microelectronics circuit. This clearly demonstrated the powerful advantage of QRN for computing.

Since generalized A-B rings, whether they are three terminals or four terminals, are divide into different classes and each class has its own distinctive transmission characteristics, the question is how to search from various classes of rings in order to find a suitable logic gates. This task can be made easy with the use next principle.

Principle 4: Use Buttiker Symmetry Rule.

Buttiker symmetry rule [6] is need in order to construct all useful logic gates. In logic applications there are usually more than one input present. When there are two coherent inputs, the transmission probability at any output terminal is the absolute value squared from the vector sum of two outputs due to two individual inputs. Classification of A-B rings has already greatly simplified our search of a particular ring for possible applications, because we know there are only limited number of varieties in their transmission characteristics. But Buttiker symmetry rule imposes an additional condition when one looks for logic applications. In the symmetry rule, the transmission probability at terminal m due to an input at terminal n at a particular applied threaded flux value is the same as the transmission probability at terminal n due to an input at terminal m at the negative value of that threaded flux.

Let us try to find an XOR-gate operation from a three terminal ring with terminals labeled as A,B and S. When input at terminal A=1 and input at terminal B=0, the transmission probability at output terminal S must be high. Similarly when A=0. and B=1, the transmission probability at terminal S must be also high at the same threaded flux value. Those two conditions imply that if there is an input placed at terminal S, this input must be transmitted equally to both terminals A and B at the negative flux value, according to Buttiker symmetry rule. Therefore the maximum allowed transmission probability at terminal S due to input from terminal A or B is then equal to 0.5 if XOR-gate is what we are looking for. Then one goes back to search for a class of A-B rings with a transmission probability that reaches the value around 0.5.

Of course, all simple logic gates are based on vector sum of two outputs. AND-gate is obtained by aligning two small vectors in the same direction to obtain an output that is four times the intensity of individual input. XOR-gate is obtained by aligning two output vectors that are equal in value but are opposite in directions. OR-gate is obtained by forming an equal-sided triangle between the three output vectors. The first two are the output vectors due to one input only . The third vector is from the output due to both inputs. It is worth noting that by rotating the phase of one of the two inputs relative to the other, the phase of output vector can be rotated exactly to the same value.

When three coherent inputs are considered, it is possible to perform higher order computation, such as those in a full adder using two simple four terminal rings. The output is then a vector sum of three vectors.

Finally, it is very important to note one of the criticisms by Landauer on quantum computing that "higher transmission occurs at narrow input parameters" [7] . That means if we try to impose too many conditions on a simple network , the result will be that a workable range of operation is so narrow to render the network practically useless. However his remarks on how to handle reflections or unwanted computations is overstated. As we have shown that junk computations inside a network can be dumped with the use of quantum circulators.

In a large QRN consists of many A-B rings, it would be very impractical to apply different flux to each ring. For a large QRN to be useful, it has to be designed in such a way that the same flux value is applied to a large region of the QRN. So the flux values applied are limited to a few varieties only.

Conclusion

In this article we summarized some of the most important mesoscopic physics principles that have been developed by others earlier and described them at a particular view point that is suitable for understanding electron wave computing. Those principles serve as the guidelines that allows one to see the huge potentials for electron wave computing. This type of quantum computing offers a much better potential than those spin-based electronics that have been reported by others. As we have shown in other publications that simple logic functions are readily available in three terminal and four terminal A-B rings. What is surprising is that higher order computing ,such as a full adder or a three-bit computation, is also available through the use of simple four terminal generalized A-B rings.

Electron wave computing is a two-dimensional vector computing. Output vector is a linear combinations of all input vectors. The coefficients of combining those input vectors depend on the structure of a QRN as well as the applied flux value, which is the control to avoid Anderson localization. A general node equation method for computing those coefficients has been developed by the author. Because of the nature of this linear combination ,electron wave computing is a lot more versatile than Fourier transform in optics or optical computing in general. With massive inputs, parallel and fault tolerant computing is now possible. In term of computing speed, the propagation time through those ultra-small rings is a lot shorter than the switching speed of any projected ultra-small transistor for the future.

Experimentally, scientists are still limited to the investigation of two terminal A-B rings. In a recent experiment [8] , electron can be shown to traverse a two terminal clean A-B ring six times before the amplitude is damped. This clearly shows that a slight modification of such ring to a three or four terminal one will allow one to test the logic functions based on electron wave computing principle.

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