

Theoretical modelling of nanofluids for heat transfer

T.G. Myers*, M.M. MacDevette** and V. Cregan*

* Centre de Recerca Matemàtica, Barcelona, Spain, tim.myerscrm@gmail.com

**CERECAM, U. Cape Town, South Africa, michelle.macdevette@uct.ac.za

ABSTRACT

There have been numerous claims concerning the remarkable heat transfer properties of nanofluids. This has driven, and still does, a great deal of research into finding the fluid composition which removes the greatest amount of heat. A popular model for nanofluids was presented by Buongiorno [2] and this model has been used by many authors for numerical and theoretical studies to demonstrate the improvement in heat transfer. We will show that in general this model in fact predicts a decrease in heat transfer with increasing solid volume fraction. This result is in line with the recent benchmark study [3]. Finally, we discuss briefly how nanofluids may lead to improvements in thermal solar collectors.

Keywords: Nanofluid, convective heat transfer, boundary layer, heat transfer coefficient, direct absorption solar collector

1 INTRODUCTION

There exists a large literature concerning the supposed remarkable heat transfer characteristics of nanofluids, see the review of Godson *et al.* [5] for example. This property has seen them proposed as a possible solution to the ever increasing need to cool modern electronic devices. However, despite the large body of evidence suggesting high heat transfer, there also exists a growing body implying that this is not the case. In particular the benchmark study of [3] uses results from a number of laboratories to demonstrate no anomalous increase in heat transfer.

Much of the theoretical and numerical work in this field has been based on the model presented in [2]. This generally leads to the conclusion of large increases in heat transfer. In the following we will show that this model in a standard flow configuration in fact predicts a decrease in heat transfer with increasing particle volume fraction, at least for the two fluid-solid systems analysed. It is then shown why some previous researchers have arrived at a different conclusion. This work is described in greater detail in [9]. Finally we mention an application, direct absorption solar collectors, where nanofluids can have a positive impact.

2 NANOFUID HEAT TRANSFER

The equations governing the flow of a compressible nanofluid may be written as

$$\frac{\partial \rho_{nf}}{\partial t} + \nabla \cdot (\rho_{nf} \mathbf{u}) = 0, \quad (1)$$

$$\rho_{nf} \left[\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right] = -\nabla p - \nabla \cdot \tilde{\tau} + \rho_{nf} \mathbf{g}, \quad (2)$$

$$\frac{\partial (\chi_{nf} T)}{\partial t} + \nabla \cdot (\chi_{nf} \mathbf{u} T) = \nabla \cdot (k_{nf} \nabla T) + \mu_{nf} \Phi, \quad (3)$$

$$\frac{\partial \phi}{\partial t} + \nabla \cdot (\phi \mathbf{u}) = \nabla \cdot \left[D_B \nabla \phi + D_T \frac{\nabla T}{T} \right], \quad (4)$$

where \mathbf{u} is the velocity vector, T the temperature, ϕ the volume fraction of nanoparticles and g gravity. Subscripts bf , nf and np refer to the base fluid, nanofluid and nanoparticle, respectively. The density, volumetric heat capacity and viscous dissipation are defined as

$$\rho_{nf} = \phi \rho_{np} + (1 - \phi) \rho_{bf}, \quad (5)$$

$$\chi_{nf} = (\rho c)_{nf} = \phi \rho_{np} c_{np} + (1 - \phi) \rho_{bf} c_{bf}, \quad (6)$$

$$\Phi = \frac{\partial u_i}{\partial x_j} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right). \quad (7)$$

The stress tensor $\tilde{\tau}$ involves the fluid viscosity: for water and ethylene-glycol based fluids we will use the relations given in [8] determined by fitting experimental data

$$\mu_{nf} = (1 + 7.3\phi + 123\phi^2)\mu_{bf}, \quad (8)$$

$$\mu_{nf} = (1 - 0.19\phi + 306\phi^2)\mu_{bf}. \quad (9)$$

The terms involving D_B and D_T represent Brownian motion and thermophoresis. The derivation of the governing equations is similar to that described in Bird *et al* [1, Ch. 3], with the exception that they take the specific heat capacity constant and neglect Brownian motion and thermophoresis. Savino & Paterna [10] write down a similar system, which includes the effect of gravity. Buongiorno [2] assumes from the beginning the fluid is incompressible.

