On the Best Performance of Interacting Agents

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ABSTRACT

Nanotechnology is dependent on harnessing effects of emergent systems, as it relies on the process of taking collections of molecules or individual atoms as building blocks and forming them into useful objects. Emergence can often be viewed when separate systems combine and form a composite system where they act harmoniously together. Most emergent systems can be modelled in terms of the interaction of agents [1]. The potential of using the mechanism involved in emergent systems is enormous as it makes possible activities and controls that are highly unlikely otherwise. But there is still no general framework within which emergent phenomena can be understood (for example [1]).

In the paper we consider the following question. Is it possible to have an optimality condition specifying when interacting agents show their best performance for a particular problem. Results of extensive computational experiments presented in the paper give strong facts to believe that such a condition exists and can be formulated in terms of a concept of structural complexity [2]. In the experiments the structural complexity of agents is increased to see how their performance changes. A remarkable result always appears for each problem tested, i.e., performance of the agents unimodally peaks at some point as their structural complexity increases. In general, the experiments allow us to formulate an optimality condition: agents show their best performance for a particular problem when their structural complexity equals the structural complexity of the problem.

Keywords: interacting agents, energy landscape, complexity, optimality condition.

1 ON OPTIMALITY CONDITION OF AGENTS BEST PERFORMANCE

The following question is considered. Is there an optimality condition specifying how properties of interacting agents must be connected with properties of a particular problem so that the agents show their best performance for the problem. The key question is what concepts describe these properties of the agents and the problem in the optimality condition.

It is well known that many important classes of practical problems are NP-hard. The energy landscape of a problem from such a class has a very rugged and complicated nature (for example [3]). The optimality condition for a class of problems, if existed, must be a constructive representation of a common property of the energy landscapes. However, there are no clear indications that the energy landscapes have such a common property. This gives serious doubts on the possibility to have the optimality condition.

Surprisingly, we find computationally that such a common property exists for a benchmark class of traveling salesman problems. Namely, the energy landscapes show their common property as each one takes a unimodal form when the configuration space is viewed in terms of structural complexity.

2 ENERGY LANDSCAPES IN TERMS OF STRUCTURAL COMPLEXITY

We associate a configuration, i.e., a $N \times n$ binary matrix, $S \in \mathbf{S}$ consisting of $N > 1$ binary sequences $s_i, i = 1, ..., N$ of length $n > 1$ with a $N \times N$ complexity matrix S_C defined by $S_C = \{C(s_i, s_j)\},\ i, j = 1, ..., N$, where $C(s_i, s_j)$ is the structural complexity of a binary sequence $s_i, i = 1, ..., N$ with respect to a sequence $s_j, j = 1, ..., N$ [2] and $\mathbf{S}$ is the configuration space. It is shown in [2] that $0 \leq C(s, s') \leq C(\eta(n), \bar{\eta}(n)) = \lfloor log_2 n \rfloor$, where s, s' are binary sequences of length n, and $\eta(n), \bar{\eta}(n)$ are the initial segments of length n of the Prouhet-Thue-Morse (PTM) sequences, and $\lfloor x \rfloor$ is the integer part of x. The structural complexity of a configuration $S \in \mathbf{S}$ is defined by $C(S) = \sum_{i=1}^{N} \sum_{j=1}^{N} C(s_i, s_j)$.

Let $S(lower), S(upper) \in \mathbf{S}$ be configurations such that the first one is composed of only $+1$ and the second one is composed of binary sequences that all are the initial segment of length n of the PTM sequence $+1-1-1+1-1+1+1-1...$. These two matrices play in an important role because they give the lower and upper bounds for the structural complexities of the configurations $0 = C(S(lower)) \leq C(S) \leq C(S(upper)) = N^2 \lfloor log_2 n \rfloor$.

We define a partial order $\prec$ in the configuration space that compares configurations in terms of structural complexity. The definition is consistent with the fact that if $S \prec S'$ then the structural complexity of S' is greater then the structural complexity of S, i.e., $C(S) < C(S')$. Configurations $S_0, S_1, ..., S_k \in \mathbf{S}$, $k \geq 1$ are called a complexity trajectory in the configuration space $\mathbf{S}$ if $S_0 \prec S_1 \prec ... \prec S_{k-1} \prec S_k$. The complexity trajectories are distinguished in $\mathbf{S}$ because each next element $S_i, i = 1, ..., k$ in a complexity trajectory $S_0, S_1, ..., S_k \in \mathbf{S}$ is more complex than the previous one S_{i-1}. If agents move along such a complexity trajectory their structural complexity increases. We are interested to know how performance of the agents to solve a particular problem changes as their structural complexity increases. In other words, the idea is to consider energy landscapes when the configuration space is represented in terms of structural complexity.

It is worth mentioning that energy landscapes appear rugged when the configuration space is viewed in terms on Cartesian order. In this representation the closest proximity of a configuration consists of configurations that are different from the configuration by a minimum value in one coordinate. The partial order above gives a different representation of the configuration space. In this representation the closest proximity of a configuration consists of configurations that are different from the configuration by minimum values in terms of structural complexity. Remarkably, computational experiments show that in this representation energy landscapes appear regular as they take the unimodal form.

3 OPTIMALITY CONDITION OF INTERACTING AGENTS

Extensive computational experiments have been made to investigate how performance of agents changes as their structural complexity increases. In the investigation a class of benchmark TSP problems is used. The TSP method used in the experiments is designed to have a control parameter that can increase the structural complexity of agents. This is realised by an algorithm that each agent uses to choose a next strategy. The algorithm, called the PTM algorithm, is about the next strategy: "win - stay, lose - consult PTM generator".

The experiments for each problem tested show a remarkable result. The agents solve the same TSP problem each next time with an increased value of their structural complexity, i.e., they move along a complexity trajectory in the configuration space. There is a value of structural complexity such that the performance of the agents increases till this value and then decreases. This means that energy landscapes of the problems appear in the unimodal form when the configuration space is represented in terms of structural complexity. Moreover, the experiments give facts to suggest that for each problem the value of structural complexity, i.e., where the performance peaks, is a characteristic of the problem itself. The characteristic can be defined as the structural complexity of the problem.

Finally, this allows us to formulate an optimality condition: agents show their best performance for a particular problem when their structural complexity equals the structural complexity of the problem.

4 CONCLUSIONS

The experiments of the paper give considerable computational facts to formulate an optimality condition in terms of a concept of structural complexity. Namely, interacting agents show their best performance for a particular problem when their structural complexity equals the structural complexity of the problem. Importantly, the optimality condition is a guide to control the structural complexity of agents in improving their performance. If their structural complexity is less than the structural complexity of a problem then it must be increased till this value. If their structural complexity is greater than the structural complexity of the problem then it must be decreased till this value for the agents to show their best.

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