The two "diffusion coefficients", D_B and D_T , (D_T has the wrong dimensions for a diffusion coefficient) involve the variable temperature and volume fraction respectively,

$$D_B = \frac{k_B T}{3\pi \mu_{bf} d_p}, \quad D_T = \frac{\beta \mu_{bf} \phi}{\rho_{bf}}, \quad (10)$$

where k_B is the Boltzmann constant and d_p the particle diameter. In many previous works the values of D_B and D_T are calculated using a reference temperature and volume fraction, whilst allowing T and ϕ to vary everywhere else in the equations, see for example [2, 10]. In the following we will retain the correct temperature and volume fraction dependence within the equations and so define $C_B = \frac{D_B}{T}$, $C_T = \frac{D_T}{\phi}$.

The above system will be solved subject to the following boundary conditions, which are relevant for flow over a flat surface located at $y = 0$. At $y = 0$

$$k_{nf} T_y = -Q \quad u = v = 0, \quad (11)$$

while the inlet values, at $x = 0$, are defined as

$$\phi = \phi_{in} \quad T = T_\infty \quad \mathbf{u} = (U, 0). \quad (12)$$

The system may be simplified with a few sensible assumptions:

1. Viscous dissipation is negligible, $\Phi \approx 0$.
2. Gravity is negligible.
3. The system is in a steady state.

We will now apply a standard boundary layer analysis, allowing us to focus on the heat transfer between the surface and the fluid. In this case we also impose the far-field conditions (for large y):

$$u = U \quad v = 0 \quad T = T_\infty. \quad (13)$$

2.1 Boundary layer analysis

Boundary layer theory is a standard way of analysing flow near a surface. The first step in a boundary layer analysis is a re-scaling that focusses on the area near the boundary. The standard scaling can be found in many fluid mechanics textbooks:

$$\hat{x} = \frac{x}{L} \quad \hat{y} = \frac{y}{L} \sqrt{Re} \quad \hat{T} = \frac{T - T_\infty}{A} \quad (14)$$

$$\hat{u} = \frac{u}{U} \quad \hat{v} = \frac{v}{U} \sqrt{Re} \quad \hat{p} = \frac{p - p_\infty}{\rho_{bf} U^2}, \quad (15)$$

where $U, Re = \rho_{bf} UL / \mu_{bf}, A$ represent the velocity in the far field, the Reynolds number and the temperature scale. We take U to be fixed, independent of particle loading, to permit comparison of subsequent results. This means that as the particle loading increases, and hence the viscosity, the pressure drop driving the flow must also increase to maintain the constant U . Physical parameters are scaled with the base fluid value

$$\hat{\mu}_{nf} = \frac{\mu_{nf}}{\mu_{bf}} \quad \hat{\rho}_{nf} = \frac{\rho_{nf}}{\rho_{bf}} \quad \hat{k}_{nf} = \frac{k_{nf}}{k_{bf}} \quad (16)$$

$$\hat{\phi} = \frac{\phi}{\phi_{in}} \quad \hat{\chi}_{nf} = \frac{\chi_{nf}}{\chi_{bf}}. \quad (17)$$

Applying this scaling to the ϕ equation leads to

$$\nabla \cdot (\phi \mathbf{u}) = \gamma \frac{\partial}{\partial y} \left[\left(T + \frac{T_\infty}{A} \right) \frac{\partial \phi}{\partial y} + \lambda \frac{\phi}{T + T_\infty / A} \frac{\partial T}{\partial y} \right] \quad (18)$$

where $\gamma = C_B A \rho_{bf} / \mu_{bf}$, $\lambda = C_T / (C_B A)$. The temperature scale is chosen based on the input of heat to the system, so we non-dimensionalize the boundary condition in equation (11), $(k_{bf} A \sqrt{Re} / L) T_y = -Q$, and choose $A = QL / (k_{bf} \sqrt{Re})$. Taking typical values for water or ethylene-glycol based nanofluids with Al_2O_3 particles [2, 9] determines that for EG $\gamma = \mathcal{O}(10^{-5})$ and for water $\gamma = \mathcal{O}(10^{-4}) \ll 1$. This signifies that the right hand side of (18) is negligible hence (after dropping the hat notation) and

$$\nabla \cdot (\phi \mathbf{u}) = \phi \nabla \cdot (\mathbf{u}) + \mathbf{u} \cdot \nabla \phi \approx 0. \quad (19)$$

This is accurate to within $10^{-3}, 10^{-2}\%$ for EG and water respectively. Combining this with the steady-state continuity equation leads to

$$\mathbf{u} \cdot \nabla \phi = 0. \quad (20)$$

Physically this equation signifies that ϕ is constant along streamlines. Assuming the fluid is well-mixed at the inlet, $\phi = 1$, then $\phi = 1$ will hold everywhere. The boundary layer scaling demonstrates clearly that *the diffusion effects due to Brownian motion and thermophoresis are so small the particles simply move with the fluid*. This finding is in agreement with the results of Evans *et al.* [4] which was obtained using molecular dynamics.

The conclusion that ϕ is constant is very important. Physically it means that quantities such as density, viscosity and conductivity also remain constant. In terms of the mathematics it means we may apply standard boundary layer techniques to analyse the flow. However, before doing this we will first discuss the heat transfer coefficient which, after all, is the true focus of this work.

2.2 Heat transfer coefficient

The heat transfer coefficient (HTC) represents the ratio of heat input at a boundary to that transferred to the fluid. Whilst it is a well-known and widely used concept it is remarkably loosely defined. If the boundary is located at $y = 0$ and an energy Q is input at the boundary then the HTC, h , is typically defined through the relation $h = Q / \Delta T$, via Fourier's law this may be expressed as

$$-k_{nf} \frac{\partial T}{\partial y} \Big|_{y=0} = h \Delta T. \quad (21)$$

However, this definition is somewhat ambiguous. The problem lies in the temperature jump ΔT . Standard choices are $\Delta T = T_w - T_\infty$, where T_w is the temperature of the solid (at $y = 0^-$) and T_∞ that of the fluid

in the far field or $\Delta T = T_{y=0^+} - T_\infty$. Sometimes the two are combined by choosing $T_{y=0^+} = T_w$. However, none of these choices will correctly represent the energy increase in the fluid for the reason that, for simplicity, they assume the temperature varies linearly from the wall to the far-field. The true energy variation depends on the fluid flow and temperature profile.

Now consider a fluid entering a system at $x = 0$, with an initial temperature T_∞ . A distance L downstream of the inlet the energy flux above the initial value is given by

$$\int_0^{\delta_T(L)} \rho c u (T - T_\infty) dy , \quad (22)$$

where $\delta_T(L)$ is the thickness of the thermal boundary layer at $x = L$. The average temperature rise in the fluid T_{av} is defined via

$$(T_{av} - T_\infty) \int_0^{\delta_T} \rho c u dy = \int_0^{\delta_T} \rho c u (T - T_\infty) dy . \quad (23)$$

This form of average is denoted the cup average in [1]. An HTC that actually represents the energy transfer from the substrate to the fluid is

$$h = \frac{Q}{T_{av} - T_\infty} = \frac{Q \int_0^{\delta_T} \rho c u dy}{\int_0^{\delta_T} \rho c u (T - T_\infty) dy} . \quad (24)$$

The boundary layer equations have been well-studied, both analytically and numerically. In [9] the boundary layer equations are solved using a semi-analytical method. In Figure 1 the HTC, as defined by equation (24), obtained by this method is presented for water and ethylene-glycol nanofluids for 3 different concentrations. The circles represent the variation of the base fluid, this is the highest curve in each set. Slightly below is a dotted line representing a 1% volume fraction, then the dashed line is 5% and the solid line 10%. In both cases the base fluid shows a higher value of HTC at any given x . As we proceed downstream the curves converge, but will not cross. Consequently we may conclude the nanofluid does not improve heat transfer. This result should in fact be more clear since we are plotting for a constant far-field velocity. As particle volume fraction increases so does the viscosity, this means it is costing more energy to pump the fluid, whilst still decreasing the HTC.

2.3 So why the positive results?

First, we should make it clear that this is not an exhaustive study of fluids or particles. It is possible that different combinations may provide a different trend, although it would require a rather large shift in parameter values. The parameters used in this study were easily accessible and for well-studied nanofluids. Further,

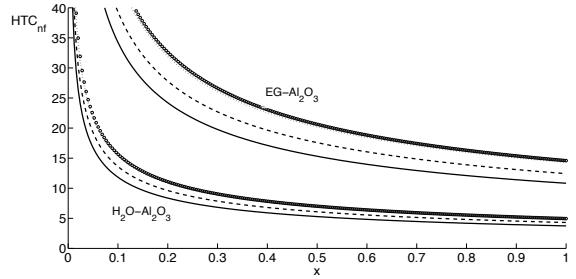


Figure 1: Variation of HTC for ethylene glycol and water with alumina nanoparticles.

the conclusions come from a specific model and it may be that some effect not considered here is important in nanofluid heat transfer.

It is impossible to go through every theoretical paper citing an improvement in HTC with increasing volume fraction. In [9] four highly cited papers are discussed, we will now summarise the explanation of that paper. All of these papers use the definition $h = Q/(T_w - T_\infty)$, so from the start they cannot accurately represent the correct heat transfer. Buongiorno's model [2] is a special case of the system studied here. Rather than carry out a boundary layer analysis he looks at a laminar sublayer and matches it to the outer turbulent region. His sublayer equations may be obtained from ours by rescaling height. For typical flow conditions it then appears that for water the sublayer equations hold in a region much smaller than 100nm from the wall, whilst the actual laminar sublayer should be of the order 0.1mm. Hence his reduced system for a laminar sublayer holds in a small part of a much larger laminar layer and should not be matched to an outer turbulent layer.

Kuznetsov & Nield [7] and Khan & Pop [6] look for similarity solutions (a standard technique for boundary layer flow). Their equations involve a Lewis number, the ratio of thermal to mass diffusivity. For a water-alumina system one would expect a Lewis number of the order 10^5 , if this value were used by these authors they would find ϕ to be approximately constant (in agreement with our result). Instead, in both papers the authors use the value $Le = 10$ and so find nanofluids have a significant effect on the heat transfer.

3 DIRECT ABSORPTION SOLAR CELLS

From the above analysis it may appear that nanofluids do not provide the hoped for miracles. However, there are other applications where they can have a more significant impact, one example is in Direct Absorption Solar Cells (DASC).

Solar thermal collectors are used to capture solar energy. Typically, a collector consists of a black sur-

face used to absorb solar radiation. The solar energy is then transferred, as thermal energy, to a fluid running in tubes embedded or fused onto the surface of the collector. The two main factors affecting the efficiency of the collector are its absorption characteristics and the mechanism by which the heat is transferred to the fluid. DASCs avoid heat transfer to the fluid by using the fluid itself to directly absorb the energy. However, due to their low absorptive properties, standard fluids are inefficient at absorbing solar radiation. Experimental research has demonstrated that dispersing trace amounts of nanoparticles into a base fluid can significantly enhance the optical and thermophysical properties of the fluid. At present NDASCs are not yet at an economically viable level of development [11]. To achieve this goal requires further experimental and theoretical research to optimise the effectiveness of the nanofluid in converting solar to thermal energy.

The mathematical description of the basic problem involves a heat equation with a source term to represent the energy absorption, coupled to a flow equation. Whilst the absorption term is complicated our preliminary results, based on the model of Veeraragavan *et al.* [12], seem to confirm that nanofluids do indeed improve the heat transfer in DASCs. In Figure 2 we show a typical result for a Therminol VP-1 graphite nanofluid, based on the data given in [12]. This result follows from a one-dimensional model, that is the flow is neglected. The solid lines represent the nanofluid, the dashed lines the base fluid. The scaling is such that the initial temperature is 1. At the top surface the temperature of the nanofluid quickly rises, at $t = 0.25$ it is close to 15. At the same time the base fluid shows a much smaller rise, with very little variation across the fluid (due to the low absorption coefficient). However, since the nanofluid reduces light penetration, close to the bottom of the layer the temperature is below that of the base fluid. By $t = 0.5$, the nanofluid has a significantly higher temperature than the base fluid everywhere. Since the absorbed heat energy is proportional to the temperature, Figure 2 makes clear the ability of a nanofluid to improve the performance of DASCs.

REFERENCES

- [1] R. Bird, W. Stewart, E. Lightfoot Transport phenomena. John Wiley and Sons, 2007.
- [2] J. Buongiorno, "Convective Transport in Nanofluids", *J. Heat Trans.* 128:240-250, 2006.
- [3] J. Buongiorno, D. Venerus, N. Prabhat, T. McKrell, J. Townsend *et al.*, "A benchmark study on the thermal conductivity of nanofluids", *J. Appl. Phys.* 106, 094312, 2009.
- [4] W. Evans, J. Fish, P. Kebinski, "Role of Brownian motion hydrodynamics on nanofluid thermal conductivity", *Appl. Phys. Lett.* 88, 093116, 2006.

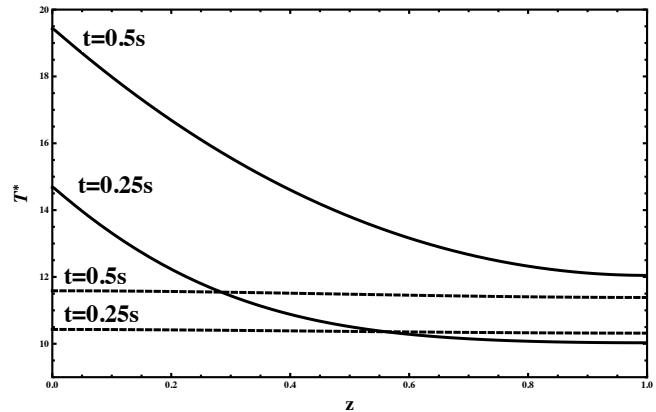


Figure 2: Temperature variation with fluid depth, z , of Therminol VP-1, with 5% graphite nanoparticles (solid line) and without nanoparticles (dashed line).

- [5] L. Godson, B. Rajab, D. Mohan Lala, S. Wongwises, "Enhancement of heat transfer using nanofluids An overview", *Renewable and Sustainable Energy Rev.* 14(2), 629-641, 2010.
- [6] W.A. Khan, I. Pop, "Boundary-layer flow of a nanofluid past a stretching sheet", *Int. J. Heat Mass Trans.* 53: 24772483, 2010.
- [7] A.V. Kuznetsov, D.A. Nield, "Natural convective boundary-layer flow of a nanofluid past a vertical plate", *Int. J. Therm. Sci.* 49:243247, 2010.
- [8] S.E.B. Maiga, C.T. Nguyen, N. Galanis, G. Roy, "Heat transfer behaviors of nanofluids in a uniformly heated tube" *Superlattices and Microstructures* 35:543-557, 2004.
- [9] M.M. MacDevette, T.G. Myers, B.R. Wetton, "Boundary layer analysis and heat transfer of a nanofluid", *Microfluid Nanofluid.*, 2014 DOI 10.1007/s10404-013-1319-1.
- [10] R. Savino, D. Paterna, "Thermodiffusion in nanofluids under different gravity conditions", *Phys. Fluids* 20, 017101, 2008.
- [11] R.A. Taylor, P.E. Phelan, T.P. Otanicar, C.A. Walker, M. Nguyen, S. Trimble, and R. Prasher, "Applicability of nanofluids in high flux solar collectors", *J. Renewable Sust. Energy*, 3, 023104, 2011.
- [12] A. Veeraragavan, A. Lenert, B. Yilbas, S. Al-Dini, E.N. Wanga, "Analytical model for the design of volumetric solar flow receivers", *Int. J. Heat Mass Trans.* 55, 556564, 2012.